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A Review of the State of the Art of Modulation Techniques and Control Strategies for Matrix Converters

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Abstract – *The reliability and stability of the Matrix Converter has improved during the last years due to the enhanced control algorithms. The traditional direct transfer function control mode has been replaced by more complex – digitally implemented control methodologies. These methodologies allow for real time calculation of the optimal switching interval of each individual switch of the matrix converter. These new switching algorithms allow optimal performances, ensuring sinusoidal outputs at any desired power factor. This paper will first revise the underlying theory of matrix converters, then review the various control limitations and finally review the current control algorithms.*

Keywords: *Matrix converter, modulation, control*

Nomenclature

$\mathbf{v_i}$ – input voltage matrix
$v_a(t)\quad v_b(t)\quad v_c(t)$ - input voltage system
$\mathbf{v_o}$ – output voltage matrix
$v_A(t)\quad v_B(t)\quad v_C(t)$ - output voltage system
S_{kj} – bilateral switch
$\boldsymbol{S}$ –switching matrix
$\mathbf{i_i}$ – input currents matrix
$i_a(t)\quad i_b(t)\quad i_c(t)$ - input current system
$\mathbf{i_o}$ – output currents matrix
$i_A(t)\quad i_B(t)\quad i_C(t)$ - output currents system
f_s, T_s – switching frequency, period

I. Introduction

A 3x3 matrix converter is a direct AC-to-AC forced commutated cyclo-converter (see Figure 1). The topology of a 3x3 matrix converter does not have an intermediate energy storing dc-link capacitor. This specific topology provides for both voltage and/or infinite frequency modification, through a single stage converter, directly connecting the input source to the output load. The specific switching elements of the matrix converter consist of a controlled bi-directional four quadrant switch. A matrix converter can provide an output of varying frequency and amplitude.

Matrix converters are growing in popularity from an interesting power electronic converter to a viable industrialised direct AC-AC converter. The term 'matrix converter' was coined by Venturini and Alensia when they presented their work on direct AC-AC converters in [1] and [2]. The authors developed and formalized the analysis and design of a 3x3 matrix converter in [3]. The paper mathematically defined the output amplitude limitations of the matrix converter. It was shown that, depending on the modulation technique, the output voltage can be either 50% or 86.6% of the maximum input voltage. This control method was later referred to as the direct transfer function approach [4]. Although [4] is a review, the discussion is limited to brief summaries of the various aspects of matrix converters. This paper places an emphasis on present modulation and control techniques.

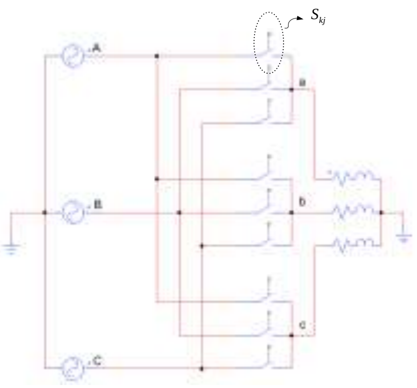

Fig. 1 – Matrix Converter

The theory and modulation of a 3x3 matrix converter was further researched. A novel control method was introduced by Rodriguez [5]. This method has been termed the indirect transfer function. Rodriguez proposed that the switching pattern should resemble that of a Voltage Source Inverter (VSI): the output line/load is switched between the most positive and most negative rail of the input using a pulse width modulation technique. This method implies the use of a virtual DC-link capacitor in the control methodology.

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Research topics progressed further with the focus shifting from a theoretical approach to more practical applications for variable speed drives for electrical motors [6], [7], [8], [9], [10] and [11]. The direct frequency conversion functionality of the matrix converter is practically useful as a variable frequency drive; due to the smaller form factor and lack of bulky energy storage capacitors. This is evidenced and documented in [12], [13] and [14].

Naturally for development of industrial applications, other operational aspects of the matrix converter were covered. Protection, commutation and operation under abnormal conditions are discussed in [15], [16], [17] and [18]. Figure 2 shows how the various subsystems are integrated for a matrix converter.

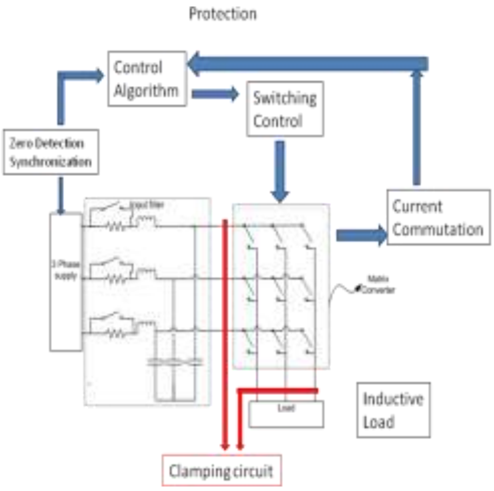

Fig. 2 – Subsystems of a Matrix Converter

II. Commutation

II.1. Fundamental Definitions for Matrix Converter

The fundamentals of matrix converters described in [1], [2] and [4] are summarised below. The switching function of the bi-directional switch may be defined as [19]:

$$S_{Kj} = \begin{cases} 1 & switch\, S_{Kj}\ closed \quad K = \{A, B, C\} \\ 0 & switch\, S_{Kj}\ open \quad j = \{a, b, c\} \end{cases} \tag{1}$$

Subject to the constraint:

$$S_{Aj} + S_{Bj} + S_{Cj} = 1 \tag{2}$$

The input and output voltages, expressed as vectors, are referenced the neutral of the supply:

$$\mathbf{v_o} = \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix}^T \tag{3}$$

and

$$\mathbf{v_i} = \begin{bmatrix} v_A(t) & v_B(t) & v_C(t) \end{bmatrix}^T \tag{4}$$

Where $v_{a,b,c}$ (t) is the respective output voltage and $v_{A,B,C}$ (t) is the respective input voltage. The relationship between the input and output voltage may be expressed as the instantaneous transfer matrix:

$$\mathbf{v_o} = \mathbf{S} \cdot \mathbf{v_i} \tag{5}$$

where

$$\mathbf{S} = \begin{bmatrix} S_{Aa}(t) & S_{Ba}(t) & S_{Ca}(t) \\ S_{Ab}(t) & S_{Bb}(t) & S_{Bc}(t) \\ S_{Ac}(t) & S_{Bc}(t) & S_{Cc}(t) \end{bmatrix} \tag{6}$$

In the same fashion the input and output current may be expressed as:

$$\mathbf{i_o} = \begin{bmatrix} i_A(t) & i_B(t) & i_C(t) \end{bmatrix}^T \tag{7}$$

and

$$\mathbf{i_i} = \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix}^T \tag{8}$$

Where $i_{a,b,c}$ (t) is the respective input currents and $i_{A,B,C}$ (t) is the respective output current.The relationship between the input and output current may be expressed as the transpose of the instantaneous transfer matrix:

$$\mathbf{i_i} = \mathbf{S}^T \cdot \mathbf{i_o} \tag{9}$$

An additional equation for the relationship between the phase voltage values and the line voltage values may be written by inspection:

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = \begin{bmatrix} S_{Aa} - S_{Ab} & S_{Ba} - S_{Bb} & S_{Ca} - S_{Cb} \\ S_{Ab} - S_{Ac} & S_{Bb} - S_{Bc} & S_{Cb} - S_{Cc} \\ S_{Ac} - S_{Aa} & S_{Bc} - S_{Ba} & S_{Cc} - S_{Ca} \end{bmatrix} \bullet \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{10}$$

II.2. Bilateral Switch Configuration

The matrix converter requires a bi-directional switch capable of blocking voltage and current in both directions. Successful attempts at manufacturing a functional bidirectional have been reported in [20] and [21]. The device characterised of a practical monolithic bidirectional switch is described in [22]. A semiconductor manufacturer has made a monolithic bi-directional switch for matrix converters commercially available [23]. The bi-directional switch may also be constructed from discrete components. The configuration may be Common Collector back-to-back, Common Emitter back-to-back or a diode-switch configuration. Both the common collector (Fig-3(a)) and the common emitter (Fig-3(b)) configurations consist of two IGBT's with two anti-parallel diodes. The diode-switch configuration (Fig-3(c)) requires four diodes and a single IGBT. A fourth switch configuration (Fig-3(d)) uses a reverse blocking IGBT. The conduction losses of the reverse blocking IGBT compares favourably with standard IGBT's, however the switching losses are high resulting in a slower switching frequency for the converter [24]. All the above configurations allow for independent control of the direction of the current.

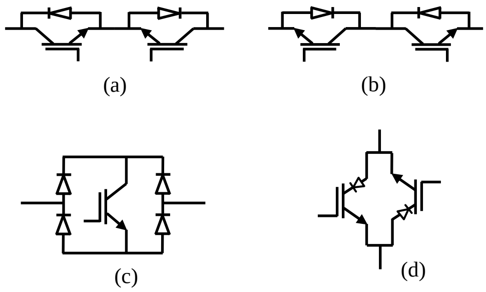

Fig. 3 – Discrete Switch layout

The common emitter configuration allows both IGBT switches to be controlled from one isolated gate drive power supply. However, in practice, the number of auxiliary power supplies required to drive the IGBT's is dependent on the method of commutation and the topology of the bi-lateral switch. This number can be between 9 and 18. The common collector configuration is preferred as only six isolated power supplies are needed to supply the gate drive signals.

Commutation is a process of switching an individual switch off and another switch on. This current transfer is called commutation. Several authors have described this problem and presented possible solutions [4], [25], [26] and [27]. Several current commutation techniques have been proposed and implemented on matrix converters. Commutation in a matrix converter must be actively controlled, since there is no natural freewheeling path, as in Voltage Source Inverters. Two fundamental constraints must be complied with, (as expressed in (2)), in order to maintain operational stability:

- No two bi-directional switches must be switched on at the same time as this would result in a potential short circuit between phases, resulting in excessively high currents, likely damaging the converter.
- The bi-directional switches for each output phase should not all be turned off at any instant as this would result in an excessively high overvoltage since there would be no path for the inductive load current.

These rules are in conflict for practical solid-state electronic switches since no device can switch on or off instantaneously owing to propagation delays and finite switching times. The various commutation techniques may be summarised from [1], [16], [25], [28], [29] and [30]. The two rules are transgressed in the basic current commutation strategy; however additional protective devices (figure 4) are required to prevent damage to the converter. Overlap current commutation requires that the incoming switch is triggered before the out-going switching element is turned off. This should cause a short circuit between phases but extra line inductance slows the rise in current so that reliable commutation is achieved.

This is not a desired method as the additional inductors would be large. Additionally the switching time for each current commutation is increased causing possible control instability.

II.3. Bilateral Switch Configuration

A clamping circuit may be used to protect the matrix converter (see figure 4). If the supply for the input is unbalanced and distorted, circulating currents may circulate in the clamping circuit and possibly affect the output current profile.

The energy stored in the capacitor of the clamping circuit could be used to drive the auxiliary circuits via a switched mode power supply.

An alternative passive protection method using varistors is presented in [24] when the matrix converter is connected to an induction machine.

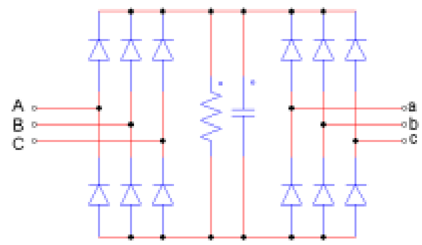

Fig. 4 – Voltage Clamp Circuit

II.4. Commutation Overview

The current direction based commutation is more reliable than current overlap commutation [27]. This strategy complies with the rules and uses a four step commutation process in which the direction of the current flow through the commutating switches can be controlled. The difficulty with this method is that the bi-directional switch must be designed in such a way so that the direction of the current flow can be actively controlled in each switch.

A dynamic switching pattern is determined from the relative magnitudes of the input voltages to control the current commutation. This method is called the relative voltage magnitude based commutation and is relatively simple to implement, [31] as only the relative amplitude of the input voltage needs to be measured to calculate the space-vector.

Resonant switching techniques are well known in general power electronic applications to reduce switching losses. In matrix converters, resonant switching techniques have the additional benefit of solving the current commutation issue. However the additional circuitry increases the component count, increasing conduction losses and requires modification of the control algorithm to operate under all possible conditions.

A matrix converter does not have automatic commutation or free-wheeling paths. This is a fundamental issue; matrix converters do not have any natural free-wheeling paths for load current.

II.5. Commutation Sequences

Commutation in a matrix converter must allow for switching to take place without violating the operational stability as described above. In [26], all 12 possible commutation steps are shown for a transition from one switch to another switch.

The diagram shows a 4-step commutation technique. In sector A V_{AB}>0 and i_{LOAD}>0, sector B V_{AB}<0 and i_{LOAD}>0, Sector C V_{AB}<0 and i_{LOAD}<0 and in Sector D V_{AB}>0 and i_{LOAD}<0. The sequences along the X axis can be followed for a voltage commutation method. In this method only the sign of the voltage needs to be determined. The sequences along the Y axis are followed for a current based commutation method. This method requires knowledge of the load current.

All other sequences require information for both the voltage and current polarity to be implemented. This may result in cumbersome control strategies. Ideally, either current or voltage commutation methods should be used [26].

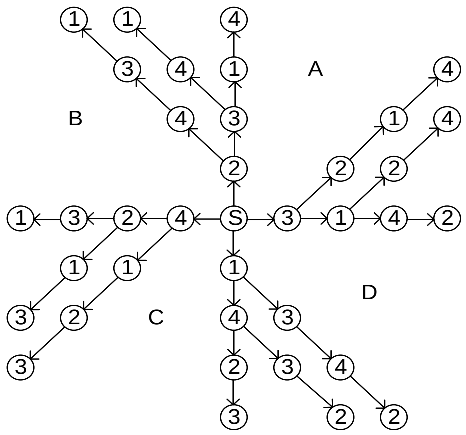

Fig. 5 – Commutation Sequence

The 4-step commutation process requires that only 1 switch may be operated at any instant [4], [26] and [32]. The 4-step method may be explained using the notation where a ‘0’ represents the off state and a ‘1’ represents the on state of the switch.

Assume that Phase A is conducting. The load current must be commutated to Phase B (or Phase C, the choice is arbitrary for the sake of explanation). The switches in Phase A are both on. If V_{AB} >0, then the switches will operate in the following sequence: 4-2-3-1 – as shown in figure 4. And if V_{AB}<0, the sequence of operation will be 3-1-4-2 in each case, when the switch number is called, the switch will change state.

For voltage controlled commutation, the critical area of stability occurs when a phase becomes more positive than another, for example V_A>V_B changes to V_A<V_B (see figure 6). In a similar manner the current commutation follows the sequence along the Y axis of figure 5. When using this method, the area of instability occurs when the load current transits through the zero crossing (see figure 7).

These commutation methods are reliable under balanced and sinusoidal supply conditions.

A new and novel method of handling the commutation problem for matrix converters is presented in [33]. A switchable freewheeling path is added to the matrix converter as shown in the diagram.

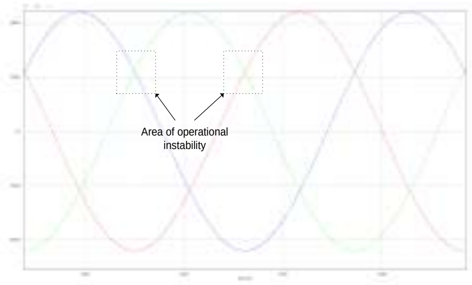

Fig. 6 - Area of voltage instability

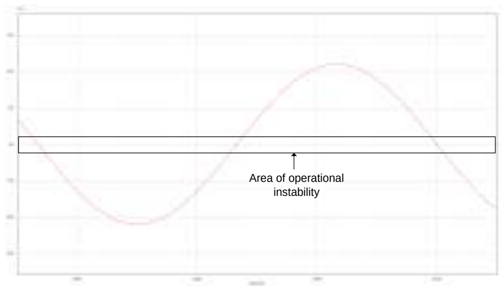

Fig. 7 - Area of current instability

II.6. Freewheeling Path

The paper [33] describes and simulates the principle of a switchable free-wheeling diode for a 3x3 matrix converter (see figure 8). Practical results have been obtained using the switchable free-wheeling diode in a low power, three-to-one matrix converter for use in wind energy extraction [34]. Using this method of a switchable free-wheeling path, commutation of the matrix converter may be implemented without using complex ‘stepping’ methods.

Using the switchable freewheeling path, it is not necessary to determine the direction of the load current. This allows for faster commutation between switches, resulting in a potential improvement of the performance of the matrix converter under unbalanced and distorted conditions.

The use of a voltage clamp may not be required when using the patented switchable freewheeling path.

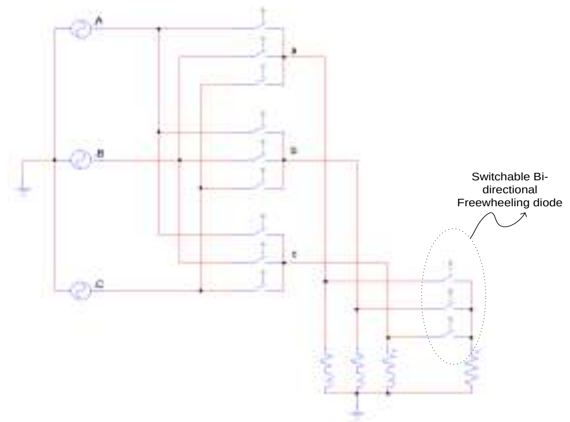

Fig. 8 – Switchable Freewheeling path

III. Modulation and Control Methods

Modulation of power electronic converters refers to a switching pattern which is applied to the converter, resulting in an output signal which is modified from the original input signal.
The duty cycle of the switching pattern must be controlled in order to achieve the desired output. The control strategy is the method used to determine the duration of the on and off times of the controlled switch. In many instances, the matrix converter modulation and control strategy are interlinked [35], [36] and [37].
A matrix converter may be modulated using either a direct or indirect modulation method. In both cases, a Space Vector Algorithm or a Pulse Width Modulation Algorithm may be applied to control the duty cycle of the switches. Other exotic control methods, such as fuzzy controllers, neural networks have been implemented in classic inverters. These control methods are novel for matrix converters.

III.1. Direct Modulation

The direct modulation strategy is when the duty cycle of each switch in the matrix converter is calculated, based on the input voltage and/or current, as well as the error between the output reference signal and the actual output voltage/current [38]. The direct method of modulation is constantly reviewed in the literature [32], [35], [36], [37], [39], [40], [41], [42], [43] and [44].

Direct modulation requires knowledge of the input voltage and the output current. These input parameters are resolved into α-β-0 components to determine the required output voltage vector. This output voltage vector is used to determine the appropriate timing of the modulation sequence for the converter.

A novel method for calculating the on-time for the matrix converter is introduced in [44]. The derived equations may be solved in real-time to determine the optimal switching time for each switch.

The Clarke Components (α and β) can be resolved for the output voltage:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{11}$$

Recalling equations (5), (6) and (8), the authors of [44] have expressed an equation for unity power factor for the matrix converter:

$$\left\{ \left[\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \right] \begin{bmatrix} v_A(t) \\ v_B(t) \\ v_C(t) \end{bmatrix} \right\}^T \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \left\{ \left[\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \right] \begin{bmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \end{bmatrix} \right\} = 0 \tag{12}$$

After some manipulation, the following expression may be written, showing the relationship between the Clarke Components, the peak input phase voltage, the output line currents and the on-time ratios of the switching elements of the converter.

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} v_A & 0 & 1 & 0 & 0 & (v_B - v_C) i_a \\ \sqrt{\frac{2}{3}} v_B & 0 & 1 & 0 & 0 & (v_C - v_A) i_a \\ \sqrt{\frac{2}{3}} v_C & 0 & 1 & 0 & 0 & (v_A - v_B) i_a \\ -\frac{1}{\sqrt{6}} v_A & \frac{1}{\sqrt{2}} v_A & 0 & 1 & 0 & (v_B - v_C) i_b \\ -\frac{1}{\sqrt{6}} v_B & \frac{1}{\sqrt{2}} v_B & 0 & 1 & 0 & (v_C - v_A) i_b \\ -\frac{1}{\sqrt{6}} v_C & \frac{1}{\sqrt{2}} v_C & 0 & 1 & 0 & (v_A - v_B) i_b \\ -\frac{1}{\sqrt{6}} v_A & -\frac{1}{\sqrt{2}} v_A & 0 & 0 & 1 & (v_B - v_C) i_c \\ -\frac{1}{\sqrt{6}} v_B & -\frac{1}{\sqrt{2}} v_B & 0 & 0 & 1 & (v_C - v_A) i_c \\ -\frac{1}{\sqrt{6}} v_C & -\frac{1}{\sqrt{2}} v_C & 0 & 0 & 1 & (v_A - v_B) i_c \end{bmatrix} \begin{bmatrix} S_{Aa} \\ S_{Ba} \\ S_{Ca} \\ S_{Ab} \\ S_{Bb} \\ S_{Cb} \\ S_{Ca} \\ S_{Cb} \\ S_{Cc} \end{bmatrix} \tag{13}$$

Further manipulation of equation (13) results in the output current been removed from the expression. The timing sequences can be determined simply from the input voltages and the output references based on the α-β components.
The equation (14) shows that the output current is not required to determine the switching time of the matrix converter. The observant reader will notice that if the output voltage references are 0.5 times the input voltage, equation (14) may result in negative timing values. This implies that the maximum transfer ratio of output voltage to input voltage is limited to 50%.

$$\begin{bmatrix} S_{Aa} \\ S_{Ba} \\ S_{Ca} \\ S_{Ab} \\ S_{Bb} \\ S_{Cb} \\ S_{Ca} \\ S_{Cb} \\ S_{Cc} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}+\frac{\sqrt{6}\left(2v_A-v_B-v_C\right)v_\alpha}{3\left\{\left(v_A-v_B\right)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}+\frac{\sqrt{6}\left(-v_A+2v_B-v_C\right)v_\alpha}{3\left\{\left(v_A-v_B\right)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}+\frac{\sqrt{6}\left(-v_A-v_B+2v_C\right)v_\alpha}{3\left\{\left(v_A-v_B\right)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(2v_A-v_B-v_C)(v_\alpha-\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(-v_A+2v_B-v_C)(v_\alpha-\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(-v_A-v_B+2v_C)(v_\alpha-\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(2v_A-v_B-v_C)(v_\alpha+\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(-v_A+2v_B-v_C)(v_\alpha+\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \\ \frac{1}{3}-\frac{(-v_A-v_B+2v_C)(v_\alpha+\sqrt{3}v_\beta)}{\sqrt{6}\left\{(v_A-v_B)^2+(v_B-v_C)^2+(v_C-v_A)^2\right\}} \end{bmatrix} \quad (14)$$

The basic Venturini transfer function also has 50% limitation on the output/input ratio. The Venturini transfer function is cumbersome and does not lend itself to real-time calculation of the switching times. However the authors of [44] have derived a set of equations which allows for the real-time solution of the switching times, using only the state of the input voltages and the Clarke Components'. This indicates that the output voltage profile is independent of possible input voltage distortions.

An additional direct control method is presented in [45]. The sliding mode control technique utilizes the space vectors (Zero, Stationary and Rotating) to ensure the output voltages and input currents track the desired reference profiles. This method allows for accurate control of the input power factor. The results obtained by the authors of [45] show a good steady state and a dynamic response as well as optimal control of the input power factor.

In 1991, authors Roy and April [46] proposed a scalar control strategy. The instantaneous output phase voltage of the matrix converter is described by the expression:

$$v_o = \frac{1}{T_s}[t_K v_K + t_L v_L + t_M v_M] \quad (15)$$

Where the subscripts, K, L, M are variable and may be assigned to any of the phase voltages A,B,C according to the following rules:

- At any instant the input phase voltage which has a polarity different from any other is assigned the subscript M.
- K and L are assigned to the remaining two; K being assigned to the smallest absolute value.

The converter switching pattern depends only on the SCALAR comparison of the input phase voltage and the instantaneous output voltage (v_o) of the desired output voltage. The authors further demonstrated that the voltage transfer ratio is as high as 0,87. This corresponds with Venturini's results. The SCALAR control method provides stable control of the output voltage and frequency. The input power factor is also controllable.

III.2. Indirect Modulation

The indirect modulation strategy was first proposed by [5] in 1983. Rodriguez proposed that the modulation of the matrix converter be modelled on a Voltage Source Inverter (see figure 9). The matrix converter is virtually managed as a cascaded rectifier, DC link capacitor and inverter. This method is also referred to the 'fictitious DC link method' [4].

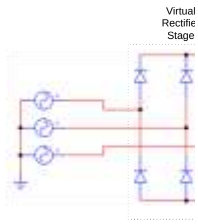

Fig. 9 – Virtual circuit of the indirect method

Each switch state of the virtual rectifier and inverter is determined by conventional control schemes for rectifiers and inverters, so that the matrix converter can have the desired output voltage and have unity input power factor. These switch states are synthizsed to yield a final control signal for the matrix converter. The indirect modulation methods are discussed in [3], [4], [19] and [47]. This method implies that the number of outputs is variable and not restricted to the classical 3 x 3 matrix converter; it could be a 3 x *m* converter. The underlying theory of a poly-phase matrix converter is discussed in [48].

As mentioned earlier, the switches of a 3 x 3 matrix converter can be represented by a nine-element matrix. The state of each switch may be indicated using a binary notation. '1' indicates that the switch is on and a '0' indicates that the switch is off; see equations (2) and (6).

The equation (2) limits the switching of the matrix converter to only 27 possible switch states from a possible 512 switch states [19], [31], [45] and [49].

The Space Vector Modulation of the matrix converter can be defined by 4 space vectors [35].

$$V_{in} = \frac{2}{3}\left(v_a + v_b e^{\frac{j2\pi}{3}} + v_c e^{\frac{j4\pi}{3}}\right) \quad (16)$$

$$V_{out} = \frac{2}{3}\left(v_A + v_B e^{\frac{j2\pi}{3}} + v_C e^{\frac{j4\pi}{3}}\right) \quad (17)$$

$$I_{in} = \frac{2}{3}\left(i_a + i_b e^{\frac{j2\pi}{3}} + i_c e^{\frac{j4\pi}{3}}\right) \quad (18)$$

$$I_{out} = \frac{2}{3}\left(i_A + i_B e^{\frac{j2\pi}{3}} + i_C e^{\frac{j4\pi}{3}} \right) \quad (19)$$

The above equations are phase values. The VSI has only six possible combinations which yield non - zero voltages from the above equations [19] and [35]. The space vector can be approximated by two adjacent switching state vectors as shown in figure 10. The authors of [35] have demonstrated that applying the equations (11)-(14) results in all possible combinations from the possible 27 switching states resulting in stationary vectors.

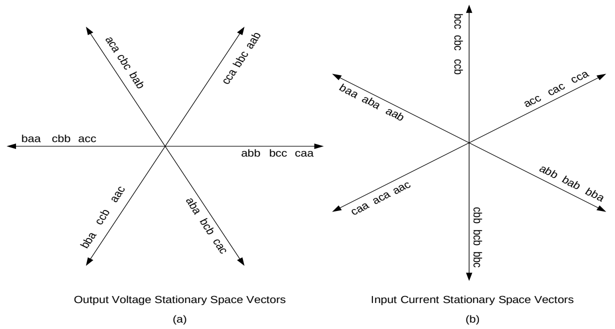

Fig. 10 – Stationary Space Vectors

This modulation method yields a maximum output voltage of 86.6% of the input voltage. The authors of [35] have additionally proposed a modified Space Vector Modulation. However the modulation index of this method results in a maximum output voltage of 57.7% of the input voltage.

III.3. Control Methods

Classical control system design techniques, such as PID controllers and traditional Root-Locus design methods, while effective, are characterised by a trial and error iterative process. Additionally, these classical methods require a transfer function to model the plant. The plant in this case, is the matrix converter.

Modern control procedures assume that the state space model may be determined for the physical system. Various techniques are available to develop a controller based on the mathematically derived plant model. In [50], the authors present a control algorithm based on the '*Least Mean Square*' principle. The permissible 27 switching permutations are pre-determined, resulting in the universal rule based "**IF** premise T**HEN** function". This particular control method, under steady state conditions, needs to sample either the input or the output to determine the next switching state from a look-up-table. The disadvantage of such a technique is that the dynamic response will be very poor under transient conditions [51].

The controller for a matrix converter requires instantaneous values of the input and output parameters in order to determine the optimal switching solution. Two popular methods are based on the instantaneous reactive power theory (*p-q* power theory) or the synchronous reference frame (*d-q* transformation) [52].

The *p-q* theory is based on a set of instantaneous powers defined in the time domain [53]. This implies that there are no restrictions on the voltage or current waveforms. The theory is valid for a balanced three-phase three-wire system as well as a balanced three-phase four-wire system. The interested reader may further pursue the derivations of the equations in [53]. The *p-q* theory transforms voltage and current from abc to αβ0 co-ordinates then to defined instantaneous powers based on the co-ordinates. A control scheme employing this theory must sample both the input voltage and output current of the matrix converter [52].

Czarnecki, [54], has questioned the validity of the *p-q* power theory under unbalanced conditions.

A *PQR* transformation is proposed in [55] and implemented for a matrix converter in [56]. *PQR* theory is a hybrid of *p-q* theory and Cross Vector theory. Cross vector theory defines one instantaneous real power and three instantaneous reactive powers. These three reactive powers are dependent on each other, resulting in only one degree of freedom for the controller. As with the *p-q* power theory, the *PQR* based controller must sample both the input voltage and the output current.

Most modulation strategies assume that the input voltages are balanced and sinusoidal. However, when the input voltage is unbalanced, these modulation methods will cause the converter to draw non-sinusoidal currents. The *PQR* based modulation method will improve the power quality as well as provide for a balanced output current. Practical results in [56] and [57] have validated this.

Another mathematical operation is the *DQ0* transformation that converts a balanced three phase electrical circuit to two orthogonal quantities. The two axes may be referred to the direct axis (*D*) and the quadrature axis (*Q*). Matrix converter controllers that employ the *DQ0* transformation require knowledge of only one input variable. The controller is also not influenced by voltage unbalance. The drawback of this approach is that it is only valid during steady state operations and for slow transient events. A genetic algorithm and experimental results that employs the *DQ0* transform is presented in [58]. The results show that the controller is valid for steady state and step voltage transients.

Neither *p-q* power theory or *d-q* transformation provides a distinct advantage or disadvantage for controller implementation. The minor variances are mentioned in the preceding paragraphs.

IV. Summarizing Comments

Many applications are based on the classical 3x3 matrix converter. Some authors have shown variations on the classical topology multi-output for machine drive applications. A similar advantage may be observed for the topologies proposed in [5], where the control algorithm treats the matrix converter as a cascaded rectifier, fictitious DC link and an inverter. A possible advantage and further research topic could be creating a generalised *n* x *m* converter without complicating the overall control. The inputs *n* may range from 2 phases upwards and the output *m* from 1 output phase to a theoretical infinite limit. The practical limitations will be based on the specific application and the cost effectiveness of the solution. Current matrix converter topologies will become more attractive once the monolithic bi-lateral switch gains acceptance for use in circuit topologies, with particular interest and application in medium and high power machine applications. The manufacturer of the bi-directional switch [23] has claimed voltages as high as 3.3kV and currents of 3.6 kA. The majority of research, proposed by various authors, considers the limitations for allowing a pathway for any inductive load current. This has been achieved by using the switches to provide a freewheeling path during commutation [25], [26] and [27].

A possible solution the commutation problem is presented in [33]. The paper proposes a switchable freewheeling path for any inductive load current. Practical results, using the switchable freewheeling path, were reported in [34]. The drawback of this switchable freewheeling path is the increased number of bi-directional switches. However, the freewheeling path is always guaranteed, without the use of multistep commutation sequences. Additionally, the potential circulating currents in the voltage clamp may be avoided. In fact the design of the voltage clamp may be simplified.

Although in the past, the control of the matrix converter proved to be complex, due to the high number of constraints and timing cycles to be calculated in real-time, control algorithms have evolved from transfer functions to Pulse-Width-Modulated solutions to Space Vector Modulation with the application of modern control techniques. This has resulted in improved stability and response of the control systems [59] and [60], making the family of matrix converters more attractive for industrial applications, especially for medium to high power.

The proposed SCALAR control algorithm was validated for balanced and undistorted input to the matrix converter.

Modern Space Vector Modulation coupled with modern control algorithms, allows for optimal input power factor control [45]. The authors of [44] demonstrated that by using a sinusoidal reference, the output current is not required to determine the switching times for each switch. This algorithm can handle a distorted voltage input but not an unbalanced supply. The two choices for simplifying the three phase inputs and outputs were also reviewed. Practical controllers [58] and [56] for the matrix converter that were implemented using either the *PQR* power theory or the *DQ0* transformation were presented. Neither method presents a distinct advantage over the other. An ongoing research by the authors of this paper is to solve for a control algorithm that is capable of drawing current at a unity power factor and delivering a sinusoidal, balanced output from an unbalanced and distorted supply.

Based on the facts presented in this paper, the reader should understand the limitations, but also the great potential of matrix converters, especially for medium to high power applications where the bulky DC capacitor is not required.

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