MODELLING GROUNDWATER FLOW UNDER RECHARGE UNCERTAINTY: AN APPLICATION TO CENTRAL LIMPOPO

by

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DECLARATION

“I hereby declare that the thesis submitted for the degree D Tech: Civil Engineering, at the Tshwane University of Technology, is my own original work and has not previously been submitted to any other institution of higher education. I further declare that all sources cited or quoted are indicated and acknowledged by means of a comprehensive list of references”.

SS Rwanga

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DEDICATION

To my parents

&

My lovely husband Hilary and my awesome daughter, Anita.
ACKNOWLEDGEMENTS

I would like to express my special gratitude and thanks to my advisor and supervisor Prof Julius Musyoka Ndambuki for his continuous support during my PhD studies. His patience, motivation, encouraging words and immense knowledge made this research possible. Your advice in both research and on my career, were invaluable, God bless you. To Dr. Yali Woyessa for accepting to be my co-supervisor, your constructive criticism added invaluable wealth of knowledge to my work. Thank you and God bless you.

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ABSTRACT

Groundwater has emerged to be one of the major sources of potable water for various purposes in both urban and rural areas. Although it is now well known that groundwater is a more economical source of water supply in water scarce areas of arid and semi-arid regions, it has been given limited attention and has not been perceived as an important water resource in South Africa.

The current study was conducted in Central Limpopo province of South Africa and focused on groundwater flow modelling under recharge uncertainty. The development of stochastic groundwater management model was aimed at treating uncertainty within decision oriented programmes, which cannot be experienced in deterministic models. Thus this research was aimed at estimating spatial and temporal variability of groundwater recharge and developing a quasi-three dimensional stochastic model for groundwater management. The novelty of this research work was that the management of groundwater was sought under recharge uncertainty.

To achieve the overall objective of the study, groundwater recharge was estimated using Geospatial information system (GIS) distributed hydrological model (WetSpass). The uncertainty in recharge was propagated to the flow model through Monte Carlo (MC) sampling technique. The study area was numerically modelled using MODFLOW 2000 while areas with potential for further groundwater development were identified through the use of Groundwater Sustainability Index (GSI). The developed model was calibrated using Parameter Estimation Program (PEST) under MODFLOW environment and later validated using head measurements data of 30 observation wells for the year 2011.

The results demonstrated that, the simulated mean annual recharge was 105 mm/year with a standard deviation of 58 mm/yr which is 22.1% of the average annual precipitation (475 mm). The simulated groundwater recharge was compared with previous studies and showed similarities. The results from distributed groundwater flow model were validated using the calibrated model. The Normalized Root Mean Square
(NMRS) for calibrated model was 1.83 % and 1.85 % for validated model. Further, the simulated groundwater flow mass balance resulted in an abstraction rate of 19.98 Mm$^3$/yr. Moreover, total water budget indicated that there is storage of 4 Mm$^3$/yr. This implies that the aquifer has potential for further exploitation of its groundwater resources.

Stochastic model results were compared to those of deterministic model. The residual mean calculated from deterministic model was 2.76 m while stochastic simulation had a residual mean of 0.82 m. This implies that by assuming recharge as deterministic, one would get results that are optimistic (i.e., results that promises more groundwater availability than one can actually get).

A GSI values close to 0.8 was used as a limit in the mapping of areas with potential for further groundwater development to avoid over-exploitation of the aquifer. All aquifer zones showed potential for higher abstraction rates of 727.6 Mm$^3$/year compared to the current abstraction rate of 17.76 Mm$^3$/year.

In conclusion, the results demonstrated the importance of developing groundwater flow models that acknowledge the existence of recharge uncertainty and hence consider it in the development of groundwater management solutions. It further demonstrates that WetSpass, Remote Sensing, GIS and MODFLOW are useful tools to be considered and linked together to develop sustainable groundwater management solutions. In addition, the study recommends the use GSI as a decision tool in the development and management of groundwater aquifers.
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<td>AOI</td>
<td>Area of interest</td>
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<tr>
<td>CMB</td>
<td>Chloride Mass Balance Method</td>
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<tr>
<td>CSIR</td>
<td>Council for Scientific and Industrial Research</td>
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<tr>
<td>DEM</td>
<td>Digital Elevation Model</td>
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<tr>
<td>DSS</td>
<td>Decision support system</td>
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<td>DWA</td>
<td>Department of Water Affairs</td>
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<td>DWAF</td>
<td>Department of Water Affairs and Forestry</td>
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<td>ERDAS</td>
<td>Earth Resources Data Analysis Systems</td>
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<tr>
<td>ETM</td>
<td>Enhanced Thematic Mapper</td>
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<tr>
<td>FAO</td>
<td>Food and Agriculture Organization</td>
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<tr>
<td>FDM</td>
<td>Difference method</td>
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<td>FEM</td>
<td>Finite Element method</td>
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<td>GIS</td>
<td>Geographic Information Systems</td>
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<td>GRA II</td>
<td>Groundwater Resource Assessment Phase II</td>
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<td>GRIP</td>
<td>Groundwater Resource Information Project (GRIP)</td>
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<td>GSI</td>
<td>Groundwater sustainability indicators</td>
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<td>IDW</td>
<td>Inverse Distance Weighting</td>
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<td>IWM</td>
<td>International Water Management Institute</td>
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<td>IWRM</td>
<td>Integrated Water Resources Management</td>
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<tr>
<td>LULC</td>
<td>Land Use and Land Cover</td>
</tr>
<tr>
<td>MAP</td>
<td>Mean Annual Precipitation</td>
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<td>MC</td>
<td>Monte Carlo (MC)</td>
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<td>RS</td>
<td>Remote sensing</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>SA</td>
<td>South Africa</td>
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<td>UTM</td>
<td>Universal Transverse Mercator</td>
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<td>WMA</td>
<td>Water Management Areas</td>
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<td>WRC</td>
<td>Water Research Commission</td>
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<td>WTF</td>
<td>Water table fluctuation</td>
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CHAPTER 1: INTRODUCTION

1.1 Background of the study

Water has always been a precious commodity for human life. Not only do we humans use it just about every day, but every living thing needs it to live. Although fresh water is a renewable resource, the world’s supply of clean, fresh water is steadily decreasing. This scenario is occasioned by continuous increase in demand for freshwater resources driven by increased population world-wide (Obuobie et al., 2012).

Increase of potable water demand worldwide leads to consistent water scarcity problem in many parts of the world. UNDP (2006) asserts that water use has been growing at a rate approximately twice the rate of population growth in the last century. Numerous countries face the water shortages although it may not be noticeable.

Groundwater sources are useful for various purposes in both urban and rural areas. In developing countries and especially in rural areas where the majority of people live, groundwater is frequently the only possible source of good potable water. Wang et al. (2008) reported that groundwater is often the only water resource, which is available around the year. This is because; groundwater has the advantage that it is frequently widely available in quantities sufficient to supply the needs of scattered communities.

Hay et al. (2012) reported that poor operation and maintenance of water supply, treatment and reticulation infrastructure in South Africa results in significant losses which, if corrected, can reverse the current water shortages being experienced. In addition, Adams et al. (2012) pointed out that over-utilisation and poor management of the groundwater resource are often due to poor or non-existent management plans and governance provisions. Often described as the ‘silent revolution’, groundwater use, mainly by agriculture (and to a lesser extent de-watering of mines, and municipal water supply), has led to declining water levels of several aquifer systems around the world.
Determination of groundwater recharge has shifted from a basic problem to an urgent and fundamental issue in hydrogeologic research for sustainable development of groundwater (Stoll, 2012). It is worth noting that most approaches for quantifying groundwater recharge measure it directly or indirectly over a limited area (point or small-basin scale) and for short periods of time (Izuka et al., 2010). However, Gieske (1992) reported that direct groundwater recharge from precipitation in semi-arid areas is generally small, usually less than about 5% of the average annual precipitation, with a high temporal and spatial variability.

Recharge happens when groundwater level infiltrates into the saturated zone (Yeh et al., 2009). Numerous factors affect the incidence and movement of groundwater recharge in a region including topography, lithology, geological structures, depth of weathering, extent of fractures, primary porosity, secondary porosity, slope, drainage patterns, landform, land use, and climate (see Yeh et al., 2009).

The rates of recharge and evapotranspiration are known to be the most difficult and uncertain components to estimate in groundwater budget. This is because they often vary spatially and temporally, especially in arid and semi-arid regions (Xu et al., 2011). The spatial and temporal variability of recharge is important in sub-humid, semi-arid and arid regions, where reference evapotranspiration exceeds precipitation over the long term (Carrera-Hernández et al., 2012).

Spatial as well as temporal difference of recharge can be determined using onsite hydrogeology experiments and geophysical survey (Yeh et al., 2009). However, these surveys focus on a single affecting factor or an indirect site-specific experiment for groundwater recharge thus reducing the reliability of the evaluation. Challenges in identifying hydrologic processes within semi-arid environments generally include insufficient physical monitoring data across large areas, difficult field conditions in remote regions, and processes such as recharge being spatially and temporally highly variable because of the intermittent nature of recharge events (see Tweed et al., 2011). In such a case, remote sensing can offer complementary datasets and guide further
investigations (Leblanc et al., 2007). Moreover, in order to enhance the accuracy of results as well as minimise bias, integrated studies that employ conventional surveys with satellite image data interpretation techniques, and geographical information systems (GIS) technology, are useful (Rao & Jugran, 2003).

For the past thirty years hydrological models have increased speedily with most of them being physical based models (Binley et al., 1997). Beven (1989) argues that “those models are simply conceptual models as the physical based quantities are actually oversimplified of reality and certainly not measurable with current point scale techniques”. Numerical-modeling efforts were quickly brought to bear upon the problems associated with recharge and the development of groundwater resources (Sanford, 2002).

The assessment of the environmental condition in water catchment requires integration of information and analysis to further decision-making. This is a requirement that can be addressed using the data-management facilities of geographical information system (GIS) (Diodato & Ceccarellib, 2006). Geographical Information System (GIS) and remote sensing are tools which may provide the basic support for many distributed hydrological models. This is because remote sensing can provide various datasets over a large remote area that can be efficiently handled and analyzed in a GIS framework. Moreover, it can provide a wide range scale of the space – time distribution of observations, and also can save time and money (Yeh et al., 2009).

Recharge can be estimated using a number of approaches depending on the availability of data and level of accuracy required. In arid and semiarid areas, recharge is relatively small and potentially more variable (Ketema & Broder, 2009). This calls for methods that can handle spatial variability. In this regard, geographic information systems (GIS) have emerged as effective tools for handling spatial data and decision making in several areas including engineering, geology and environmental fields. GIS techniques have many advantages over older and improved geo-referenced thematic map analysis and interpretations. Unlike conventional methods, GIS methods are able to take into account
the diversity of factors that control groundwater recharge. Moreover, GIS utilities can be used to create a hydrogeological database (Krishnamurthy et al., 2000) to be used for groundwater recharge mapping and simulation.

Many researchers have applied GIS and remote sensing techniques in delineation of groundwater resources and potential zones (Rao & Jugran, 2003). Sener et al., (2005) pointed out that remote sensing can effectively identify the characteristics of the surface of the earth (such as lineaments and geology) and can also be used to examine groundwater recharge.

Geospatial techniques can be used in conjunction with groundwater modelling to define the space-time distribution of groundwater recharge and discharge (Leblanc et al., 2007). GIS techniques and numerical modelling create a unique opportunity to improve groundwater management. The integration of such techniques can provide preliminary spatial distribution of the recharge zone.

Several researchers have demonstrated that GIS with its sophisticated analytical capabilities, can provide a powerful platform for developing GIS based model, calibrating and validating them, and presenting their results (Jha et al., 2007). Emphasis has been placed on the use of conceptual and physically based models for the forecast of groundwater recharge. However, the quantitative spatial descriptions of the phase of the hydrologic cycle remain always very complicated and subject to a great deal of uncertainty.

Increasing use of remote sensing and GIS has significantly shown success in different studies on assessment of groundwater recharge and degree of uncertainty. For example, Tweed et al., (2007) verified that the integration of remote sensing and GIS reduces the uncertainty of hydro-geological data in terms of both macroscopic (climate, change of land utilization) and microscopic (preferential flow) factors. Their data can be used to analyse groundwater numerical models or water balance. Thus, RS and GIS being potential tools, have been used to quantify groundwater recharge in this study.
In this research, rather than formulating a complex model requiring a vast number of parameters, the aim was to develop a distributed groundwater flow model with simplified components that can be used in areas with minimal data hence minimal parameterisation considering recharge as uncertain input parameter.

1.2 Problem statement

Groundwater is a precious resource of limited extent both in quantity and in space. In order to ensure its judicious use, proper evaluation of its availability is required. Groundwater recharge is one of the most important parameters which aids in proper evaluation of the resource. Groundwater quantity predictions made through deterministic approaches are inherently based on the belief that the present is the key to the future. However, this assumption commonly proves invalid when applied to groundwater system because properties can be estimated only with a large degree of uncertainty. The majority of groundwater flow numerical models still provide only deterministic predictions with no supplementary information on uncertainty predictions. Stochastic groundwater management aims at treating uncertainty within decision oriented models in a logical and systematic manner which cannot be experienced in deterministic modelling approach. Thus, this research aimed at estimating groundwater recharge and developing distribution model for groundwater flow. The novelty of this research work was that the determination of groundwater flow models was sought under recharge uncertainty.

1.3 Research Motivation

Increasing population, urbanization and expansion in agriculture over the years has led to unscientific exploitation of groundwater resources. The consequence of this practice is in efficient and unsustainable use of our water resources especially groundwater. To remedy this situation, it is imperative that exploitation of these resources be carried out with the aid of scientific tools which would inform on how best to utilise the resource optimally. Geographical Information System (GIS) and
Remote sensing (RS) provide the basic support for distributed water balance hydrological models and numerical modelling. Unfortunately, despite the availability of GIS and RS and its tools, numerical values of many parameters within distributed groundwater flow models remain uncertain for application to real world problems. It is therefore necessary to quantify the uncertainty or unreliability in model parameter estimates. Thus, this study aimed to develop a groundwater flow model with simplified components which can be used in areas with minimal data hence minimum parameterisation considering recharge as uncertain input parameter.

1.4 Research Questions

1. Does consideration of groundwater recharge based on GIS approach have significant impact on groundwater exploitation potential?
2. Does consideration of uncertainty improve groundwater flow model?
3. How does parameter uncertainty affect decision making process?
4. Does the use of stochastic method in groundwater recharge estimation improve groundwater management?

1.5 Research objectives

The main objective of the study was to quantify the spatial and temporal variability of groundwater recharge in order to develop a quasi-three-dimensional distributed flow management model for groundwater recharge under uncertainty. The specific objectives are;

i. To estimate groundwater recharge using GIS-based distributed hydrologic models (WetSpass);

ii. To develop a distributed groundwater flow model for the study area;

iii. To develop a stochastic solution methodology to solve a groundwater flow problem through simulation considering recharge as uncertain;

iv. To develop a groundwater management model that explicitly considers recharge as uncertain input parameter.
1.6 Thesis outline

The thesis is arranged into five main chapters. Chapter one details the general description and background to the study revealing the need and importance of this research. Chapter two includes literature review associated to groundwater, groundwater recharge, methods for recharge estimations and groundwater flow modelling. The methodology followed is presented in Chapter 3. The chapter also includes information on data processing both primary and secondary which involve screening of field data and manipulation. Later in the chapter methodologies on groundwater flow development concepts, water budget flow, model calibration and validation are also discussed. In Chapter four results and discussion are presented. It commences with discussion of results on estimated recharge, followed by groundwater flow simulation, calibration and validation and later on, discussion on the results of deterministic and stochastic groundwater flow models are presented. Chapter five deals with conclusion and recommendation of the study. Finally, a list of references cited is presented followed by appendices.
CHAPTER 2 : LITERATURE REVIEW

2.1 Introduction on water resources and groundwater

Water resource protection is an issue globally and the situation is under excessive pressure from increasing population, rapid urbanization, and dietary changes as due to excessive abstraction rates. Today, water concerns remain high on many national agendas especially in developing nations since past ‘compartmentalised approaches’ to water management have generally failed to accomplish sustainable outcomes (Foster & Ait-Kadi, 2012). Water resources experts have learned how to plan, design, build and operate structures that, together with non-structural measures, increase the benefits people can attain from the water resources. However, there is a limit to the services one can expect from these resources because of the climate unpredictability and lack of in depth understanding of the uncertainty brought about by our inability to quantify precisely the various parameters involved in water resources management and planning.

With growing population worldwide, emphasis is put on the water resources due to the threat of overexploitation, resulting in the need for sound groundwater management. This in turn requires a full consideration of groundwater in terms of sustainability of the resource (Stadler et al., 2010). However, long term availability of groundwater supplies for escalating populations can be ensured only if effective management schemes are developed and put into practice (Healy, 2010). At present, about 40% of the world’s population uses groundwater as a main source of water supply and about 50% of the global food production depends on irrigated agriculture related to groundwater (Seiler & Gat, 2007).

It has been estimated that about 98.7% of all liquid freshwater is available as groundwater, while only 0.98% is available in rivers and lakes (Ayibotele, 1992). During the last few decades, groundwater has become a vital source of freshwater throughout the world (Rejani et al., 2007).
Groundwater has emerged as an extremely essential water resource and its increasing demand in agriculture, domestic and industrial uses ranks it as a resource of strategic importance. Groundwater resources are dynamic in nature and affected by factors like growth of irrigation activities, industrialisation, and urbanisation. For this reason, monitoring and conserving this important resource is essential (Ketata et al., 2011).

Moreover, groundwater development has for a long time provided drinking water to urban and rural populations of developed and developing countries. At present, groundwater is estimated to provide about 50% of the world’s drinking-water supplies (UN WWAP, 2003). In developing countries and especially in rural areas where the majority of people live, groundwater is often the only possible source of good potable water. In arid and semi-arid regions, where water shortage is almost endemic, groundwater has played a major role in accommodating domestic and irrigation demands. Groundwater also has advantage that it is frequently broadly available in quantities sufficient to supply the needs of scattered communities.

Economic studies undertaken by the United Nations indicate that it is five times expensive to build a dam and reservoir than to develop an equal quantity of water from aquifers. The advantage of groundwater over surface water lies, however, in its availability in practically every part of the country, though, in varying quantities. The cost of development of groundwater is cheaper than that of surface water and the quality of the water is mostly reasonably good requiring only minimal treatment, and in most areas readily potable. However groundwater quality varies from one rock type to another and also within aquifers along groundwater flow paths. Groundwater quality can be altered as a result of anthropogenic activities such as landuse change, agriculture, as well as urbanisation.

In many parts of South Africa groundwater allocation and protection has become an important issue, as groundwater represents an important source of freshwater (Münch & Conrad, 2007). The sustainability of this crucial resource is greatly dependent on the amount of groundwater recharge. Thus, accurate assessment of the spatial and
temporal distribution of recharge is essential for optimal operation of the groundwater system.

**Groundwater situation in South Africa**

Groundwater contributes about 13% to the total water supply in South Africa although it is unboundedly a significant strategic water resource. Moreover, groundwater recharge can differ from catchment to catchment in space and time especially in dry areas like the study area. Hence it is important to consider both factors (spatial and temporal).

**Groundwater as a natural resource**

Groundwater is an important resource that supports human health, economic development and ecological diversity. Calow *et al.* (1999) defined groundwater as a recurrent source of water, a source that can be developed for content use. Groundwater is suitable for human consumption because of its good quality (Jha *et al.*, 2007). Many rural communities rely on groundwater source thus it is important to acknowledge that effective management of this resource is a key factor in socio-economic development in Southern Africa.

**2.2 Groundwater recharge**

Recharge, defined as the entry of water into the saturated zone, varies spatially and temporally and depends on a wide variety of factors (e.g. Vegetation, precipitation, topography, geology, and soil type), making it one of the most difficult, complex, and uncertain hydrologic parameter to quantify in the water budget (Dripps & Bradbury, 2007). Recharge can be classified as direct (diffuse), indirect (non-diffuse) and localised (focused). According to Haile (2015), groundwater recharge can be defined as water that enters into the groundwater system from any directions (up, down or laterally) (Lerner, 1997; Adem & Batelaan, 2006; Russell, 2010). The main sources of recharge are both natural and artificial. The natural sources include precipitation, lakes, rivers, ponds and other aquifers. Artificial recharge sources include reclaimed wastewater,
injection of (treated) surface water, leaking water supply lines and irrigation losses from fields and canals.

Groundwater recharge has a fundamental role in sustainable groundwater resources development and management. In order to optimal in quantify groundwater recharge; other hydrologic, social and economic factors have to be considered. For sustainable management of groundwater resources, the amount of recharge received by an aquifer is by far the most important input required. Yet this is usually the least well-known quantity in hydrogeology, especially in arid and semi-arid environments (Kinzelbach et al., 2002). Recharge estimates are made particularly difficult in arid and semi-arid lands by the vast variability of hydrological events in time and space. Thus, as far as groundwater management is concerned, it is essential that decision makers have a satisfactory understanding of the space-time changes of groundwater levels.

2.2.1 Factors affecting groundwater recharge

Climate change

The challenges of understanding climate-change effects on groundwater are unprecedented because climate change may affect hydrogeological processes and groundwater resources directly and indirectly (Green et al., 2011). The relation between climate variables and groundwater is considered more complicated than with surface water (Holman, 2006; IPCC, 2001). This understanding is confused by the fact that groundwater-residence times can range from days to tens of thousands of years, which is likely to delay and disperse the effects of climate change. This challenges efforts to immediately detect responses in the groundwater (Chen et al., 2004). Hence the principle focus of climate change research with regard to groundwater has been used on quantifying the likely direct impacts of changing precipitation and temperature patterns (Holman, 2006).
Land cover

Land use is an important factor in groundwater recharge. Shaban et al. (2006) concluded that vegetation cover benefits groundwater recharge in the following ways: (i) Biological decomposition of the roots helps loosen the rock and soil, so that water can percolate to the surface of the earth easily (ii) Vegetation prevents direct evaporation of water from soil and (iii) the roots of a plant can absorb water, thus preventing water loss. It is well recognized that land cover and land use change have significant effects on hydrological processes such as evapotranspiration (ET), soil moisture and groundwater recharge (Zhang et al., 2004). According to Jyrkama & Sykes (2007), Vegetation influences recharge through the processes of interception and transpiration, and other less commonly characterized, yet potentially significant processes such as stem flow and through fall (Le Maitre et al., 1999; Taniguchi et al., 1996). Plant roots also play an important role in the recharge process not only by enabling plants to draw water from deep in the vadose zone (and even from the saturated zone) thereby reducing the amount of percolating water that reaches the water table, but also by creating preferential flow paths and channels that aid water flow through the soil profile (Jyrkama & Sykes, 2007).

Evapotranspiration

Evapotranspiration is a major source of water depletion in arid and semiarid environments. Approximately one-third of the worldwide land surface occurs in regions with arid or semiarid climates, and about 20% of the world’s population presently lives in these regions (Coonrod & McDonnell, 2001)

In sub-humid, semi-arid and arid regions, evapotranspiration (ET) is the second largest component of the water balance and so is affected not only by the availability of water but also by any changes in other aspects of climate (e.g. temperature). Recharge is often the smallest component of the water balance and is often calculated as the residual after subtracting ET and runoff from precipitation (Crosbie et al., 2010).
Precipitation

Precipitation and evapotranspiration are particularly important because they directly affect groundwater recharge and indirectly affect groundwater withdrawals or discharge. Even small changes in precipitation may lead to large changes in recharge in some semiarid and arid regions (Woldeamlak, 2007; Green et al., 2011). Precipitation is the largest term in the water balance of a catchment and as such the hydrological cycle is most sensitive to changes in rainfall. Thus, increased variability in precipitation, temperature, and evapotranspiration that is predicted under many climate-change scenarios have varied effects on different aquifers and different locations within an aquifer.

For groundwater recharge estimation, changes in precipitation amount and intensity (in so far as it affects runoff and infiltration) are much more important than changes in temperature. Many studies suggest that precipitation intensities tend to increase, particularly for larger events (Holman, 2006).

### 2.2.2 Types of recharge

Direct (diffuse) recharge: This is the water added to the groundwater system in excess of soil moisture deficits and evapotranspiration by direct infiltration of precipitation and percolation through the unsaturated (vadose) zone (Lerner et al., 1990; Jyrkama & Syke, 2007). This type of recharge occurs throughout the entire vadose zone (diffuse). Direct recharge forms the highest contribution in humid climates as the vadose zone has high water content and little additional storage capacity due to regular precipitation (Sophocleous, 2004).

Indirect or non-diffuse recharge: This is the flow of water to the groundwater system through the beds of surface water bodies (Lerner et al., 1990). Indirect recharge forms the most extensive and significant recharge in arid areas, which spreads flood water over large areas on both sides of watercourses (Sen, 2008).
Localized or focused recharge: This is an in-between form of recharge obtained from horizontal surface concentration of water in local joints, swamps, potholes and depressions. In terms of total aquifer refill, indirect and localized recharges are more essential than direct recharge as aridity increases (Sophocleous, 2004).

Another type of recharge is artificial recharge. As defined by Senanayake et al. (2016), artificial recharge is a type of controlled recharge where surface water is injected in the ground and subsequently flows to the aquifer to augment the groundwater resources. Bhattacharya (2010), defined artificial recharge as the ‘practice of increasing the amount of water entering to the subsurface reservoirs by artificial means’. The application of artificial recharge is becoming increasingly important in groundwater management and especially in conjunctive use of surface and groundwater resources.

2.2.3 Spatial and temporal variability of groundwater recharge

Recharge in arid and semi-arid areas is primarily driven by the occurrence of periods of above normal rainfall. The climate of southern Africa is inherently variable in time and space and changes through seasonal, annual, decadal, millennial and even longer variations (Meadows, 2001). Recharge is also influenced by this variability. The temporal variability of recharge is a result of the incidences of rainy days as well as the intensity and duration of the rainfall events. Rainfall in dry areas is usually in the form of short intense rainstorms. It is generally documented that rainfall below a certain value, usually 400 mm, will not result in any significant recharge (Gustafson & Krasny, 1994; Singhal 2003). This is because for short duration high intensity rainfall, it would imply less infiltration. However, for long duration, more recharge would be realised. Therefore, the spatial and temporal variability of groundwater recharge are key factors that need to be quantified to determine the sustainability of groundwater resources (Berehanu et al., 2017).
2.2.4 Groundwater recharge estimation methods

Reliable recharge estimation is extremely important for efficient and sustainable management of groundwater systems. Various methods exist to estimate recharge. These methods can be used over different spatial and temporal scales as well as over different complexity and expense. Besides, each method has its own advantages and limitations in terms of applicability and reliability. For instance, water resources assessment requires recharge information at large spatial and temporal scales whereas contaminant transport would require recharge information at local and shorter time scales.

Recharge estimation methods can be classified according to three hydrological zones, which are surface water, unsaturated zone and saturated zone (Scanlon & Cook, 2002; Beekman et al., 1999). Within each zone, the methods can be classified into physical, tracer and numerical modelling methods (Scanlon & Cook, 2002). The physical and tracer methods can further be classified into direct versus indirect, water balance and Darcian methods, and chemical, isotopic and gaseous tracer methods (Lerner et al., 1990; Beekman et al., 1999). Other methods of recharge estimation include base flow separation, zero-flux plane, inverse groundwater modelling, remote sensing, temperature and electromagnetic (Stephens, 1996).

A wide variety of methods for estimating recharge can be found in Lerner et al. (1990) and Stephens (1996). Most of the methods in the literature focus on recharge in arid and semi-arid regions, where recharge is vital environmentally and economically. Generally, it is recommended to use multiple methods to estimate recharge due to the uncertainty associated with each method (Scanlon & Cook, 2002). Here, a brief description of the water table fluctuation method, Darcy’s law, water budget and inverse groundwater modelling methods are presented. Moreover, choosing a suitable method for a particular site is not straight forward (Scanlon et al., 2002) and depends on numerous factors including, field constraints such as availability of field data. However, techniques based on water table fluctuation, water budget, hydraulics heads and numerical methods are among the most widely used methods (Healey & Cook, 2002).
Water Table Fluctuation (WTF)

The water table fluctuation method (WTF) method is based on the assumption that water level rise in unconfined aquifers are caused by recharge. If the water level rises and the specific yield is known, then recharge can be calculated as; (Healey & Cook, 2002)

\[ R = S_y \frac{d\phi}{dt} \approx S_y \frac{\Delta \phi}{\Delta t} \]  \hspace{1cm} (2.1)

Where \( S_y \) is specific yield [-], \( \Delta \phi \) is change in hydraulic head [L], and \( \Delta t \) is change in time [T]. Equation (2.1) implicitly assumes that there is no horizontal flow in the saturated zone and \( \Delta \phi \) is caused only by recharge and that the recharge goes immediately into storage \( \frac{d\phi}{dt} \). This approach has been applied in various studies (Rasmussen & Andereasen, 1959; Hall & Risser, 1993; Crosbie et al., 2005; Delin et al., 2007) because of its simplicity, ease of use and availability of water level data. The WTF method is ideal for estimating recharge over short periods in shallow aquifers with a clear rise and fall of hydraulic heads. The main downside of this approach is determining a constant representative specific yield and ensuring that the hydraulic head fluctuations are caused by recharge. The hydraulic head fluctuation can be caused by pumping, barometric pressure fluctuation, evapotranspiration, entrapped air (Lisse effect). Consequently, ensuring that the hydraulic head fluctuation is caused by recharge alone can be a difficult task.

Darcy’s law

Darcy’s law can be used both in the unsaturated and saturated zones to estimate recharge (Nimmo et al., 1994). Darcy’s law in the unsaturated zone is written to estimate recharge as;

\[ R = -k(\theta) \left( \frac{d\psi}{dz} + 1 \right) \]  \hspace{1cm} (2.2)
Where $k(\theta)$ is the unsaturated zone hydraulic conductivity [LT$^{-1}$], $\psi$ is the matric pressure head [L], $z$ is elevation [L] and $\theta$ is water content [-]. Equation (2.2) can be used to estimate recharge over a wide range of time scales if the hydraulic conductivity and head gradient are measured accurately (Scanlon et al., 2002). Equivalently recharge can be estimated in the saturated zone using Darcy’s law. In this case, Darcy’s law can be easily applied by assuming steady state condition and flow net analysis. Two-dimensional (2D) vertical flow nets can provide an approximate way of estimating recharge. However, recharge estimated by Darcy’s law can have high uncertainty due to the high spatial variability of hydraulic conductivity (Scanlon et al., 2002).

**Water budget**

The water budget approach is one of the most widely used methods to estimate recharge (Healey, 2010). In this approach, all components of the water budget equation are measured or estimated with groundwater recharge being the residual. The water budget equation can be written as:

$$R = P - ET - RO - \Delta S \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.3)$$

Where $P$ is precipitation [L], $ET$ is actual evapotranspiration [L], $RO$ is surface runoff and $\Delta S$ is the change in water storage in the unsaturated zone. The accuracy of recharge estimated using this approach depends on the accuracy of estimation or measurement of the other components. Due to the propagation of errors in the water budget analysis, the uncertainty of recharge is very high when it is a small fraction of precipitation (Stephens, 1996; Jyrkama & Sykes, 2007).

**Inverse groundwater modelling**

Inverse groundwater modelling can be used to estimate recharge based on known hydraulic heads, hydraulic conductivity and other flows into and out of the groundwater system. The accuracy of the recharge estimated is highly dependent on the accuracy of hydraulic conductivity. During inversion, mostly recharge and hydraulic conductivity are
estimated simultaneously as the modeller tries to constrain both parameters (Sanford, 2002). As a result, only the ratio of recharge to hydraulic conductivity can be estimated. As different ratios can produce the same hydraulic head distribution, the estimated recharge will be non-unique. To obtain a unique solution, additional flux observations should be used to constrain the model (Sanford, 2002; Scanlon et al., 2002).

The concept of determining a unique recharge from hydraulic head, hydraulic conductivity and flux data can be extended to determine a unique hydraulic conductivity field if the other parameters are known. One of the challenges of determining the hydraulic conductivity is to find a model that can simulate both flux and hydraulic head data.

2.2.5 Recent development in recharge estimation

Recharge can be estimated using a number of approaches depending on the availability of data and level of accuracy required. In areas mainly characterized by arid and semi-arid conditions, recharge is usually small and potentially more variable (Ketema & Broder, 2009) as such the use of techniques that can handle spatial variability is necessary. In this regard, Geographic Information Systems (GIS) have emerged as effective tools for handling spatial data and decision making in several areas including engineering, geology and environmental fields.

For example, Tweed et al. (2007) used “remote sensing (RS) and GIS for mapping of groundwater recharge and discharge areas in salinity prone catchments in southeastern Australia”. By using GIS and remote sensing mapping techniques, the resolution and extent of information was enhanced for the regional catchment, and hence new hydrogeological information was provided for numerical modelling, water budget analysis, and salinity mitigation programs. Sander et al. (1996) used GIS which focused on the identification of phenomena that contributed to successful wells, for the purpose of developing optimal strategies for future well siting.
The concept of integrating remote sensing and GIS is comparatively new. Many assessments of groundwater conditions made with remote sensing techniques have been reported (Krishnamurthy & Srinivas, 1996; Bastiaansen et al., 1998). Geographic Information System techniques have many advantages, for example GIS can take into account the diversity of factors that control groundwater recharge. Moreover, hydrogeological database can be created by using GIS tools in order to be used for groundwater recharge mapping and simulation (Krishnamurthy et al., 2000).

Several researchers have demonstrated that GIS because of its sophisticated analytical capabilities, can provide a powerful platform for developing GIS based model, calibrating as well as validating them (Jha et al., 2007). However, the use of RS has demerits in that it requires a special kind of training to analyse the data (i.e satellite images), the instruments used in remote sensing may sometimes be un-calibrated which may lead to un-calibrated remote sensing data (uncertain data). Thus, accuracy assessment is necessary in all remote sensed data to avoid classification error. Moreover, GIS has limitation as it requires enormous amount of input data to be practical for some tasks and geographic error is increased as you get into a larger scale domain.

2.3 Groundwater recharge studies in South Africa

Internationally, there is a vast amount of literature on groundwater recharge and, according to Simmers (1998); the key in recharge studies is the project objective. Xu & Beekman (2003) stated that the first systematic recharge studies carried out in South Africa was done back in the early 1970s in the western “Transvaal” (now the province of Gauteng) and in the Northern Cape. Recharge studies were mostly carried out at a local scale and not as part of larger groundwater resources assessment. The growing need for reliable recharge estimation in South Africa was based on improved management of limited water resources. Verhagen et al. (1979) carried out extensive studies on the application of natural isotopes to the Kalahari region and to some semi-arid dolomite areas. Moreover Fleisher (1981) provided a major contribution to quantitative estimations of recharge based on water balance studies at West Rand dolomites. The
other research was done by Connelly \textit{et al.} (1989) who estimated recharge by rainfall method. This study covered three different recharge areas and attempted to apply a conceptual model to infer recharge from the physical nature of the catchments and the characteristics of the aquifer. Although some results were obtained the study was curtailed due to the complexity of the systems to which the techniques had been applied. Further, Kirchner \textit{et al.} (1991) studied recharge of the Karoo formations, incorporating water balance methods. Moreover Sami (1991) provides a good summary and evaluation of the methods available and favours the use of natural chloride in semi-arid environments. Method by Chloride mass balance (CMB) was used by different researchers. For example, Gieske (1992) studied recharge in semi-arid regions of South Africa using chloride method.

Wright & Burgess (1992) have reviewed the hydrogeology of crystalline basement aquifers in Africa and include some useful quantitative estimates of recharge according to the baseflow from small sub-humid catchments in Zimbabwe and Malawi. Major contribution of groundwater maps was done by Vegter (1995). In his explanation a set of national groundwater maps that was developed described the method which was used to calculate groundwater recharge on each area. Moreover Bredenkamp \textit{et al.} (1995) produced a manual on recharge and storability estimation, which includes many case studies and data. Again, publication by Xu & Beekman (2003) provides an overview of recharge methods and studies carried out in southern Africa, particularly within arid and semiarid environments.

Based on the previous studies carried out in South Africa, majority of them used old methods to estimate groundwater recharge which are; chloride mass balance method (CMB), rainfall recharge relationship and mean annual precipitation (MAP) approach. However, the use of new approaches of incorporating GIS and remote sensing in recharge estimation in South Africa is new. No study in South Africa using water balance GIS hydrological model (WetSpass) for groundwater recharge estimation has been reported. As a result, this research used a new approach of WetSpass on estimating groundwater recharge.
2.4 Recharge estimation using GIS hydrological model (WetSpass)

Reliable recharge estimation is extremely important for efficient and sustainable management of groundwater systems. Various methods exist to estimate recharge as discussed in sub-section 2.2.4. Among these methods is WetSpass.

WetSpass stands for Water and Energy Transfer between Soil, Plants and Atmosphere under quasi-Steady State (Batelaan & De Smedt, 2001). It is a physically based model for the estimation of long-term average spatial patterns of groundwater recharge, surface runoff and evapotranspiration employing physical and empirical relationships. Regional groundwater models used for analysing recharge - discharge relations are often quasi-steady and need long-term average recharge input that accounts for the spatial variability of the recharge. The original WetSpass model is a quasi-steady state spatially distributed water balance model scripted in Avenue and used to predict hydrological processes at seasonal and annual time step. With rising popularity of Python programming language in the scientific and research areas, the model has been scripted in Python and using own spatial library Hydrology and Hydraulic Programming Library (H2PL). This newer version of the model has ability to simulate interception from vegetated surfaces, runoff from the landscape, evapotranspiration, soil water balance, and recharge at monthly time step or annually.

Previous Applications of WetSpass Model

Some recent work that used WetSpass includes that of Abu-Saleem et al. (2010) which aimed at estimating the water balance components including surface runoff, actual evapotranspiration and groundwater recharge by using Geographical information system (GIS), Remote sensing (RS) and land use information. Another recent research was done by Al Kuisi & El- Naqa (2013) at Jafr basin in Jordan to estimate groundwater recharge. Results obtained from the WetSpass model were compared with the estimated recharge to groundwater carried out by Howard (1984) and JICA (1990). The
results demonstrated that the estimation of groundwater recharge using WetSpass was in good agreement with those obtained by other studies.

Furthermore, WetSpass was used for the analysis of the effect of landuse changes on groundwater discharge areas for Grote Nete basin, Belgium (Asefa et al., 2000; Batelaan & Kuntohadi, 2002; Batelaan et al., 2003). WetSpass was also used in the analysis of the hydrological characteristics of the Kikbeek sub-basin of the Border Meuse River, Belgium (Van Rossum et al., 2001). Fast and slow discharge coefficients were calculated from WetSpass output data as these discharge coefficients were related to the total surface runoff and groundwater recharge, respectively. By using the WetSpass model of the actual hydrological situation in the Kikbeek sub-basin as a starting point, the sensitivity of the discharge coefficients of the Kikbeek sub-basin towards climate and land use changes were analysed.

The WetSpa model (similar to WetSpass) was also used to analyse the effects of topography, soil type, and land use cover on the runoff characteristics for upper Biebrza catchment (Porretta-Brandyk et al., 2010). The derivation of parameter maps and analysis of the daily runoff as reaction of the catchment on rainfall was performed. The values for the catchment justified the assessment of the model quality measurements as very good.

**Limitation on WetSpass model**

In every model limitations must be considered in running the model and interpreting its output. Thus, the following are the limitations associated with WetSpass model simulation;

1) WetSpass model use the GIS tool for data preparation, hence the modeller should have knowledge of GIS in order to process input data and also for analysis of output data.

2) WetSpass model use land use map as among the important input of the model. The land use categories are grouped, for which some of the categories might be
somewhat unclear. Thus, proper land use classification is required by using an appropriate software and recent Landsat Images.

3) Values assigned to any raster or grid cell each represent an average value over the area of each cell. The greater the unevenness over the cell, the greater will be the error induced through the use of an average value. Therefore, the grid size should be well defined.

4) The time of simulation should be well defined. If it is for two seasons, the seasons should be well defined and if it is annually it means the model will run one simulation annually only.

2.5 Groundwater modelling

Models are conceptual descriptions and not exact descriptions of the physical processes. Models are not anticipated to substitute for field investigation but are rather to be used as additional tools. Two types of models exist which are physical and mathematical models (numerical). Brooks et al. (1991) and Salarpour et al. (2011) defines physical model as a scaled-down form of a real system, while mathematical models include clear chronological set of relation, numerical and logical steps that change numerical inputs into numerical outputs.

Groundwater models are constructed across various spatial and temporal scales. Spatially, they vary from very small-scale flow systems to regional or national scale flow systems (e.g., Srinivas et al., 1999). Steady state models have a constant head representing the long-term equilibrium between recharge and discharge, and transient state models allow the head to change over time. Recharge should also be represented in line with the scale of the groundwater models. In order to represent recharge efficiently in groundwater models, both the physical processes controlling recharge and the objective of the modelling study should be taken into account (Sanford, 2002). For example, in arid climates, where the saturated zone can transmit away more recharge than provided by the climate and soil, the climate and soil conditions control the rate of recharge. Sanford (2002) reported that controlling factors on recharge are the basis for
deciding the type of mathematical boundary condition used to represent recharge in groundwater models.

For quite a while now, groundwater models have been constructed with the assumption of single geological unit representing best possible geology of the unknown reality (Refsgaard et al., 2012). Due to local scale geological heterogeneity of groundwater system, the hydraulic properties exhibit unknown spatial heterogeneity.

**Types of models in groundwater flow**

**Analytical Models**

Analytical solution of the mathematical equation provides exact solution to the problem, i.e., the unknown variable is solved continuously for every point in space (steady-state flow) and time (transient flow) (Igboekwe & Amos-Uhegbu, 2011). Analytical models are best used for: (i) evaluations which require low level degree of accuracy (ii) designing data collection plans before beginning field activities (iii) an independent check of numerical model simulation results and/or (iv) situations in the field which are in line with simplifying assumptions embedded in the analytical models. Hence to solve 3D groundwater flow and transport equations, numerical solution techniques would be required.

**Numerical Models**

Numerical models are capable of solving the more complex equations that illustrate groundwater flow. These equations usually describe multi-dimensional groundwater flow. Numerical models use approximations (e.g. finite differences, or finite elements) to solve the differential equations describing groundwater flow. The approximations require that the model domain and time be discretized. In this discretization process, the model domain is represented by a network of grid cells or elements, and the time of the simulation is represented by time steps.
It is noted that, the accuracy of numerical models depends upon the accuracy of the model input data, the amount of data available, and time frame of the available data and also the numerical method used to solve the model equations. The advantages of numerical models are that; they may be used to simulate very simple flow conditions which can be easily simulated using an analytical model as well as simulating complex problems which cannot be accurately described using analytical models.

On the other hand, most numerical models still provide only deterministic predictions with no accompanying information on forecasted uncertainty. Li et al. (2003) reported that, accuracy of numerical models is typically based on after-the-fact comparisons of predictions with measurements rather than on information generated as an intrinsic part of the modelling process. Two techniques which can be used to solve groundwater governing equation numerically are Finite Element Method (FEM) and Finite Difference Method (FDM).

**Finite element method**

Finite element method is basically adopted from solid mechanics application and is relatively new in the groundwater modelling application (Istok, 1989). The method has been used more often due to its potential of handling non – regular shaped domain and providing smooth flow along various directions (Rabbani, 1994). Finite elements method has several advantages (1) anisotropic, heterogeneous, irregular or curved aquifer boundaries can be incorporated in numerical modelling through this method (2) various types of problems can be solved using a small set of identical computer procedures. Moreover, this method has the following disadvantages, including its requirements for great amount of mathematical and programming even for a simple problem.

**Finite Difference Modelling**

The finite difference method is one of a numerical method which is used to solve partial differential groundwater flow equations. The analytical solutions are intricate to use
because it is based on exact solution of one or two-dimensional flows thus the input parameter and aquifer boundaries should be highly precise.

2.6 Uncertainty in Groundwater Models

Uncertainties in groundwater system may be attributed to lack of perfect knowledge about the aquifer, intrinsic variability of system parameters both in space and time, errors in historical data used to model the groundwater system, incapacity to accurately predict future events such as water demand, and other factors like economic factors of projects (Ndambuki, 2001a).

The existence of uncertainties limits our ability to predict system behaviour with definitiveness under various decisions. In literature, several methodologies to account for uncertainties arising from inputs (forcing data), parameters and the definition of alternative conceptual models have been proposed (Beven & Binley, 1992; Neuman, 2003; Poeter & Anderson, 2005).

Bredehoeft, (2003), Neuman, (2003), Neuman & Wierenga, (2003) suggested that uncertainties in groundwater model predictions are largely dominated by uncertainty arising from conceptual models and that parametric uncertainty solely does not allow compensating for conceptual model uncertainty. Moreover, different external factors such as climatic conditions or groundwater abstraction policies, on the other hand, increase the uncertainty in groundwater model predictions due to unknown future conditions (Rojas et al., 2010).

2.6.1 Deterministic Modelling

Deterministic models assume that future response of the system (aquifer) is prescribed a priori by physical laws governing groundwater flow. As a result, these methods require full characterization of aquifers which is neither practical nor economically feasible, hence different approaches are required which are capable of utilizing the available insufficient data.
Deterministic methods assume that all the input data are known without error. This assumption is illogical since usually only a few set of data are available to define the current situation of the system with certainty (Ndambuki, 2001a). For example, natural geological formations that form aquifers are heterogeneous (i.e., their hydraulic properties display variability in space), complex and uncertain (due to lack of sufficient data). As a result, the existence of uncertainties limits our ability to forecast system behaviour with definitiveness under diverse management decisions.

This study considers the fact that uncertainty exists in determination of groundwater recharge. Consider the usual groundwater flow equation (equation 2.4).

\[
S_s = \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) \pm w \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.4)
\]

Where:

\( K_x, K_y \) and \( K_z \) are hydraulic conductivity along the x, y and z coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity \([\text{ms}^{-1}]\);

\( h \) is the potentiometric head \([\text{m}]\);

\( w \): Volumetric flux per unit volume, representing sources and/or sinks of water \([\text{m}^3\text{s}^{-1}]\);

\( S_s \): the specific storage of the porous material \([\text{m}^{-1}]\); \( t \): time \([\text{s}]\).

\(- w = \text{discharge}\)

\(+ w = \text{recharge}\)

Equation (2.4) can be used to simulate the unknown variable (e.g. recharge). If the parameters in equation (2.4) are known with certainty, then the groundwater simulation is said to be deterministic, otherwise stochastic. If \(- w \) can be correctly estimated, it means equation (2.4) can be used to accurately determine the amount of groundwater storage.
However, equation (2.4) is a partial differential equation (complex) which cannot be solved analytically; hence we need to call upon numerical methods which will convert the partial differential equation to ordinary differential equation which is solvable through numerical methods. By disaggregating $w$ term in equation (2.4) into $-w$ and $+w$, equation (2.4) changes into the following (equation 2.5):

$$S_s = \frac{dh}{dt} = \frac{d}{dx} \left( K_x \frac{dh}{dx} \right) + \frac{d}{dy} \left( K_y \frac{dh}{dy} \right) + \frac{d}{dz} \left( K_z \frac{dh}{dz} \right) - w_1 + w_2 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS...
In this research stochastic approach was used to solve the uncertainty problem. For instance, if we take recharge as uncertainty ($\mu$); equation (2.6) becomes equation (2.7).

$$S_s = \frac{dh}{dt} = \frac{d}{dx} \left( K_x \frac{dh}{dx} \right) + \frac{d}{dy} \left( K_y \frac{dh}{dy} \right) + \frac{d}{dz} \left( K_z \frac{dh}{dz} \right) - w_1 + w_2((x,y),t,\mu) \ldots \ldots \ldots (2.7)$$

This is now a fairly complex equation to solve because the problem is not only heterogeneous and dynamic, but also uncertain and hence stochastic. Because its solution depends on the outcomes of realisations of the random values of recharge.

Nowadays numerical methods used to solve stochastic groundwater problems are divided into Monte Carlo (MC) and classical perturbation methods. However, both of these methods need more computation time than traditional deterministic methods. In this research Monte Carlo approach was used as a solution methodology.

**Monte Carlo (MC) method**

Monte Carlo method is the most common method used to solve problems with uncertain parameters. This is because; this method is capable of working with many classes of uncertain parameters (Ndambuki, 2001b). According to Neuman & Wierenga (2003), this method is popular because it is straightforward and to some extent nonlinear. However, Cooley (1997) showed that for highly nonlinear models, this type of Monte Carlo based presumption on predictions can be highly inaccurate, and that such a Monte Carlo based confidence interval can be as inaccurate as the linearised confidence interval.

Beven (2008) discussed Monte Carlo (MC) simulation as an enormously flexible and robust sampling based method which is extensively used for uncertain problems in hydrological applications. The uncertain parameters in the method are portrayed by probability distributions, and in the situation of lack of enough information, model parameters are assumed independent. In the process, random values of each of the uncertain parameters are generated according to their respective probability
distributions and the model is run using each random sample. In so doing, samples of model outputs are generated giving statistics (e.g. mean, standard deviation, skewness) and estimated probability distribution of the model output can be determined (Pechlivanidis et al., 2011). On the other hand, MC method has its disadvantages that a huge number of parameters are frequently required to consistently represent all probable results particularly when there are a number of random variables.

Monte Carlo technique (also known as simulation method) is a method that relies on repeated random sampling to get results. These methods (MC) are stochastic method and uses random sampling based on underlying cumulative distribution function (cdf) and probability density function (pdf). The Monte Carlo simulation (MCS) approach deals with repeated large numbers of trial or simulations with uncertain variables which are selected at random from the supposed parent probability distribution for the purpose of creating expected values of model uncertainty (Bobba et al., 1996).

To illustrate MCS approach, consider there exist a probability space $(\Omega, F, P)$, where $\Omega$ is a set of all possible outcomes, $F$ denotes a set of collection of all subsets of $\Omega$ possible outcomes, and $P$ denotes a probability measure. Let’s reconsider a random continuous variable vector $X$ having probability mass function (pmf) or probability density function (pdf) $f(X)$ which is greater than zero on a set of values $X$. Assuming that the mean values of realisations of the uncertain recharge $(\mu)$ of aquifer system (simply denoted as $(\bar{\mu})$ such that $\bar{\mu} \in \bar{\Omega}$, where $\bar{\mu} = 1, 2, \ldots, \infty$ and $\bar{\Omega}$ is the total possible number of realisations, hence each realisation $\mu$ leads to different infinite groundwater flow simulation model solutions (hydraulic heads). Thus the expected value of a function $g$ of $X$ can be defined as:

$$E(g(X)) = \int_{\Omega} G(x, \bar{\mu}) dP(\bar{\mu})$$ ................................................................. (2.8)

It should be noted that to solve such equation (2.8) above to an exact solution is computationally impossible or prohibitively expensive. Normally, a simple way is to revert to Monte Carlo numerical methods by assuming a set of finite independent
identically distributed (i. i. d.) samples of the $n$ realisations. In this study, we assume the $n$ – samples of $X$’s ($x_1, x_2, \ldots, x_n$) and compute the mean of $g(x)$ of the samples, then through the Monte Carlo method based on equation (2.9) estimate the expected function $\mathbb{E}(g(X))$ as follows:

$$\bar{g}(x) = \frac{1}{n} \sum_{i=1}^{n} g(x, \overline{p})$$

(2.9)

Now, we assume that if the $\mathbb{E}(g(X))$ exists, then based on the weak law of large numbers deviation which says that for any arbitrarily small $\epsilon$;

$$\lim_{n \to \infty} P(|\bar{g}(X) - \mathbb{E}(g(X))| \geq \epsilon) = 0$$

(2.10)

Hence, this implies that as $n$ gets large (2.10) there is probability that $\bar{g}_n(X)$ deviates from $\mathbb{E}(g(X))$. Moreover, on the other hand based on the law of large numbers and central limits theorem which applies that as $n$ gets large enough, the function $\bar{g}_n(X)$ estimated values (i.e., MC procedure estimates), we can claim that the estimated value shall be closer to $\mathbb{E}(g(X))$ value as required.

### 2.7 Groundwater Management

Groundwater management in many countries has progressed from virtually nil to a more regulatory regime today. Valuable tools to study and develop groundwater management strategies are numerical groundwater simulations, which are widely adopted (Mao et al., 2005). Management of groundwater resources requires that recharge to the groundwater be known on finer scales, therefore modelling is often the chosen method of estimating recharge (Crosbie et al., 2010). Thus, effective groundwater management requires the identification of the mechanisms of recharge generation (e.g. diffuse vs. focused), the evaluation of the uncertainties in recharge estimation, the spatio-temporal assessment of recharge patterns, and the integration of groundwater studies with other studies, such as saltwater intrusion (Andreu et al., 2011).
Loaiciga (2003) defined Groundwater management as the continuing performance of coordinated actions related to the replenishment and withdrawal of water to achieve long-term sustainability of the resource without detrimental effects on other resources. Groundwater management all over the world often lacks sustainability as evidenced by falling water tables, drying wetlands, increasing sea-water intrusion and general deterioration of water quality.

According to Loucks (2000) sustainability in water resource management means: 'Water resource systems that are managed to satisfy the changing demands put on them, now and into the future, without system degradation. Groundwater resources can best be managed through protection of aquifers to ensure long – term, high quality supplies. An aquifer management plan can help achieve this by developing an understanding of how an aquifer works and answer questions such as: How much water is available? Are there any practices that could possibly pollute the aquifer? Is the aquifer being over pumped? What can be done to protect this resource? These questions lead most of water resources planners and managers to work in an environment of change and uncertainty. This is because; most of the values of parameters used on modelling process to predict the hydrological, economic, environmental, ecological and social impacts are also changing and uncertain.

2.8 Groundwater sustainability indicators

Groundwater sustainability indicators (GSI) can be used as a tool to manage groundwater resources development and protection. It is very important to control aquifer exploitation because intensive abstraction from aquifers may affect springs, base-flow of streams, groundwater levels, and groundwater storage.

Groundwater indicators are considered as important tools in instigating various aspects of water resource management. These tools underline the state of development, stress and other aspects connected to condition of aquifers and help significantly in the struggle for sustainable water supply solutions. Also, groundwater indicators offer appropriate platform with high-level similarity between various regions and where
applied at the national level, they have potential to support international dialogue on great environmental matters.

Groundwater indicator compares the amounts of the abstracted groundwater to total groundwater recharge. Therefore, it is necessary to indicate the intensity of renewable groundwater use or stress on the resource. GSI indicate if groundwater is used in a sustainable way or if there is any indication of over abstraction. When abstraction is less than recharge, groundwater is considered sustainable (Lavapuro et al., 2008).

The GSI can be calculated from the information on total groundwater abstraction and groundwater recharge (Vrba & Lipponen, 2007) as follows:

$$GSI = \frac{GW_{ab}}{GW_{rech}}$$  \hspace{1cm} (2.11)

Where;

GSI is Groundwater sustainability index

GW_{ab} is Groundwater abstraction

GW_{rech} is Groundwater recharge

Equation (equation 2.11) indicated that if the amount of groundwater abstraction reaches the amount of groundwater recharge then over-exploitation could occur. The value of GSI closes to unity (for example 0.8) and above indicates overexploitation (Vrba & Lipponen, 2007).

**2.9 Concluding remarks**

Groundwater has emerged to be one of the major sources of potable water for various purposes in both urban and rural areas. This is because groundwater has the advantage that it is frequently widely available in quantities sufficient to supply the needs of scattered communities. Evaluation of groundwater involves several factors of which, recharge is paramount. Recharge can be estimated using a number of
approaches depending on the availability of data and level of accuracy required. In arid and semiarid areas, recharge is relatively small and potentially more variable. This calls for methods that can handle spatial variability. Spatial distribution of recharge is seldom taken into account in groundwater simulations, although poorly parameterised recharge can increase significantly the uncertainty in modelling results and calibration. The GIS-based WetSpass methodology is a tool that can be used to simulate groundwater recharge and later the results can be used in groundwater flow modelling. The simulated groundwater flow characteristics can then be used by policy makers and water managers as a decision support system (DSS) for water resource management.

Uncertainty assessments in groundwater modelling applications typically attribute all sources of uncertainty to errors in parameters and inputs, neglecting what may be the primary source of uncertainty. However, simulations can be very sensitive to models, thus the uncertainty in projecting future groundwater recharge rates lie not only on climate change predictions, but also in characterisations of other recharge control parameters. In this research recharge was considered as uncertain parameter.

Future groundwater quantity predictions that are realised through deterministic approach are made with the assumption that the present situation plays a major role in determining the future. However, this supposition commonly proves invalid when applied to groundwater system because properties can be estimated only with a large degree of uncertainty. Hence, we should expect uncertainty in the values of the state variables predicted by the model. Thus, this study aimed at developing a model with simplified components which can be used in areas with minimal data hence minimum parameterisation considering recharge as uncertain input parameter.
CHAPTER 3 : METHODOLOGY

3.1 Introduction

This chapter discusses the materials and methods that were used for this study. The methodologies were designed in order to achieve the objectives of the study. The descriptions of these methods are split into four objectives;

The first objective dealt with estimation of groundwater recharge using GIS-based distributed hydrological model (WetSpass). To achieve this objective various primary and secondary data were collected. These include hydrogeology, climatology, digital elevation data, soil parameters and groundwater level data. Data input preparations are discussed in this chapter.

The second objective involved the development of a distributed groundwater flow model of the study area. The development of groundwater model was performed using Visual MODFLOW software. Conceptual and numerical models of groundwater flow were developed for aquifer of the study area. The model was later calibrated using the trial and error method together with automatic approach using PEST method. The descriptions of this objective method are presented herein.

The third objective entailed solving a groundwater flow problem through the use of stochastic methods. In this case, recharge was considered as stochastic. Groundwater recharge uncertainty was propagated into simulation model through the use of Monte Carlo approach. The method includes the generation of random numbers of recharge. One thousand realisations of recharge values were generated. Mean recharge of 50, 100, 300, 500, 800 and 1000 realisations were used in the calibrated model (MODFLOW) for simulation of heads and problem analysis.

The fourth objective entailed the design of groundwater management model (stochastic) which explicitly considers recharge as uncertain parameter. Further the research used groundwater sustainability indicator (GSI) as a management tool for the study area.
In this study various tools (software) were used during processing and manipulation of data which includes (i) ArcGIS for data preparation and analysis (ii) ERDAS used for Land use/cover classification (iii) MODFLOW used for groundwater flow simulation (iv) MATLAB used to generate random numbers of recharge.

3.2 Description of the study area

The study area is situated in central Limpopo, South Africa. The study area covers an area of about 19188 km². The choice of the study area was influenced by (i) the need for groundwater supply and (ii) the availability of data. The area falls under latitude -23 0 31.0956 S, 29 30 48.5697 E and 24 2 48.3007 S and 29 32 16.9088 E (Geographic (Lat/Lon WGS 84). The major portion of the study area is covered by mixed agriculture, built-up, bare soil and water bodies. The study area is shown in Figure 3.1.

![Study area location map](image)

**Figure 3.1:** Study area location map
3.2.1 Climate and rainfall

The climate of the study area is semi-arid subtropical. The rainfall data was obtained from South Africa weather services (SAWS). The summer's average temperature varies from 32.2°C to 27.3°C and winter temperature varies from 9.2°C to 4.3°C. The mean annual rainfall of the study area amounts to 453 mm as recorded at five stations found in the area. Annual rainfall varies from 475 mm in the south East to 205 mm in the Northern portions.

3.2.2 Surface hydrology

The main river found in the study area is Sand River. The river flows from south of Polokwane to the north through the Soutpansberg mountain range; inflowing into Limpopo River (DWAF, 2003). The main tributaries of Sand River are Hout and Brak rivers. Other rivers include Seepabana and Mogalakwena. The major dams found in the study area are summarised in Table 3.1 and Figure 3.2. Due to the seasonal nature of rainfall occurrence in the study area the river discharge is highly inconsistent hence the necessity to consider groundwater as an alternative source for both domestic and socio-economic development.

Table 3.1: Dams found in study area

<table>
<thead>
<tr>
<th>S/N</th>
<th>Dam_Name</th>
<th>Site_Name</th>
<th>Elevation</th>
<th>Surface Area (Ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glen Alpine Dam</td>
<td>Mogalakwena</td>
<td>1100</td>
<td>469.8</td>
</tr>
<tr>
<td>2</td>
<td>Gert Combrink Dam</td>
<td>Dorps River</td>
<td>1161</td>
<td>20.7</td>
</tr>
<tr>
<td>3</td>
<td>Chuniespoort Dam</td>
<td>Chunie River</td>
<td>1145</td>
<td>61.2</td>
</tr>
<tr>
<td>4</td>
<td>Molepo Dam</td>
<td>Tlhabasane</td>
<td>724</td>
<td>100.0</td>
</tr>
</tbody>
</table>
3.2.3 Geology and Lithology of the study area

South Africa digital Geological maps were obtained from Council of Geosciences. Figure 3.3 shows the geological structures found in the study area. Geological structures (faults, fractures and lithologic contacts) contribute greatly in the movement and occurrence of groundwater.

3.2.4 Aquifer Characteristics

The study area is characterized by major and minor aquifers as described by Parsons (1995). Major aquifers cover about 50.6 % of the study area which is a good indication that water can be abstracted in large quantity for public supplies. On the other hand, 49.6 % of study area is covered by minor aquifers. These aquifers do not have high
permeability although water can be abstracted in limited amounts. Aquifer class types are shown in Figure 3.4.
The main aquifers in the study area are considered to be fractured and weathered aquifers in the Alma formation rocks and along the sill and dyke contact zones. It is important to note that the availability of dykes in area act as barrier for groundwater abstraction. Dykes diabase and dyke dolerite are found in the study area.

3.3 Estimation of groundwater recharge

The first specific objective of the study was to estimate groundwater recharge using GIS-based distributed hydrologic model (WetSpass). During the process, several steps were undertaken for preparation of input data for WetSpass model which includes (i) Preparation of meteorological input maps like; rainfall, temperature, evapotranspiration and wind speed (ii) preparation of grids maps; soil, elevation, slope, groundwater depth (iii) Preparation of Land use/cover grid map. All these datasets were incorporated in WetSpass model for estimation of groundwater recharge.

Figure 3.5: Integration of input data in the WetSpass model (Batelaan & De Smedt, 2001)
WetSpass simulation was divided into two seasons; summer (December to February) and winter (June to August). All climatic variables, were classified for the two-seasons input for the case of summer and winter, while for annual recharge estimation, the variables were prepared annually. All the maps were prepared using ArcGIS 10.4.1 and saved as ASCII files for seasons (summer and winter) and annual. WetSpass required all the maps to have the same number of columns and rows, number of bands, cell size, extent and spatial reference as shown in Table 3.2.

### Table 3.2: WetSpass grid maps resolutions

<table>
<thead>
<tr>
<th>Raster information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns and rows</td>
</tr>
<tr>
<td>Cell size (X, Y)</td>
</tr>
<tr>
<td>Extent</td>
</tr>
<tr>
<td>Top</td>
</tr>
<tr>
<td>Left</td>
</tr>
<tr>
<td>Right</td>
</tr>
<tr>
<td>Bottom</td>
</tr>
<tr>
<td>Spatial Reference</td>
</tr>
<tr>
<td>Angular Unit</td>
</tr>
<tr>
<td>Datum</td>
</tr>
</tbody>
</table>

### 3.3.1 Preparation of meteorological input maps

Climatic variables inclusive of maximum and minimum temperature, rainfall, wind speed and humidity were acquired from South African Weather Service (SAWS) database. The study used a long-time series of data from 2000 until 2015. Meteorological stations maintained by DWA and University of Limpopo were used for rainfall monitoring.
purposes at different locations of the study area and for this reason were used to supplement the data obtained from SAWS. Based on spatial distribution of meteorological stations (Figure 3.6) and availability of full dataset in relation to number of years selected, five weather stations were selected to provide the meteorological data. Since the data obtained were in monthly values, manipulation of data was done through excel to get average data for each year. These included meteorological data for rainfall, temperature, evapotranspiration and wind speed.

![Spatial distribution of meteorological station in the study area](image)

**Figure 3.6:** Spatial distribution of meteorological station in the study area

After manipulation of data through excel, the data for each station was imported in ArcGIS 10.4a. All the data were merged to create one rainfall map of the study area. In order to have the value of data at each point in the study area, the maps were interpolated using Inverse distance weighting (IDW) technique. Grid maps were
prepared for summer and winter season as well as annually. Thereafter the grid maps were converted to ASCII files for use in WetSpas model.

### 3.3.2 Preparation of grid maps

#### Soil

The soil physico-chemical and hydrological properties data for South Africa was obtained from Food and Agriculture Organization - UNESCO database (FAO, 2002; 2005) soil map. The shapefile soil database contains attributes like (i) soil type (ii) soil texture (iii) Hydrological Soil Group (HSG) and (iv) soil polygon area. FAO soil databases are considered as the most comprehensive available resource with enormous database on soil attributes for Africa and the world (Batjes, 2002). By using ArcGIS 10.4.1, the subset of soil map of the study area was extracted from FAO soil map using the coordinates of the boundary and extent of the study area. The attributes of the soils (soil texture, Hydrological Soil Group (HSG), soil type and WetSpas serial number) were captured into a database using ArcGIS (Figure 3.7) which provided a link for use in WetSpas (Table 3.3). WetSpas serial number was the crucial value fields that ensured the proper recall of gridded soil map into WetSpas model. Afterward the soil grid map was converted to ASCII files and used in WetSpas model.

**Table 3.3:** Properties of major soils in the study area

<table>
<thead>
<tr>
<th>SOIL_TYPE</th>
<th>FAO code</th>
<th>Hydrological group</th>
<th>WetSpas Serial No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy Clay Loam</td>
<td>Lk4-2ab</td>
<td>C</td>
<td>7</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>Qc42-1a</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>Sandy Loam</td>
<td>Lc65-1/2ab</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>Sandy Clay Loam</td>
<td>Lc3-2ab</td>
<td>C</td>
<td>7</td>
</tr>
<tr>
<td>Sandy Clay Loam</td>
<td>Lc3-2ab</td>
<td>C</td>
<td>7</td>
</tr>
<tr>
<td>Sandy Loam</td>
<td>Lc65-1/2ab</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>Clay</td>
<td>Qc42-1a</td>
<td>C</td>
<td>12</td>
</tr>
<tr>
<td>Clay</td>
<td>Vc23-3a</td>
<td>D</td>
<td>12</td>
</tr>
</tbody>
</table>
Figure 3.7: Distribution of soil types in the study area

Digital Elevation Map (DEM) and slope

Another grid map required as input in WetSpass model is elevation map. Digital Elevation Model (DEM) dataset of South Africa was obtained from Department of Geoscience, Pretoria. The dataset acquired had a resolution of 30 m. The DEM was processed to prepare topographic and slope maps of the study area. The subset elevation map of the study area was clipped from South Africa DEM dataset using clip function in raster processing tool of ArcGIS. The map was then resampled in order to have the required format as indicated in Table 3.2. A slope grid map was generated from elevation map through surface function in spatial analyst tool by using ArcGIS. Both the grid maps were converted to ASCII files and used in WetSpass model.
Groundwater level

Borehole data was obtained from different sources including; Department of water affairs (DWA) (currently known as Department of water and Sanitation), National Groundwater Archive (NGA), VSA Leboa Consulting (PTY) Limited, Limpopo and GRIP Limpopo. Boreholes database included information like water level, water strike level, yield, well identification number, longitude and latitude. Although Limpopo have huge database of boreholes, most of the boreholes lack information required for this study. Therefore, thorough scrutinization of data was done using Excel and later on the manipulated data was imported into ArcGIS. The demarcation of boreholes falling within the study area was done using clipping in Geoprocessing tool of ArcGIS. The groundwater level grid map was then converted to ASCII format and used as grid map input in WetSpass model.

3.3.3 Preparation of Land use/cover grid map for WetSpass model

Land use/cover mapping was developed through two major steps (i) the land use/cover supervised classification and (ii) accuracy assessment through utilization of ground control points in ERDAS IMAGINE (image processing software). The main goal was to obtain the classified land use/cover map of the study area which has minimum error of accuracy and acceptable range for WetSpass model. Later on, the classified map was used as input in WetSpass simulation model for groundwater recharge estimation. Figure 3.8 shows the systematic workflow for land use/cover and accuracy assessment.

Image pre-processing

The classification and analysis of the various LULC classes were done using two Landsat satellite images covering the Landsat 8 OLI/TIRS acquired on 16 September 2015. These images includes; L8 OLI/TIRS (path 170, rows) and L8 OLI/TIRS (path 170, rows 77). The Landsat images were downloaded from United States Geological (USGS) Earth Explorer (https://earthexplorer.usgs.gov/). The choice for the selection of the dates was influenced by the image quality in terms of those with limited or low cloud
cover. Each Landsat was georeferenced to the WGS_84 datum and Universal Transverse Mercator Zone 35 South coordinate system.

An intensive pre-processing including geo-referencing, mosaic, and layer-stacking were carried out in order to Ortho-rectify the satellite images. The image was then processed in ERDAS IMAGINE 2015 software. The satellite image of each band was stacked in ERDAS Hexagon within interpreter main icon utilities with layer stacked function. Then,

**Figure 3.8**: Layout of accuracy assessment approach for LULC
from the stacked satellite image the study area image was extracted by clipping the study area using ArcGIS 10.3 software.

For this study, only supervised classification was performed. Supervised classification according to Eastman (2003) is where “the user develops the spectral signatures of known categories, such as urban and forest, and then the software assigns each pixel in the image to the cover type to which its signature is most comparable”. “Supervised classification is the process most frequently used for quantitative analyses of remote sensing image data” (Richard & Xiuping, 2006). The supervised classification was applied after defined area of interest (AOI) which is called training classes. More than one training area was used to represent a particular class. The training sites were selected in agreement with the Landsat Image, Google Earth and Google map (Rwanga & Ndambuki, 2017). Further, the selection of the training sites was based on those areas clearly identified in all sources of images. After the entire signature was created, the signature (SIG) file was saved as dialog. Thereafter the classification process was done on ERDAS classification tool.

Classification Accuracy assessment

Accuracy assessment is an important final step of the classification process. The aim was to quantitatively determine how effectively pixels were grouped into the correct land cover classes (Pravada, & Rahila, 2013.). The primary emphasis for accuracy assessment pixel selection was on areas that could be clearly identified on Landsat high resolution image, Google earth and Google Map. A total of 550 points (locations) were randomly created in the classified image of the study area. The Accuracy Assessment Cell Array Reference column was filled according to the best guess of each reference point. Hydrogeological map series of the republic of South Africa, topographic map, Google earth and Google Map were used as reference source to classify the selected points. In order to estimate the classification accuracy and Kappa coefficient, various statistics were computed based on Jenness & Wynne (2007) equations as follows;

\[
Sensitivity = \frac{a}{a + c} \text{ (equivalent to Producer's Accuracy)} \quad (3.1)
\]
Where:

\[ \text{Positive Predictive Power} = \frac{a}{a + b} \] (Equivalent to User's accuracy) \hspace{1cm} (3.2)

\[ \text{Negative Predictive Power} = \frac{d}{c + d} \] \hspace{1cm} (3.3)

\[ \text{Specificity} = \frac{d}{b + d} \] \hspace{1cm} (3.4)

\[ \text{Commision error} = 1 - \text{Specificity} \] \hspace{1cm} (3.5)

\[ \text{Ommision error} = 1 - \text{Sensitivity} \] \hspace{1cm} (3.6)

Where:

\( a \) = number of times a classification agreed with the observed value
\( b \) = number of times a point was classified as X when it was observed to not be X
\( c \) = number of times a point was not classified as X when it was observed to be X.
\( d \) = number of times a point was not classified as X when it was not observed to be X

Total points = \( N = (a + b + c + d) \)

In order to measure the accuracy of the assessment done on classification of landuse map, the Kappa statistic (K) was computed as:

\[ K = \frac{N \sum_{i=1}^{r} x_{ii} - \sum_{i=1}^{r} (x_{i+} \times x_{+i})}{N^2 - \sum_{i=1}^{r} (x_{i+} \times x_{+i})} \] \hspace{1cm} (3.7)

Where:

\( r \) = number of rows and columns in error matrix,
\( N \) = total number of observations (pixels)
\( x_{ii} \) = observation in row i and column i,
\( x_{i+} \) = marginal total of row i, and \( x_{+i} \) = marginal total of column i
A Kappa coefficient equal to 1 means perfect agreement where as a value close to zero means that the agreement is no better than would be expected by chance.

3.4 Modification of Lookup Parameters tables for WetSpass model

The WetSpass model uses four parameters tables that are linked through their attributes. However, since this model differs from place to place in terms of climatic conditions, it was necessary to modify the parameter tables to suit conditions of semi-arid environment of the study area. The modified parameters were adopted from (Willis, 2002; and Bashir & Batelaan, 2011). The season variations were adjusted to accommodate the wet and dry months. Hence the months falling under dry season include; April to October (opted as winter in a model) while wet season was from November to March (opted as summer in a model). In this respect winter is the dry season while summer is the main rainy season in the study area. The modifications of the parameter tables are presented in appendices 1 to 3. The significance of these tables was to allow easy definition of new land-use and soil type as well as changes to each parameter value.

3.5 WetSpass water Balance Components Estimation

The total water balance of a specified area was calculated as the summation of the water balances of each raster cell (Batelaan & Woldeamlak, 2007). Every land-cover class was characterized by four types of fractions: bare soil, impervious surface, vegetation and open water. This information was stored in the land-use parameter tables. The components of the water balance within the study area were estimated using the water balance equations;

\[
ET_{\text{raster}} = avET_v + asE_s + aoE_o + aiE_i
\]
\[
S_{\text{raster}} = avS_v + asS_s + aoS_o + aiS_i
\]
\[
R_{\text{raster}} = avR_v + asR_s + aoR_o + aiR_i
\]
Where $ET_{raster}$, $S_{raster}$, $R_{raster}$, are the total evapotranspiration, surface runoff, and groundwater recharge of a raster cell, respectively, each having a vegetated, bare-soil, Open-water and impervious area component denoted by $a_v, a_s, a_o,$ and $a_i$, respectively. Precipitation was taken as the starting point for the computation of the water balance of each of the above-mentioned components of a raster cell.

The water balance for a vegetated area depends on the average seasonal precipitation ($P$), interception fraction ($I$), surface runoff ($S_v$), actual transpiration ($T_v$), and groundwater recharge ($R_v$);

\[ P = I + S_v + T_v + R_v \] \hspace{1cm} \text{(3.11)}

Surface runoff was calculated in relation to precipitation amount, precipitation intensity, interception and soil infiltration capacity. In considering water losses through evaporation and transpiration, the calculation of seasonal evapotranspiration and a reference value of transpiration were obtained from open-water evaporation and a vegetation coefficient. Recharge was the major and last component to be simulated in WetSpass model. The groundwater recharge was calculated as a residual term of the water balance, i.e,

\[ R_v = P - S_v - ET_v - E_s - I \] \hspace{1cm} \text{(3.12)}

hereby $ET_v$ is the actual evapotranspiration given as the sum of transpiration $T_v$ and $E_s$ (the evaporation from bare soil found in between the vegetation). The spatially distributed recharge was therefore estimated from the vegetation type, soil type, slope, groundwater depth, and climatic variables of precipitation, potential evapotranspiration, temperature, and wind-speed.

After all the inputs data and lookup table were prepared, the information was uploaded to the relevant WetSpass model after which the model run was initiated. After every run
the Model indicated the completion of each water balance component simulated until the last component (i.e. recharge). The output files were automatically saved in output folder.

3.6 Development of a distributed groundwater flow model

The second objective entailed the development of a distributed groundwater flow model for the study area. The development of this model was done through Visual MODFLOW 2000 software. The development of the model consisted of a set of possible assumptions that simplify the real problem and come up with a conceptual model that was suitable in view of the objective of the modelling.

For modelling purpose, the weathered and fractured dolerite and the limestone units were combined to form a single aquifer system. In order to accommodate the complexity of the real hydrogeological system; some basic assumptions about the study area were made. The following assumptions about the modelled area were made (i) the system was considered to be in a steady-state throughout the year (ii) The geological formations of concern were considered horizontal in extent. The development of a distributed numerical groundwater flow model for the study area comprised of three steps; (i) construction of numerical model (ii) model calibration (iii) model validation. To achieve this objective, three software were used (a) ArcGIS 10.4 (b) Excel and (c) Visual MODFLOW 2000. A flow chart of inputs and methodology followed is shown in Figure 3.9.

Construction of numerical model

Construction of steady – state, numerical model was done using MODFLOW – 2000 (Harbaugh et al., 2000). The model was discretized into 117 columns and 164 rows resulting in 19188 active cells. The grid cell size was 1000 m in both the x and y-directions. Due to insufficient log data, a single model of confined aquifer layer 60 m thickness (obtained from GRIP dataset) was used to simulate flow and mass balance of the study area. The modelled domain covered an area of 19188 km$^2$ which was projected with UTM projection system.
Elevation data was imported in the model domain, the elevation was gathered from elevation map processed in ArcGIS. After importing the elevation, the model executed interpolation of the imported data through Neighbouring Interpolation technique. Construction of the model in MODFLOW includes three input packages (i) Wells (ii) Model properties and (ii) model boundary conditions.
Wells

Boreholes data were obtained from GRIP Limpopo and National Groundwater Archive (NGA). In this process two types of wells data were prepared; (i) pumping wells and (ii) observation wells.

Pumping wells

The pumping wells data were used to generate water levels during aquifer pumping as well as drawdowns at each well through the groundwater sustainability index (GSI). A total of 203 boreholes were captured on the model. Data processing was done through ArcGIS using extraction and clipping on Geoprocessing tool and later imported in MODFLOW through import tool.

Observations wells

Observation wells were added in the model for the purpose of model calibration. 30 observation wells were used for this task. The choice of observation wells was based on (i) recent data availability (ii) completeness of the data and (iii) spatial distribution of the wells. Observation wells were imported in MODFLOW through import tool.

Model Properties

A groundwater flow model requires many different types of data to simulate the hydrogeological processes influencing the flow of groundwater. In MODFLOW the hydrogeological characteristics of the model are classified into inputs such as flow properties; hydraulic conductivity ($K_y$, $K_x$ and $K_z$) and storage ($S_s$, $S_y$, $P_l$, $P_{tot}$). The inputs for model properties include aquifer parameters and initial heads.

Aquifer parameters

The aquifer parameters (transmissivity, hydraulic conductivity and storage coefficient) data was obtained from log test data. Since the groundwater flow model was single layered; only horizontal hydraulic conductivities were significant. The model domain was
divided into five hydraulic conductivity zones based on their geological formation and point hydraulic conductivities. The aquifer hydraulic data were mapped through ArcGIS and imported to MODFLOW.

**Initial heads**

The data gathered had information on static water level, thus for this study, static water levels records of the wells were used as initial heads. The initial heads were interpolated within the model to obtain the initial heads for the entire model. The observation heads were interpolated using the nearest neighbour technique.

**Boundary conditions**

Three types of boundary conditions were incorporated in this study which include; constant head, river and recharge.

**Constant head boundaries**

The constant head in this study was used to fix the head values thus acting as an infinite source of water entering the system, or as infinite sink for water leaving the system. Dirichlet boundary conditions (fixed boundary condition) of the study area were used. The constant head was allocated within a minimum distance of 2 km from pumping to boundary condition.

**River**

The river boundary condition was used to simulate the influence of water body on the groundwater flow. MODFLOW simulated the interaction between the surface and groundwater via seepage layer separating the surface water from groundwater system. Four rivers were included in the model; Sandriver, Houriver, Seepabana and Brak rivers. All rivers allocated within the model domain were imported from ArcGIS as shapefile. Stage elevation, river depth and thickness data were obtained from DWA through Georeguest service. River bottom elevations were obtained through deduction of river depth from stage elevation. The conductance was automatically calculated in MODFLOW using Equation (3.13).
\[ C = \frac{K \cdot L \cdot W}{M} \] (3.13)

*Where*: \( C \) is Conductance value, \( L \) is length of a reach through a cell, \( W \) is Width of the river in the cell, \( M \) is thickness of the riverbed, \( K \) is the vertical hydraulic conductivity of the riverbed material.

**Recharge**

Another input to MODFLOW is recharge. In this study, recharge was computed using WetSpass as explained in sub-section 3.5. The estimated recharge polygon shapefile map was imported in MODFLOW under boundaries.

**3.7 Model Calibration**

A total of 30 observation wells from GRIP database were selected for the calibration of steady state model. In this study, the hydraulic conductivity was used as parameter of interest for calibration. Five zones of hydraulic conductivities were used during automatic calibration process.

Initially, PEST executed MODFLOW with the initial hydraulic conductivities values of five zones (Figure 3.10) and output of MODFLOW were compared with observed data. The evaluation of calibration was done by comparing predicted and observed heads through scatter plot and by applying the three common ways of error quantifying methods which are (i) Mean Error (ME) (ii) Mean absolute error (MAE) and (iii) Root Mean Square Error (RMSE) adopted from Anderson & Woessner (1992).

**3.8 Model Validation**

The model was validated using 30 boreholes. In order to validate the model, dataset of 2011 boreholes was selected (appendix 4). This is because most of the data needed for the model was available in this year. The preparation of validation wells was done using Excel and ArcGIS and later on imported to MODFLOW using Well import tool. The
model performance statistics were evaluated using ME, MAE, RMSE and Normalised Root Square Mean (NRMS).

![Hydraulicconductivitieszones](image)

**Figure 3.10: Hydraulic conductivities zones**

### 3.9 Development of a stochastic solution methodology

The third objective entailed the development of a stochastic solution methodology to solve a groundwater flow problem through simulation considering recharge as uncertain. This study considered recharge as the only uncertainty parameter in the groundwater flow model (equation 2.7). Random numbers were generated using Pseudo-random number generators. Mean recharge and standard deviation estimated from WetSpass were used to generate random numbers using MATLAB code (appendix 5) that was written to solve stochastic groundwater flow model (see equation 2.7). These realisations were then used to compute expected mean which was then used as
input into MODFLOW (deterministic model). A Monte Carlo (MC) technique was used to solve stochastic groundwater problem. The flow chart of stochastic solution methodology is shown in Figure 3.11.

**Figure 3.11**: Flow chart for stochastic solution methodology
For purposes of clarity on how uncertainty was evaluated in the simulation model, (see equation 2.6 developed in sub section 2.6.1), assume recharge is a random variable denoted by μ, hence, by considering the source/sink term in equation (2.6) as being stochastic, equation (2.6) can be transformed into equation (2.7) which becomes stochastic because its solution depends on the outcomes of realisations of the random values of recharge (This study dealt with two dimensional therefore the $K_z$ dimension falls off). Thus, a large number of realisations of recharge were generated and Monte Carlo (MC) approach used to solve the problem using methodological steps shown in Figure 3.11.

3.10 Development of Groundwater Management Model

The fourth objective entailed the design of groundwater management model that explicitly considers recharge as uncertain input parameter. To achieve this objective, the Groundwater Sustainability Index (GSI) was used as a tool to develop a management model. The GSI was calculated by using the total groundwater abstraction and groundwater recharge as shown in equation (2.11). The amount of groundwater was obtained by summing all the pumping rates of a particular cell. Recharge values were calculated from mass balance results for each cell. The GSI was then calculated using equation (2.11).
CHAPTER 4 : RESULTS AND DISCUSSIONS

In Chapters one, two and three, the objectives, concepts and methodological structure of this study were presented and discussed. The overall study effort was devoted towards the development of a quasi-three-dimensional distributed flow management model for groundwater taking into account uncertainty due to recharge. This chapter presents and discusses results of the study.

4.1 Study area characteristics

Rainfall

The results of analysis performed for minimum, maximum and mean annual rainfall in the study area for each station is shown in Table 4.1. The average rainfall varies from 11.5 mm to 56.8 mm in winter (Figure 4.1), and 93.8 to 418.9 mm in summer (Figure 4.2). Moreover, average annual rainfall varies from 105.3 mm to 475.8 mm (Figure 4.3). This indicates that even though there was little rainfall in winter, about 56.9 mm of rainfall accumulated on annual basis.

![Rainfall Distribution](image)

Figure 4.1: Distribution of winter rainfall pattern for five stations in study area
**Table 4.1:** Rainfall (mm) data for five stations of the study area

<table>
<thead>
<tr>
<th>Year</th>
<th>Station 1</th>
<th>Station 2</th>
<th>Station 3</th>
<th>Station 4</th>
<th>Station 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>644.8</td>
<td>682.5</td>
<td>65.55</td>
<td>81.28</td>
<td>54.83</td>
</tr>
<tr>
<td>2001</td>
<td>497.4</td>
<td>502.4</td>
<td>97.7</td>
<td>88</td>
<td>107.1</td>
</tr>
<tr>
<td>2002</td>
<td>257.3</td>
<td>301.2</td>
<td>109.1</td>
<td>167.1</td>
<td>200.9</td>
</tr>
<tr>
<td>2003</td>
<td>302.1</td>
<td>301</td>
<td>73.9</td>
<td>86</td>
<td>65.2</td>
</tr>
<tr>
<td>2004</td>
<td>478.9</td>
<td>398.5</td>
<td>88.4</td>
<td>93.2</td>
<td>96.6</td>
</tr>
<tr>
<td>2005</td>
<td>472.4</td>
<td>475</td>
<td>32.3</td>
<td>72.3</td>
<td>47.4</td>
</tr>
<tr>
<td>2006</td>
<td>677.7</td>
<td>621</td>
<td>137.3</td>
<td>101</td>
<td>88.3</td>
</tr>
<tr>
<td>2007</td>
<td>422.9</td>
<td>435.7</td>
<td>91.4</td>
<td>57.1</td>
<td>87.9</td>
</tr>
<tr>
<td>2008</td>
<td>386</td>
<td>370</td>
<td>113.6</td>
<td>136</td>
<td>97.2</td>
</tr>
<tr>
<td>2009</td>
<td>541</td>
<td>442</td>
<td>73.7</td>
<td>104.7</td>
<td>85.3</td>
</tr>
<tr>
<td>2010</td>
<td>469.2</td>
<td>593.3</td>
<td>112</td>
<td>90</td>
<td>115.6</td>
</tr>
<tr>
<td>2011</td>
<td>541.7</td>
<td>425</td>
<td>98.4</td>
<td>109.3</td>
<td>129.4</td>
</tr>
<tr>
<td>2012</td>
<td>419.4</td>
<td>226.3</td>
<td>101.1</td>
<td>135.4</td>
<td>106.6</td>
</tr>
<tr>
<td>2013</td>
<td>636.3</td>
<td>793.5</td>
<td>101.3</td>
<td>109.6</td>
<td>102.8</td>
</tr>
<tr>
<td>2014</td>
<td>456.5</td>
<td>355.4</td>
<td>159.8</td>
<td>144.5</td>
<td>140.1</td>
</tr>
<tr>
<td>2015</td>
<td>408.4</td>
<td>344.2</td>
<td>106.9</td>
<td>110.7</td>
<td>132.2</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>475.8</td>
<td>454.2</td>
<td>97.7</td>
<td>105.4</td>
<td>103.6</td>
</tr>
<tr>
<td><strong>Max.</strong></td>
<td>677.7</td>
<td>793.5</td>
<td>159.8</td>
<td>167.1</td>
<td>200.9</td>
</tr>
<tr>
<td><strong>Min.</strong></td>
<td>257.3</td>
<td>226.3</td>
<td>32.3</td>
<td>57.1</td>
<td>47.4</td>
</tr>
</tbody>
</table>

**Figure 4.2:** Distribution of summer rainfall pattern for five stations in study area
Aquifer Characteristics

Log test data acquired from GRIP Limpopo and VSA Leboa Limpopo indicated that the area is generally under regional confined aquifer. Under natural conditions, groundwater drains through localised springs via fractures to the perennial tributaries from the Soutpnesberg rocks found in study area.

Geology and Lithology of the study area

The top six lithostrata of the study area are provided in Table 4.2. Basic volcanic rocks are mostly found in south north part of the study area, while the central part is mostly covered by Goudplaats gneiss. Sandstone and Mudstone is mostly found in south west of the study area. Geological structures like faults, fractures and lithologic contacts are also found in the study area and contribute greatly in the movement and occurrence of groundwater (Figure 3.2).
Table 4.2: Dominant Geology and Lithology types found in the study area

<table>
<thead>
<tr>
<th>Lithology_Type</th>
<th>Area (Km²)</th>
<th>% Covered by area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Goudplaats-hout river gneiss</td>
<td>12088.2</td>
<td>48.7</td>
</tr>
<tr>
<td>2 Letaba</td>
<td>2705.5</td>
<td>10.9</td>
</tr>
<tr>
<td>3 Matlabas</td>
<td>1635.3</td>
<td>6.6</td>
</tr>
<tr>
<td>4 Wyllie’s poort</td>
<td>1087.7</td>
<td>4.4</td>
</tr>
<tr>
<td>5 Pietersburg</td>
<td>913</td>
<td>3.7</td>
</tr>
<tr>
<td>6 Malala drift gneiss</td>
<td>866.9</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Groundwater levels

Groundwater level data was obtained from GRIP Limpopo database and 203 boreholes falling within the study area. Through analysis and site visit, not all the boreholes were found to be operational due to: (i) boreholes running dry (ii) pumps getting stolen (iii) poor water quality and (v) pumps breaking down and not getting repaired. Moreover, analysis showed that groundwater table varied spatially and temporally with water level rising following rainy season. The groundwater depth ranged from 23 to 291 m. Figure 4.4 shows a sample representative drill log of the study area. The well log indicates that the area has multiple confined aquifers ranging from a depth of 24 m to a depth of about 96 m while their thickness ranges from 20 m to 120 m.
4.2 Estimation of groundwater recharge

The first objective was to estimate groundwater recharge using GIS-based distributed hydrologic model (WetSpass). WetSpass model was used to estimate the long-term average spatially varying water balance components. The components estimated were groundwater recharge, surface runoff, evapotranspiration, interception, transpiration and soil evaporation. However, in this research, the key objective of using WetSpass model was to estimate groundwater recharge. The estimated recharge was then used as input into groundwater flow model. Several steps were undertaken in the preparation of input
data for WetSpass model. The results of (i) meteorological input data (ii) grid maps (iii) Land use/cover grid map are discussed herein.

4.2.1 Meteorological input data

Rainfall and temperature

Rainfall and temperature pattern for summer, winter and annual distribution for five stations in the study area were used as input in WetSpass model as presented in sections 3.2.1 and 4.1.

![Temperature_Summer map](image-url)

**Figure 4.5:** Summer temperature (°C) map
4.2.2 Grids maps

Digital Elevation Map (DEM) and slope

This map was produced based on DEM model obtained from Geoscience office. The elevation in the study area decreases in a South to North and East to West direction (Figure 4.7). The minimum elevation is 673 m above mean sea level (a.m.s.l) and the maximum elevation is 1922 m a.m.s.l at South part of the study area.

The slope grid map of the study area was extracted from digital elevation model using ArcGIS spatial analyst tool. The slope of the area obtained dictates the actual flow of surface runoff, and hence recharge in the area. The steeper the slope the greater the velocity of flow, and hence the lesser the recharge. The study area falls under flat and
gently sloping surface, thus moderating the flow velocity and hence encouraging recharge.

Figure 4.7: Elevation map

Soil Map

The soil grid map was created from Food and Agriculture Organization (FAO) soil map. The study area is composed of clay, sandy clay loam and sandy loam soils as shown in Figure 4.8. The percentage of the type of soil from the study area was 0.07% clay, 79.7% sandy loam and 20.23% of sandy clay loam. As shown in Table 4.3, most parts of the study area are covered by sandy loam (19785.2 km²) followed by sandy clay loam (5030.6 km²) and clay (18.2 km²). This implies that the soil in the study area promotes recharge.
Table 4.3: Classified area under different Land use classes in study area

<table>
<thead>
<tr>
<th>WetSpass Code</th>
<th>Soil texture</th>
<th>Percentage covered %</th>
<th>Area covered, km²</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Sandy Clay Loam</td>
<td>20.3</td>
<td>5030.6</td>
</tr>
<tr>
<td>3</td>
<td>Sandy Loam</td>
<td>79.7</td>
<td>19785.2</td>
</tr>
<tr>
<td>12</td>
<td>Clay</td>
<td>0.1</td>
<td>18.2</td>
</tr>
</tbody>
</table>

Figure 4.8: Soil texture map

4.2.3 Land use/cover grid map

The land use/landcover (LULC) of the area provides essential indications of the extent of groundwater requirements and utilization. Groundwater recharge strongly depends on landuse/cover of the study area. Land use/cover mapping was developed through
two major steps: (i) land use/cover supervised classification and (ii) accuracy assessment through utilization of ground control points in ERDAS IMAGINE (image processing software). The main goal of this exercise was to obtain the classified land use/cover map of the study area which has minimum error of accuracy and acceptable range for further analysis.

**Land use/cover classification**

Supervised classification of land use/cover of the study area was carried out. The area of each class was calculated taking into account the pixel count and total study area. Results of the allocations of each classified area (percentage) are shown in Table 4.4 while their distribution is shown in Figure 4.9.

<table>
<thead>
<tr>
<th>WetSpass code</th>
<th>Landuse/cover Classified</th>
<th>Area in km²</th>
<th>Percentage (%) area</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Build up</td>
<td>2963.9</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>Excavation_mining</td>
<td>103.75</td>
<td>0.4</td>
</tr>
<tr>
<td>6</td>
<td>Airport</td>
<td>1431.1</td>
<td>5.8</td>
</tr>
<tr>
<td>7</td>
<td>Bare land</td>
<td>5151.1</td>
<td>31.4</td>
</tr>
<tr>
<td>21</td>
<td>Agricultures</td>
<td>3262.8</td>
<td>21.2</td>
</tr>
<tr>
<td>33</td>
<td>Mixed forest</td>
<td>696.83</td>
<td>2.8</td>
</tr>
<tr>
<td>36</td>
<td>Shrubs</td>
<td>3675.5</td>
<td>14.8</td>
</tr>
<tr>
<td>51</td>
<td>Dam</td>
<td>53.21</td>
<td>0.2</td>
</tr>
<tr>
<td>55</td>
<td>River</td>
<td>377.27</td>
<td>1.5</td>
</tr>
<tr>
<td>201</td>
<td>Highway</td>
<td>521.76</td>
<td>2.1</td>
</tr>
<tr>
<td>202</td>
<td>District road</td>
<td>950.99</td>
<td>3.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>19188</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>
Bare land was found to be the dominant type of land use classified which covers about 31.4% of the total study area, followed by build-up areas with 16.0% while the least classified was dams which accounts for 0.2%. During the classification, among the water body classified were rivers (sand river, brak, seepebana and Houriver) and dams (Seshego, Chuenespoort, Nkumidamme and Houriver dam). Due to high percentage of agriculture land classified (21.2%), it was assumed that infiltration percolates the aquifers through irrigation hence increases water storage in aquifers. Moreover, the presence of water bodies in the study area could lead to loss of water through evaporation and discharge to aquifers.
Classification Accuracy Analysis

Another important step of the classification process is accuracy assessment. The aim of accuracy assessment is to quantitatively assess how effectively the pixels were sampled into the correct land cover classes. The key emphasis for accuracy assessment pixel selection was on areas that could be clearly identified on Landsat high resolution image, Google earth and Google Map. Classification of accuracy analysis for land use/cover classified image was done using error matrix. The results obtained are presented in Table 4.5.

Table 4.5: Theoretical error matrix of LULC classification

<table>
<thead>
<tr>
<th>S.No</th>
<th>Classified</th>
<th>Build up</th>
<th>Excavation - mining</th>
<th>Airport</th>
<th>Bare Land</th>
<th>Agriculture</th>
<th>Mixed forest</th>
<th>shrubs</th>
<th>River</th>
<th>Highway</th>
<th>District road</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Build up</td>
<td>84</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Excavation - mining</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Airport</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>17</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Bare Land</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>184</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Agriculture</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>93</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Mixed forest</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>shrubs</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>83</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>River</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>Highway</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>District road</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4.5 shows a theoretical confusion matrix (error matrix) of a LULC classification. The columns of the confusion matrix show to which classes the pixels in the validation set belong (ground truth) and the rows show to which classes the image pixels have been assigned in the image. The diagonal (gray in colour) shows the pixels that are
classified correctly. Pixels that are not assigned to the proper class do not occur in the diagonal and give an indication of the confusion between the different land-cover classes in the class assignment. The results for user’s accuracy, producer’s accuracy and overall accuracy are presented in Table 4.6. The measure of producer’s accuracy (sensitivity, i.e., equation 3.1) reflects the accuracy of prediction of the particular category.

**Table 4.6: Summary of overall classification accuracy and kappa coefficient**

<table>
<thead>
<tr>
<th>Classified</th>
<th>Totals</th>
<th>Numbers correct</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>Build up</td>
<td>100</td>
<td>84</td>
<td>0.8074</td>
</tr>
<tr>
<td>Excavation</td>
<td>3</td>
<td>2</td>
<td>0.6655</td>
</tr>
<tr>
<td>mining</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Airport</td>
<td>34</td>
<td>5</td>
<td>0.1392</td>
</tr>
<tr>
<td>Bare Land</td>
<td>186</td>
<td>184</td>
<td>0.9496</td>
</tr>
<tr>
<td>Agriculture</td>
<td>95</td>
<td>93</td>
<td>0.9731</td>
</tr>
<tr>
<td>Mixed forest</td>
<td>18</td>
<td>17</td>
<td>0.9420</td>
</tr>
<tr>
<td>Shrubs</td>
<td>90</td>
<td>82</td>
<td>0.9028</td>
</tr>
<tr>
<td>River</td>
<td>4</td>
<td>2</td>
<td>0.4917</td>
</tr>
<tr>
<td>Highway</td>
<td>6</td>
<td>1</td>
<td>0.1651</td>
</tr>
<tr>
<td>District road</td>
<td>14</td>
<td>5</td>
<td>0.3477</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>550</td>
<td>476</td>
<td>0.8183</td>
</tr>
</tbody>
</table>

Overall accuracy = 86.5 %
Overall Kappa = 0.82

*K=Kappa statistics; PA=Producer accuracy; UA=User accuracy

The User’s accuracy (equation 3.2) reflects the reliability of the classification to the user. User’s accuracy is a more relevant measure of the classification’s actual utility in the field. For this exercise agriculture was found to be more reliable with 97.89% of user’s accuracy (Table 4.7). Moreover, overall classification accuracy (Table 4.6) was
calculated as 86.5% (i.e., Number of correct points divided by total number of points = \(\frac{476}{550} = 86.5\%)\).

In order to assess how perfect, the classification was conducted, a Kappa coefficient was calculated (equation 3.7). According to Landis & Koch (1977), a Kappa coefficient of 1 means perfect agreement where as a value close to zero means that the agreement is no better than would be expected by chance. Categorization of Kappa statistic (Landis & Koch, 1977) is presented in Table 4.7. A Kappa coefficient (equation 3.7) of 0.82 was obtained (See Table 4.7). Thus, the classified landuse/cover (LULC) map was found to be appropriate as input in WetSpass model for recharge estimation.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Kappa statistics</th>
<th>Strength of agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 0.00</td>
<td>Poor</td>
</tr>
<tr>
<td>2</td>
<td>0.00 - 0.20</td>
<td>Slight</td>
</tr>
<tr>
<td>3</td>
<td>0.21 - 0.40</td>
<td>Fair</td>
</tr>
<tr>
<td>4</td>
<td>0.41 - 0.60</td>
<td>Moderate</td>
</tr>
<tr>
<td>5</td>
<td>0.61 - 0.80</td>
<td>Substantial</td>
</tr>
<tr>
<td>6</td>
<td>0.81 - 1.00</td>
<td>Almost perfect</td>
</tr>
</tbody>
</table>

### 4.3 WetSpass model simulation

Using WetSpass model we estimated the long-term average recharge as a spatial variable dependent on the soil texture, land-use, slope and meteorological conditions. Summer, winter and annual average values of recharge, surface runoff and evapotranspiration were generated.

### Groundwater recharge

Recharge is promoted by vegetation cover, flat topography, deep water table, porous soils and the absence of confining beds. The simulated seasonal groundwater recharge varied from 1 mm to 218 mm with a mean of 76 mm and standard deviation of 52 mm.
for summer (Figure 4.10). For winter, the amount of recharge rate ranged from a minimum of 0.19 mm to a maximum of 1.98 mm (Figure 4.11). Analysis also showed that annual recharge ranged from 1 to 283 mm/annum, with a mean of 105 mm and standard deviation of 58 mm (Figure 4.12). The average annual groundwater recharge was calculated to be 22.1% of the average annual precipitation (475.8 mm) in the study area. About 88.04% of the annual groundwater recharge occurs during the wet season (summer), and the remaining 11.96% during dry season (winter).

The simulated recharge in some places in the western part of the study area resulted in low recharge rates. This could be due to the fact that western part of the study area experiences high temperature ranging from 30.4°C to 32.2°C (Figure 4.5) and thus causes high evapotranspiration and therefore less recharge. Similarly, some of the places on the western side have negative values which indicate that there is no groundwater recharge (i.e. discharge).

Furthermore, results show that South - east part of the study area receives relatively higher rainfall (ranging from 408 mm to 636 mm) and have substantial groundwater recharge compared to other areas with values ranging from 133 mm to 283 mm/annum. Rainfall data for five stations were used to assess the correlation between the rainfall and recharge. Figure 4.13 shows the correlation of all stations while depicts the correlation ratio on each station in relation to the mean recharge.
Figure 4.10: Groundwater recharge (summer) map

Figure 4.11: Groundwater recharge (winter) map
From Table 4.8 and Figure 4.13 (a-e), all stations showed good correlation ranging between 0.9721 and 0.9894 of rainfall and recharge. This indicates direct relationship between rainfall data obtained on site and recharge estimated using MAP.

**Table 4.8: Correlation results for rainfall – Recharge relationship**

<table>
<thead>
<tr>
<th>Stations</th>
<th>Station 1</th>
<th>Station 2</th>
<th>Station 3</th>
<th>Station 4</th>
<th>Station 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.979</td>
<td>0.9733</td>
<td>0.9454</td>
<td>0.978</td>
<td>0.9478</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.9894</td>
<td>0.9864</td>
<td>0.9721</td>
<td>0.9889</td>
<td>0.9737</td>
</tr>
</tbody>
</table>
Figure 4.13 (a-e): Graph of rainfall – recharge relationship correlations for five stations.
Comparisons of Groundwater recharge results with previous studies

A comparison was made on estimated groundwater recharge from this study with results of previous scholars (Table 4.9). The values of annual mean recharge was obtained by demarcating the same areas falling within the study area from South Africa recharge maps using ArcGIS.

Table 4.9: Comparisons of estimated recharge with previous studies

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean annual Recharge (mm/a)</th>
<th>Method used</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>75</td>
<td>Chloride Mass Balance (CMB)</td>
<td>Vegter, 1995</td>
</tr>
<tr>
<td>2006</td>
<td>113</td>
<td>Rainfall Recharge relation (Woodford Approach)</td>
<td>DWAF, GRA II report, 2006</td>
</tr>
<tr>
<td>2006</td>
<td>98</td>
<td>Chloride Mass Balance (CMB)</td>
<td>DWAF, GRA II report, 2006</td>
</tr>
<tr>
<td>2016</td>
<td>105</td>
<td>GIS hydrological model (WetSpass)</td>
<td>This study, 2016</td>
</tr>
</tbody>
</table>

In 1995, Vegter determined groundwater recharge using Chloride mass balance (CMB) method which resulted in an annual recharge value of 75 mm/year. Same method was used by GRA II (DWAF, 2006) to estimate groundwater recharge resulting in a value of 98 mm/year. Both results were lower than recharge value of 113 mm/year determined by GRA II (DWAF, 2006) using Mean annual precipitation approach (MAP). It is worth noting that the recharge value obtained by GRA II (DWAF, 2006) shows a good relationship between rainfall and recharge. This is because; in 2006 rainfall intensity recorded was higher (Table 4.1) compared to previous three years in the study area. This implies that the amount of rain percolating in the ground contribute highly recharge.
values in the aquifers. The average rainfall recharge recorded for all five stations was 325.06 mm in year 2006 while the highest recorded value for all stations for fifteen years was 348.7 mm. Moreover, in this study groundwater recharge estimated using WetSpass model resulted in a mean recharge value of 105 mm/year which demonstrates close similarity with previous studies. This implies that WetSpass is a potential tool that can be used to estimate recharge even in areas where data is scarce and hence uncertain.

4.4 Development of a distributed groundwater flow model

The development of a distributed groundwater flow model for the study area aimed at providing information on the groundwater aquifer system as well as the water budget associated with it. It also helped in understanding the behaviour of the groundwater system in response to stress due to groundwater abstractions. The developed model can be used for management of groundwater through scenarios analysis and which will aid in decision making.

4.4.1 Conceptual model development

The first step in the modelling was the construction of a conceptual model in relation to aquifer domain of the study area. The developed model (Figure 4.14) consists of a set of possible assumptions that diminish the real problem and come up with a conceptual model that is suitable in view of the objective of the modelling. As mentioned by Yihdego (2005) the reasons for developing a conceptual model are to formulate a better understanding of site conditions of the modelled area, to define the groundwater flow problem, to develop a suitable numerical model and to help in selecting a suitable computer code. In this study the conceptual model was developed by integrating wells logs data, hydrogeological maps and hydrostratigraphy data. In order to accommodate the complexity of the real hydrogeological system, some basic assumptions about the study area were made. The assumptions were as follows: (i) the system was considered to be in a steady-state throughout the year and (ii) the geological formations of concern were considered horizontal in extent.
The development of a distributed numerical groundwater flow model for the study area comprised of three steps; (1) construction of numerical model (2) Model calibration and (3) model validation. Three inputs were used in model developments and included: (i) model properties (ii) model boundary conditions and (iii) wells.

4.4.2 Construction of numerical model

Steady - state numerical model was developed using MODFLOW – 2000 (Harbaugh et al., 2000). Identical discretization was assigned in all models in order to decrease partiality caused by different numerical errors in each model. The details of model discretization are presented in section 3.6. Figure 4.15 shows the discretized groundwater model of the study area.
Model properties

A groundwater flow model requires hydraulic conductivity, storage and initial heads values for each grid cell in order to run a flow simulation. The values of each property used for model input are shown in Table 4.10.
Table 4.10: Properties parameters value for model inputs

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic conductivity, $K_x$ (m/d)</td>
<td>0.003568 - 30.78</td>
</tr>
<tr>
<td>Specific storage, $S_s$ (m$^{-1}$)</td>
<td>0.0015 – 0.00201</td>
</tr>
<tr>
<td>Specific yield, $S_y$</td>
<td>0.20</td>
</tr>
<tr>
<td>Effective porosity</td>
<td>0.15</td>
</tr>
<tr>
<td>Total porosity</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Initial heads

For initial/starting heads, static water levels records of the wells were interpolated within the model to obtain the initial heads for the entire area in the model. The initial heads were interpolated using a nearest neighbour technique. Figure 4.16 shows the initial heads of the model. The initial heads ranged from 1353 m to 801 m decreasing from south to north part of the study area.

![Initial groundwater head with contours used for model setup](image)

Figure 4.16: Initial groundwater head with contours used for model setup
**Boundary conditions**

Boundary conditions used in model setup included constant head, river and recharge. A constant head boundary was allocated at a distance of 2 km from pumping wells found nearest to the boundary of the study area. A boundary located at a distance of 2 km will ensure that effect of pumping from the nearest well will not be felt at such a distance. Four rivers incorporated in the model included Sand river, Hout river, Seepabana and Brak river. Figure 4.17 shows the boundary conditions used as input in groundwater flow model.

**Recharge**

Another important boundary condition for this study was recharge. In this study, recharge was estimated using GIS – based distributed hydrological model (WetSpass) as discussed in sub-section 3.2. The estimated annual groundwater recharge ranged from 1 to 283 mm/year with a mean of 105 mm/yr and standard deviation of 58.

![Figure 4.17: Section of model area showing boundary condition](image)
**Pumping wells data**

The abstraction information of pumping wells was obtained from GRIP Limpopo database. Pumping wells distribution and their abstraction rates are presented in Figure 4.18. From Figure 4.18, it is apparent that most wells in the study area have pumping rates of between 5.2 and 127.9 m$^3$/d with a few having pumping rates of between 127.9 and 345.6 m$^3$/d. The rest have high pumping rates. Thus, the study area can be said to have high yielding aquifer, and hence a good candidate for groundwater exploitation. As reported by Environmentek, CSIR (2003), the highest well yields, excluding the alluvium associated with sand rivers, occurs in southern portion of Limpopo river basin. Furthermore, Woodford et al. (2005) pointed out that, over 80% of South Africa is underlain by relatively low yielding, shallow, weathered and/or fractured-rock aquifer systems. However, considerable quantities of groundwater can be abstracted at relatively high-rates from dolomitic and quartzitic aquifer systems located in the northern (i.e. Limpopo) and southern parts of the country. The study area is characterised by fractured geology which introduces secondary porosity leading to improved recharge and hence high potential for groundwater exploitation. It is to be noted that fractures can either introduce secondary porosity or can act as no flow boundaries. In this current study, the fractures seem to be interconnected with the aquifer, thus introducing secondary porosity that leads to a high yield aquifer.

![Figure 4.18: Pumping wells distribution and their pumping rates](image)
4.5 Calibration of distributed groundwater flow model

Before the calibration procedure was carried out, there was a need to set the calibration target. For this study heads from groundwater level measurement data were used as calibration values. The target was to compare the heads observed from site to simulated heads. Water level data used for calibration process was recorded at different times. Thus, depending on the date when the borehole was monitored, variations in water levels may exist. These variations referred to as errors could be due to: (i) errors due to static water levels measurements taken soon after well-construction (ii) errors due to measurement instruments (iii) errors due to lack of independent monitoring wells and (iv) some monitoring wells measurements were taken from pumping wells.

In this study, the criteria indicated that, for RMSE the collective effect should not be more than 10 meters difference between the observed and calculated heads (Tsou et al., 2006) and Normalised Root Mean Square Error (NRMS) should be less or equal to 10% (Coelho et al., 2017). This is because NRMS it is a more representative measure of the fit than the standard RMSE, as it accounts for the scale of the potential range of data values. Other studies used these criteria’s include; Loudyi (2005); Surinaidu et al. (2014). The study by (Loudyi, 2005) used NRMS for calibration analysis to compare model results obtained by MODFLOW and groundwater flow finite volume model (GWFV). The NRMS for MODFLOW was 8.21%, whereas the GWFV gave 7.15%. Another study by Surinaidu et al. (2014) used NMRS to assess the calibration and validation results using MODFLOW and the NMRS obtained was 2.1 %. They concluded that the model was well calibrated and validated for observed field hydrological conditions.

During calibration procedure each of the 5 zones (Figure 3.10) was handled as a separate calibration parameter of hydraulic conductivity which was varied within a possible range of values. To assess the performance of the model, essential calibration statistics were determined. The final automated hydraulic conductivities are presented in Table 4.11.
Table 4.11: Initial and calibrated hydraulic conductivities for five zones

<table>
<thead>
<tr>
<th>Zone</th>
<th>Initial value</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>3.521199</td>
<td>3.291028</td>
</tr>
<tr>
<td>Zone 2</td>
<td>4.59171</td>
<td>21.22475</td>
</tr>
<tr>
<td>Zone 3</td>
<td>5.820491</td>
<td>0.7698707</td>
</tr>
<tr>
<td>Zone 4</td>
<td>2.282853</td>
<td>3.310401</td>
</tr>
<tr>
<td>Zone 5</td>
<td>5.091464</td>
<td>2.980593</td>
</tr>
</tbody>
</table>

The achieved calibration residual values were used to calculate statistics such as (i) Mean Error (ME) (ii) Mean absolute error (MAE) (iii) Root Mean Square Error (RMSE) and (iv) Normalized Root Mean Square (NRMS). The following formulas were used for calculation of the statistics. ME (the mean of the difference between measured head \(h_m\) and simulated \(h_s\)) was calculated as:

\[
ME = \frac{1}{n} \sum_{i=1}^{n} (h_m - h_s)_i
\]  

MAE (the mean of the absolute value of the difference between the measured head \(h_m\) and the simulated head \(h_s\)) was calculated using equation (4.2) as follows:

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |(h_m - h_s)_i|
\]  

The RMSE (the square root of the average of the squared difference between measured head \(h_m\) and the simulated head \(h_s\)) was calculated using equation (4.3) as follows:

\[
RMSE = \left[ \frac{1}{n} \sum_{i=1}^{n} (h_m - h_s)_i^2 \right]^{0.5}
\]  

Where \(n\) is the number of calibration measurements points. Moreover, the Normalized Root Mean Square (NRMS) was introduced to further evaluate the calibration. This is because it is a more representative measure of the fit than the standard RMS, as it
accounts for the scale of the potential range of data values. NRMS is expressed as a percentage; and was calculated as follows:

\[ NRMS = \frac{RMS}{(X_{obs}\text{max} - (X_{obs}\text{min})} \] (4.4)

Where;

RMS is a Root mean square

\((X_{obs}\text{max})\) is maximum value of observed results

\((X_{obs}\text{min})\) is minimum value of observed results

The results from calibrated groundwater flow model yielded acceptable calibration statistics in line with target criteria. Calibration statistics calculated after the calibration are given in Table 4.12. The Normalised Root mean square (NRMS) was 1.828%, while the Mean Error (ME) and Mean absolute error (MAE) were within the set criteria. The calculated linear correlation coefficient of 0.998 indicates a high positive correlation between observed and simulated head values. As reported by Zheng & Bennett (2002), well calibrated models are expected to have linear regression coefficients close to 1, hence our groundwater model can be said to have been well calibrated and hence can be used for predictions.

**Table 4.12:** Statistics of calibrated groundwater flow model

<table>
<thead>
<tr>
<th>Criteria</th>
<th>PEST results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised Root mean squares (NRMS)</td>
<td>1.85%</td>
</tr>
<tr>
<td>Mean Error (ME)</td>
<td>0.433 m</td>
</tr>
<tr>
<td>Mean absolute error (MAE)</td>
<td>6.734 m</td>
</tr>
<tr>
<td>Root Mean Square Error (RMSE)</td>
<td>8.499 m</td>
</tr>
<tr>
<td>Correlation coefficient (Cor)</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Moreover, scatter plot graph was plotted to illustrates the comparison between observed and calculated heads at 95% confidence interval (i.e. is the interval where
95% of the total number of data points are expected to lie (see Figure 4.19). The scatter plot shows a good distribution of residual around zero. This means that the calibrated values do not show any bias and hence can be used for further analysis of the groundwater flow model.

![Calculated vs. Observed Head: Steady State](image)

**Figure 4.19**: Scatter plot graph for calculated head and observed head for calibration model

Further analysis was performed to show the relationship between calculated heads and observed heads for the 30 observation wells used for the calibration (Figure 4.20). Calculated head which is higher than observed heads indicates residual less than zero while calculated heads lower than observed heads indicate residual greater than zero.
For some wells simulated heads were slightly higher than the observed heads. This may be due to effect of model boundary condition.

**Figure 4.20:** Graph of observed and simulated heads for the calibrated model

In conclusion, assessment of the calibrated model results indicate that; (i) the calculated errors were within acceptable range as per predefined error criteria (ii) the difference between observed heads and simulated heads through MODFLOW were within the pre-established calibration criteria (iii) groundwater flow direction and heads were in agreement with those of the conceptual model (Figure 4.21).
4.6 Model Validation

Groundwater flow model was validated using independent head measurements for the year 2011. Groundwater levels at 30 monitoring wells were used as target for validation process. The performance statistics for validated model are shown in Table 4.13 and Figure 4.22.
Table 4.13: Statistics for Validation model

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Calibration results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised Root mean squares (NRMS)</td>
<td>1.828 %</td>
</tr>
<tr>
<td>Mean Error (ME)</td>
<td>0.999 m</td>
</tr>
<tr>
<td>Mean absolute error (MAE)</td>
<td>6.701 m</td>
</tr>
<tr>
<td>Root Mean Square Error (RMSE)</td>
<td>8.601 m</td>
</tr>
<tr>
<td>Correlation coefficient (Cor)</td>
<td>0.997</td>
</tr>
</tbody>
</table>

The NMRS for validated model was 1.850% which is slightly higher than that of the calibrated model (1.828%) by 0.022%. The linear correlation was calculated as 0.997 which was similar to that of the calibrated model. Moreover, most of the performance
statistics for validation model were within acceptable set criteria. The residuals obtained from validated model were distributed randomly around zero as shown in Figure 4.23, which mean that the validated model yielded comparable water elevations for most of the wells. This implies that the validated model can be used for prediction purposes and hence suitable for management of the groundwater aquifer of the study area.

Further analysis was performed to compare the residual heads for all 30 wells used for calibration and validation (it important to note validation used independent dataset of 2011). Even though the RMSE and NMRS resulted with agreeable results, some of the wells showed level of discrepancy for calibrated and validated model as presented in Figure 4.24.

Figure 4.23: Comparison of residuals values for validated model
In summary, the validation results fits very well with the calibrated ones. This good agreement may be due to the dataset of 2011 used for this purpose. It could be that if another dataset (say a few years after 2011) was available and used, the results might have been different. This is because groundwater moves slowly and effects of pumping are only felt some years after pumping the reservoir.

Moreover the calibrated and validated model can be used to derive components of groundwater budget and to estimate the response of the regional system to new stresses such as, increased groundwater withdrawals due to increasing demand of water for the development of domestic, agriculture, and industrial. These results may lead to sufficient information, and water-resource managers can use this information to make informed decisions when planning for future groundwater development.
4.7 Groundwater mass balance

In this study groundwater balance was used to evaluate the groundwater components within the aquifer. The principle behind groundwater mass balance is that, a change in storage should be equal to the sum of difference between inflow and outflow in the system. The groundwater inflow to the study area includes; recharge from rainfall, indirect recharge from river, water bodies leakages and groundwater inflows across the study area boundary. The system loses water through well abstractions, discharge to water bodies, and outflow outside the study area boundary. A summary of water balance for the study area is presented in Table 4.14.

The total river leakage from the aquifer was found to be 127.3197 mm/yr which is 57% of the total area (19188 km$^2$) runoff (Table 4.14).

<table>
<thead>
<tr>
<th>Components</th>
<th>IN (m$^3$/day)</th>
<th>OUT (m$^3$/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wells</td>
<td>0</td>
<td>54729</td>
</tr>
<tr>
<td>Constant head</td>
<td>10040252</td>
<td>9034549</td>
</tr>
<tr>
<td>River Leakage</td>
<td>5508300</td>
<td>12201481</td>
</tr>
<tr>
<td>Recharge</td>
<td>5742211</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>21290763</td>
<td>21290759</td>
</tr>
<tr>
<td>IN – OUT (m$^3$/day)</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Percent Discrepancy</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

The model found the river leakage slightly higher than 50%, this could be due to the following; (i) the river stage elevation was found to be lower than water level elevations observed in study area and (ii) the presence of perennial rivers flows in the catchment comes from groundwater discharge. Nevertheless, detailed study is required to determine the characteristics of catchment base flow in the study area.
Moreover, an abstraction value of 19.98 Mm$^3$/yr was obtained from the simulation results. The results agree with those from a study done by Van Vureen et al. (2003). They estimated total abstraction as 19.76 Mm$^3$/yr which is slightly lower than the current estimate. The total water budget indicated that there is storage of 4 m$^3$/day. This implies that the aquifer has potential for further exploitation of its groundwater resources. However, such exploitation should be carried out within a sustainable framework so as to avoid over-exploitation which would otherwise damage the aquifer.

**Limitation of the groundwater flow model**

The results of this model can be used to study the behaviour of the study area aquifer system against continuous exploitation. The model may also be used to evaluate different management scenarios. However, since the model did not consider uncertainties in other model parameters, the model prediction should be taken as an approximation at this stage. For this reason, all model predictions should be expressed as a range of possible outcomes that reproduce the assumptions involved and uncertainty in model input data and parameter values. In this study, the model calibration was done using the obtained data thus the accuracy of a model aquifer responses is directly related to the accuracy of the input data used in the model calibration. The model was calibrated only to conditions that are supposed to be steady state. Therefore, to calibrate a transient model detailed records of data changes with time is required. In addition, the model simulated general area-wide responses to stress, thus should not be used where detailed site-specific projections are required.

**4.8 Development of a stochastic solution methodology**

The third objective aimed at using stochastic methods to solve a groundwater flow problem considering recharge as uncertain, hence stochastic. Stochastic uncertainty analyses were performed by randomly sampling recharge realisation and running a series of Monte Carlo (MC) simulations. A series of realisations ranging from 50 to a maximum of 1000 realisations of recharge fields were generated. A series of MC experiments was done in order to determine the number of realisations for which the
residual mean was insensitive to increasing number of realisations used. Appendix 6 and Figure 4.25 show the results of a number of realisations against residual mean.

![Graph](image)

**Figure 4.25: Residual mean against number of realisations**

The results demonstrate that, the model started to stabilise at 800 realisations with residual mean of 0.302 m. The results remained relatively unchanged up to 1000 realisations. This implies that, as the number of realisations increases, the model becomes stable. Thus, 1000 realisations were used for the MC simulations.

Another experiment done was to examine how the standard deviations of expected mean recharge performed with increasing number of realisations (Table 4.15). The motive behind this experiment was to assess again if 1000 realisations would be enough to capture the recharge uncertainty and hence lead to solutions that are robust (solutions that will not change with increasing number of realisations).
Results indicated that there is a considerable change of standard deviation from 50 realisations (0.743 mm) to 1000 realisations (0.131 mm) as shown in Table 4.15. The results agree with the law of large numbers as illustrated in equation 2.10. Moreover, it is important to note that typical number of realisations required to give reliable statistical measures ranges between few tens and thousands according to Ndambuki, (2001a); Leeuwen et al. (1998); Wagner et al. (1994); Wagner & Gorelick, (1989). Furthermore, Ballio & Guadagnini, (2004) indicated that increasing the number of realisations from 1000 to 2000 has basically no effect on the overall convergence. This implies that 1000 realisations were sufficient to propagate recharge uncertainty in the groundwater flow problem leading to solutions that are stable. Hence the developed stochastic model qualifies to be used for further analysis.

**Comparison of deterministic and stochastic model results**

Stochastic model results were compared to those of deterministic model (developed in objective two) in order to evaluate the impact of parameter uncertainty (in this case recharge) on the groundwater flow simulation results. These comparisons were based on three approaches (i) variations of residual heads (ii) net recharge and (iii) zone budget results.
Deterministic and Stochastic residual heads

The goal of this analysis was to compare the residual (difference between measured and simulated head) of deterministic and stochastic model. This is because, it is important to examine whether some areas of the model have biased results (either too high or too low values). Thus, the closeness of fit between simulated and observed head are important in evaluating the appropriateness of the model in addressing groundwater management problems. Figure 4.26 shows a 3D representation of the modelled area for deterministic and stochastic model.

![3D representation of modelled area for deterministic and stochastic model](image)

**Figure 4.26:** 3D representation of modelled area for deterministic and stochastic model

The residual mean calculated from deterministic model was 2.76 m while stochastic simulation had a residual mean of 0.82 m. This implies that by assuming recharge as deterministic, one would get results that are optimistic (i.e., results that promises more groundwater availability than one can actually get). This situation can lead to destruction of the groundwater aquifer. On the other hand, stochastic results are pessimistic and hence cautious. This means that by taking uncertainty in recharge into consideration,
the solutions obtained are more realistic and will protect the aquifer from over-exploitation, thus leading to sustainable groundwater management. This is also confirmed by results shown in Figure 4.27 which indicates that stochastic model simulated heads where lower than the observed head compared to deterministic model results. This implies that the stochastic model can be used as an appropriate tool to manage the groundwater resources in the study area.

![Comparison of residual for deterministic and stochastic simulations](image)

**Figure 4.27:** Comparison of residual for deterministic and stochastic simulations

**Net recharge for deterministic and stochastic models**

Comparison of net recharge for both deterministic and stochastic simulations was done. In this study, net recharge represents the specified recharge values minus the calculated evapotranspiration simulated through MODFLOW. Analysis was performed to show the results obtained when considering the recharge to be known with certainty and when recharge was considered as uncertain. Net recharge for deterministic model was 106.01 mm/year with standard deviation of 43.4 mm/yr while for the stochastic case
was 109.2 mm/year with standard deviation of 39 mm/yr. Results implies that the amount of groundwater available for exploitation determined through deterministic approach was 2033 Mm$^3$/yr while that for stochastic method gives about 2091 Mm$^3$/yr. This implies that results obtained through deterministic approach are pessimistic compared with those obtained through stochastic approach. Hence, this suggests that, as we move from deterministic to stochastic modelling, the value of recharge tends to be better estimated with values higher than those of a deterministic case. This finding is supported by study done by Holman et al. (2009). This in turn implies that there is more groundwater available for exploitation.

Moreover, these results suggest that; if this information was to be used to design a groundwater management plan, one would consider choosing stochastic model over deterministic one. For management option, deterministic models should be taken as starting point in the process of searching for better management solutions which recognizes the existence of uncertainty (i.e. stochastic approach).

**Zone budget for deterministic and stochastic model**

Recharge zones were demarcated based on calibrated hydraulic conductivities (Figure 4.28). The water budget analysis for recharge zones were done for both deterministic and stochastic situations. Table 4.16 shows the simulated recharge for five zones for both deterministic and stochastic simulations.
Table 4.16: Simulated recharge values for the five zones of the models

<table>
<thead>
<tr>
<th>Zones</th>
<th>Area [km$^2$]</th>
<th>Deterministic [mm/year]</th>
<th>Stochastic [mm/year]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>8791.677</td>
<td>4.93</td>
<td>4.95</td>
</tr>
<tr>
<td>Zone 2</td>
<td>1366.934</td>
<td>193.45</td>
<td>192.76</td>
</tr>
<tr>
<td>Zone 3</td>
<td>3323.288</td>
<td>125.47</td>
<td>127.17</td>
</tr>
<tr>
<td>Zone 4</td>
<td>4018.728</td>
<td>99.08</td>
<td>101.52</td>
</tr>
<tr>
<td>Zone 5</td>
<td>1687.373</td>
<td>533.03</td>
<td>534.76</td>
</tr>
</tbody>
</table>

The results from Table 4.16 shows that there were higher values of simulated recharge for zones 1, 3, 4 and 5 for stochastic model compared to the deterministic one. However, zone 2 shows an insignificant decrease of recharge of 0.69 mm/year for
stochastic model. The simulated recharge for zone 3 agrees with the results obtained from WetSpass application (Figure 4.12), thus the annual recharge ranged between 114 – 283 mm/yr. The least recharge values of 4.93 and 4.95 mm/year for deterministic and stochastic simulation were found in zone 1. This could be due to geological characteristics that do not promote recharge. Shockingly, Zone 5 resulted in high recharge of about 55.6% (Table 4.16). This is because; this zone has no abstraction wells as per modelled area. It is also located at the south part of the study area where aquifers are assumed to be recharged through infiltration and stream flow. In addition, several faults and fractures are observed in this zone. However, this zone grossly overestimated the amount of potential recharge considering the mean annual rainfall of the area, hence further investigation is required for this zone. For this reason, zone 5 was eliminated from further analysis.

**Table 4.17:** Comparisons of deterministic and stochastic groundwater potential

<table>
<thead>
<tr>
<th>Zones</th>
<th>Deterministic [Mm³/yr]</th>
<th>Stochastic [Mm³/yr]</th>
<th>Difference [Mm³/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>43.34</td>
<td>43.52</td>
<td>0.18</td>
</tr>
<tr>
<td>Zone 2</td>
<td>264.43</td>
<td>263.49</td>
<td>-0.94</td>
</tr>
<tr>
<td>Zone 3</td>
<td>416.97</td>
<td>422.62</td>
<td>5.65</td>
</tr>
<tr>
<td>Zone 4</td>
<td>398.18</td>
<td>407.98</td>
<td>9.81</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1122.92</strong></td>
<td><strong>1137.61</strong></td>
<td><strong>14.7</strong></td>
</tr>
</tbody>
</table>

*Comparisons of simulated recharge volume for the two models*

Further analysis was performed to determine if there was any significance change of using stochastic model over deterministic model. Table 4.17 shows that there is significant change in the amount of groundwater available for exploitation from the stochastic model for zones 3 (5.65 Mm³/yr) and zone 4 (9.81 Mm³/yr). On the other hand, the amount of groundwater available in zone 2 (0.94 Mm³/yr) was reduced in the case of the stochastic model. This zone was observed to have dams within the area and
hence the possibility of groundwater discharges to the dam occurring rather than recharge. In as far as zone 1 is concerned, there was no significance change observed.

In general, the total amount of water for all demarcated zones of the study area showed an increase of 14.7 Mm$^3$/year for stochastic model compared to deterministic model. Hence these observations emphasize the use of stochastic model for groundwater management.

4.9 Development of a Groundwater Management Model

The fourth objective entailed the development of a groundwater management model that explicitly considers recharge as uncertain input parameter. The motive behind this objective was to examine how much water can be withdrawn from an aquifer without producing non-reversible impacts on groundwater quantity. This is because; intensive extraction of groundwater from aquifers may affect groundwater levels, base-flow of streams and groundwater storage. The development was done through the computation of Groundwater Sustainability Index (GSI) by using equation 2.11.

In this study total groundwater abstraction means the total withdrawal of water from a given aquifer by means of pumping wells. The data for pumping wells together with their pumping rates were obtained from GRIP Limpopo and VSA Leboa Limpopo. Groundwater recharge results from the stochastic model developed in objective three were used. The groundwater index based on total groundwater abstraction and groundwater recharge was adopted in this study (refer section 2.8). The analysis started by randomly increasing the pumping rate from initial pumping rates (obtained from primary data) until calculated GSI reached 0.8. GSI analysis was done by using hydraulic zones as shown in Figure 3.10. However, in this objective only four zones were used. Zone 5 was eliminated from analysis as discussed in section 4.8. Figure 4.29 shows the demarcations of four zones used in GSI analysis.
The aim of this exercise was to examine how much water is currently available in each zone and how much is extracted. Pumping rate and recharge of each zone was obtained by demarcating the wells falling in each zone. By using ArcGIS (statistics tool), the total amount of pumping rate and recharge was obtained. Figure 4.28 presents the amount of recharge in each zone while Figure 4.29 shows the relationship between pumping rate and recharge for each zone.

**Figure 4.29:** Demarcation of GSI zones
Figure 4.28: Amount of recharge in percentage for four GSI zones
Figure 4.29: Recharge and Pumping rate for four GSI zones

Figure 4.28 and Figure 4.29 demonstrate that, there is still potential for further groundwater abstraction in the study area. This is because the amount of abstraction in relation to recharge is almost negligible for zones 2, 3 and 4 (see Figure 4.29).

**Groundwater Sustainability Index (GSI) results**

Groundwater Sustainability Index (GSI) was used to examine the effect of pumping rate on the stochastic groundwater flow model. This exercise was done by randomly increasing the amount of pumping rate while observing GSI index. Table 4.18 and Figure 4.30 show the results of this analysis.
### Table 4.18: Groundwater sustainability Index results

<table>
<thead>
<tr>
<th>Zones</th>
<th>Original Pumping rate (m$^3$/day)</th>
<th>Recharge (m$^3$/d)</th>
<th>GSI Original</th>
<th>GSI 7*Q</th>
<th>GSI 45*Q</th>
<th>GSI 55*Q</th>
<th>GSI 160*Q</th>
<th>GSI 200*Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZONE 1</td>
<td>14218.85</td>
<td>118750</td>
<td>0.1</td>
<td>0.8</td>
<td>6.6</td>
<td>6.6</td>
<td>19.2</td>
<td>23.9</td>
</tr>
<tr>
<td>ZONE 2</td>
<td>13150.94</td>
<td>724490</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>1.1</td>
</tr>
<tr>
<td>ZONE 3</td>
<td>4740.768</td>
<td>1148800</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>ZONE 4</td>
<td>16536.96</td>
<td>1090900</td>
<td>0.0</td>
<td>0.1</td>
<td>0.8</td>
<td>0.8</td>
<td>2.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Figure 4.30: GSI analysis for four zones**
Table 4.19 and Figure 4.30 show the results of GSI in relation to exploitation level for each zone (ie. Exploitation starts from 0.8). Zone 1 (comprising of sections of Seshego, Mokoreng, Soutpansberg and Dzanani districts) have 54 wells within an area of 8791.677 km$^2$ and total pumping rate of 14218.85 m$^3$/d. Through GSI analysis, this zone can tolerate pumping rate of up to 99531.95 m$^3$/d an increase of about 14.3% from the current pumping rate.

The existing extraction rate of Zone 2 is 724490 m$^3$/day; however, through GSI this zone has a potential groundwater abstraction rate of 264 Mm$^3$/year. This zone was observed to be on south part of the study area where the infiltration rate is high compared to the other zones. Moreover, it also has a high potential of recharge as shown in Figure 4.30.

Zone 3 which is located in eastern part of study area comprises of districts of Thabamoopo (394.10 km$^2$), Pietersburg (2045.13 km$^2$), Sekgosese (174.36 km$^2$) and Seshego (709.39 km$^2$). According to data obtained from GRIP Limpopo only 42 wells had complete dataset. Therefore, the analysis was based on 42 pumping wells (Table 4.19). Current abstraction rate of this zone is 4740.768 m$^3$/d (1.73 Mm$^3$/yr) while results of GSI analysis indicates that this zone has potential for groundwater abstraction of up to 346 Mm$^3$/yr.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of wells</th>
<th>Pumping rate total (m$^3$/d)</th>
<th>Zone Area, km$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>14218.85</td>
<td>8791.677</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>13150.94</td>
<td>1366.934</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>4740.768</td>
<td>3323.288</td>
</tr>
<tr>
<td>4</td>
<td>69</td>
<td>16536.96</td>
<td>4018.728</td>
</tr>
</tbody>
</table>

Table 4.19: Pumping wells, total area and abstraction rate
Zone 4 located at northwest of the study area (Figure 4.29) can tolerate pumping rates of up to approximately 80.64 Mm$^3$/yr.

In general, all zones showed potential for higher abstraction rates compared to what is currently abstracted. This observation is supported by Limpopo DFED (2004) report which stated that, the largest concentration of high potential groundwater (i.e. 70% of the average borehole yields per strategic resource type range > 2 l/s) occurrence is located in the central areas of the province (i.e. current study area), Pietersburg Plain and to the east in the direction of Thoyandou (DWAF, 1998). Moreover, the study established that, pumping wells available are very few compared to the size of the study area (Table 4.19). This implies that there is need to increase the number of pumping wells in the study area in order to accommodate increasing of water supply demands from the communities living in the study area. Moreover, this research has demonstrated that the groundwater resource is available throughout the study area in varying quantities that can be exploited for bulk-water supply.

**Comparisons of findings with previous studies**

The results were compared with recent studies done in the study area. The study by Limpopo Water Management Area North Reconciliation Strategy (2015) presents the finding of Limpopo Water Management Area (WMA) (See appendix 7). Table 4.20 shows comparison of current study results with those of other recent studies done in the same area.
The results indicate that about 659 Mm³/yr estimated by Limpopo Water Management Area North Reconciliation Strategy (2015) is available for exploitation while this study estimates a value of about 1138 Mm³/yr using the stochastic approach. This implies that there is excess of 478.61 Mm³/yr which was not captured on the previous study. This could be due to the fact that deterministic approach was used in the estimation of groundwater exploitation potential. For these reasons, this study aimed at providing a methodology which can be used to manage the groundwater resources of the Central Limpopo area and ensure that its use is sustainable.

4.10 Contribution of Research

i. This study has come up with a framework that links WetSpass, Remote Sensing (RS), Geospatial information system (GIS) and MODFLOW in order to develop a distributed groundwater management model that explicitly considers recharge as uncertain input parameter. The developed stochastic model can then be used as a decision support system (DSS) for water resource management.
ii. The use of non-commercial software for estimating groundwater recharge (WetSpass) was also initiated in this study. This provides water researchers with a free accessible tool that can be used to effectively estimate spatial and temporal groundwater recharge.

iii. In the area of water resources management, this study mapped out groundwater potential areas using Groundwater Sustainability Index (GSI). This provides water managers and policy makers with fundamental information on where future water developments can be carried out.
CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

This study was conducted with the main aim of quantifying the spatial and temporal variability of groundwater recharge for the purpose of developing a quasi-three-dimensional distributed groundwater flow model under recharge uncertainty. The main aim was accomplished through four specific objectives that required: (1) to estimate groundwater recharge using GIS-based distributed hydrologic models (WetSpass); (2) to develop a groundwater management model that explicitly considers recharge as uncertain input parameter; (3) to develop a stochastic solution methodology for solving groundwater flow problems considering recharge as uncertain input parameter and (4) to develop a groundwater management model that explicitly considered recharge as uncertain. These objectives assisted to answer the research question as stipulated on section 1.4. Generally, objectives one and two were designed to facilitate execution of objective three while objective three facilitated the implementation of objective four. Based on the findings of the study, subsections 5.1 and 5.2 discuss the conclusions and recommendations, respectively.

5.1 CONCLUSIONS

Estimation of groundwater recharge using GIS-based distributed hydrologic models (WetSpass)

Groundwater recharge of a semiarid study area was determined using a GIS-based distributed hydrologic model, WetSpass. Long-term average spatial and temporal distributed groundwater recharge was obtained. The average simulated summer and winter recharge were 76 mm and 1.98 mm, respectively. The annual average recharge calculated was 105 mm with standard deviation of 58 mm which is 22.1% of the average annual precipitation (475 mm) in the study area. The calculated annual average recharge compared well with values reported by other studies, hence GIS-based WetSpass can be used to estimate spatial recharge for use in stochastic groundwater management.
Development of a distributed groundwater flow model for the study area

A steady state distributed groundwater flow model of the study area was developed using MODFLOW. Calibration resulted in normalised residual mean square (NRMS) value of 1.828%, mean error (ME) of 0.999 m, Mean absolute error (MAE) of 6.701 m and Correlation coefficient (Cor) of 0.997. The model was validated using head measurements for the year 2011. Groundwater levels at 30 observation wells were used as target for validation process. The validated model resulted in a NMRS value of 1.85% which compares very well with the value of the calibrated model of 1.83 %. All set criteria were within acceptable limits for both calibrated and validated model. Thus, the model can be used to simulate future aquifer responses to changes in groundwater pumping rates, thus acting as a decision support tool.

Development of a stochastic solution methodology to solve a groundwater flow problem through simulation considering recharge as uncertain

Stochastic uncertainty analyses were performed by running a series of Monte Carlo (MC) simulations. Recharge values obtained through stochastic simulation were higher than those obtained through deterministic simulations. The stochastic model resulted groundwater potential of about 5.65 Mm³/yr for zone 3 and 9.81 Mm³/yr for zone 4. This suggests that there is potential for further groundwater resource development in the study area. The total amount of water for the four demarcated zones of the study area showed a significant groundwater potential of 14.7 Mm³/year for the stochastic model compared to the values of the deterministic model. At the end, these observations emphasize the use of stochastic model for groundwater management.

Development of a groundwater management model that explicitly considers recharge as uncertain.

Development of groundwater management model was achieved through groundwater sustainability index (GSI) approach. GSI values close to unity (i.e. 0.8) and above indicated overexploitation. Using this approach, results indicated that the potential for higher abstraction rates compared to the current abstraction rates is real. Thus, GSI
approach can be used in the management of groundwater aquifer sustainably. This tool can be used by water managers and policy makers to inform them on where to drill boreholes and how much to extract from them while protecting the resource. Moreover, comparison of results with previous studies indicate that about 659 Mm$^{3}$/yr estimated by Limpopo Water Management Area North Reconciliation Strategy (2015) is available for exploitation while this study estimates a value of about 1138 Mm$^{3}$/yr using the stochastic approach. This implies that there is excess of 479 Mm$^{3}$/yr which was not captured on the previous study. This could be due to the fact that deterministic approach was used in the estimation of groundwater exploitation potential.

5.2 RECOMMENDATIONS

i. The study recommends the use of GIS distributed hydrological model (WetSpass) for quantification of groundwater recharge in other parts of South Africa. This is because the model is capable of estimating spatially distributed and long-term average recharge.

ii. It is recommended that the methodology developed in this study that links RS, GIS and numerical modelling in the solution of groundwater flow problems be applied in other areas with a view to sustainably manage our scarce resources.

iii. It is further recommended that the developed methodology be used to study groundwater pollution management with the ultimate desire of coupling the quantity and quality aspects of groundwater. This will ensure that the resource is protected in terms of quantity and quality, hence providing a holistic groundwater management tool.
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VAN ROSSUM, P., BATELAAN, O., GHEBREMESKEL, O. & DE SMEDT, F. 2001. Discharge Coefficients for the Kikbeek Sub-basin, a Brook Sub-basin at the Belgian Side of the River Border Meuse (Grensmaas).


## Appendix 1

WetSpass Soil parameters

<table>
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<td>1187.8</td>
</tr>
<tr>
<td>OBW26</td>
<td>763775</td>
<td>7346010</td>
<td>VW26</td>
<td>1300</td>
<td>365</td>
<td>1307.8</td>
</tr>
<tr>
<td>OBW27</td>
<td>772557</td>
<td>7369760</td>
<td>VW27</td>
<td>1175</td>
<td>365</td>
<td>1182.8</td>
</tr>
<tr>
<td>OBW28</td>
<td>709298</td>
<td>7460550</td>
<td>VW28</td>
<td>835</td>
<td>365</td>
<td>842.8</td>
</tr>
<tr>
<td>OBW29</td>
<td>706128</td>
<td>7326200</td>
<td>VW29</td>
<td>1146.46</td>
<td>365</td>
<td>1154.07</td>
</tr>
<tr>
<td>OBW30</td>
<td>706681</td>
<td>7333200</td>
<td>VW30</td>
<td>1151.46</td>
<td>365</td>
<td>1152.87</td>
</tr>
</tbody>
</table>
APPENDIX 5: MATLAB CODE

%pd = makedist('Lognormal')
pd = makedist('lognormal','mu',105.00,'sigma',log(58.00));
Mm = mean(pd);
Mnv = Mm;

%The mean of the lognormal distribution is not equal to the mu parameter.
%Generate random numbers from the lognormal distribution and compute their log values.
rng(1);  % for reproducibility
Y = random(pd,117,164,500);
X = log(Y);
R1 = X(:,1);
RN1 = R1(:);
R2 = X(:,2);
RN2 = R2(:);
R3 = X(:,3);
RN3 = R3(:);
R4 = X(:,4);
RN4 = R4(:);
R5 = X(:,5);
RN5 = R5(:);
R6 = X(:,6);
RN6 = R6(:);
R7 = X(:,7);
RN7 = R7(:);
R8 = X(:,8);
RN8 = R8(:);
R9 = X(:,9);
RN9 = R9(:);
R10 = X(:,10);
RN10 = R10(:); 
Continues up to 1000............R1000 = X (:,1000); 

minR = min([RN1,RN2,RN3,RN4,RN5,RN6,RN7,RN8,RN9,RN10,RN11,RN12,RN13,RN14,RN15,RN16,RN17,... 
RN18,RN19,RN20,RN21,RN22,RN23,RN24,RN25,RN26,RN27,RN28,RN29,RN30,RN31,RN32,RN33,RN34,RN35,RN36,RN37,RN38,RN39,RN40,RN41,RN42,RN43,... 
RN44,RN45,RN46,RN47,RN48,RN49,RN50]); 
minRN = minR(:); 
maxR = max([RN1,RN2,RN3,RN4,RN5,RN6,RN7,RN8,RN9,RN10,RN11,RN12,RN13,RN14,RN15,RN16,RN17,... 
RN18,RN19,RN20,RN21,RN22,RN23,RN24,RN25,RN26,RN27,RN28,RN29,RN30]); 
maxRN = maxR(:); 
X = X(:); 
Xp1 = X(1:180,1); 
Xp2 = X(181:360,1); 
Xp3 = X(361:540,1); 
Xp4 = X(541:720,1); 
Xp5 = X(721:900,1); 
%x = permute(x,1); 
%Compute the mean of x values. 
%Mm = mean(x); 
%XX = log(X); 
Mmm = mean(X); 
%The mean of x values is equal to the mu parameter of x, since x has a lognormal distribution. 
%Plot x. 
subplot(2,1,1)
scatter(Xp1,Xp2,10,'ko')
scatter(X,10,'ko')
xlabel('x')
ylabel('y')
hold on
subplot(2,1,2)
hist(X,50)
xlabel('x')
ylabel('y')

figure
scatter(Xp1,Xp2,12,'bo')
xlabel('Recharge [mm/year]')
ylabel('Recharge [mm/year]')

m = mean(logX);
figure
subplot(2,1,1)
scatter(logX1,logX2,10,'ko')
scatter(logx,10,'ko')
xlabel('logx')
ylabel('logy')
subplot(2,1,2)
hist(logX,50)
xlabel('logx')
ylabel('logy')

figure
scatter(logX1,logX2,12,'bo')
xlabel('Recharge [mm/year]')
ylabel('Recharge [mm/year]')

% The plot shows that the log values of x are normally distributed with a mean equal to 5 and a standard deviation equal to 2.
% options = statset('MaxIter',1000,'Display','final');
% gm = fitgmdist(logx,3,'Options',opts);
% gm = fitgmdist(logx,3);
y = logX(:,:,);
options = statset(statset('gmdistribution'), ...
    statset('MaxIter',30,'Display','final'));
gm = fitgmdist(X,5,'Options',options);
%S = gmcluster(X(:,:,),5,'Replicates',3,'Options',options);
figure
% hold on
y = logX(:,:,);
n = length(y);
XX = mean(y);
%n = round(n);
z = 1:n;
%ezsurf(@(x,y)pdf(gm,[x y]),[-1 1],[-1 1]);
%ezsurf(@(logx,y)pdf(gm,[logx y]),[-10 10],[-10 10])
% y = x(:,2);
%ezcontour(@(logx,y)pdf(gm,[logx y]),[-0.2 0.2],[-0.2 0.2]);
%ezcontour(logX, pdf(gm,[logX y]),[-5 -3.5],[-5 -3.5]);
% contour(logX);
% hold off
%[idx,ctrs] = gmcluster(logX,5,'Options',options);
%S = gmcluster(logX,5,'Options',options);
idx = cluster(gm,y);
cluster1 = (idx == 1);
cluster2 = (idx == 2);
cluster3 = (idx == 3);
cluster4 = (idx == 4);
cluster5 = (idx == 5);
plot(y(idx==1,1),y(idx==1,1),'r.','MarkerSize',12)
plot(y(idx==2,1),y(idx==2,1),'b.','MarkerSize',12)
plot(y(idx==3,1),y(idx==3,1),'g.','MarkerSize',12)
plot(y(idx==4,1),y(idx==4,1),'y.','MarkerSize',12)
plot(y(idx==5,1),y(idx==5,1),'m.','MarkerSize',12)
ctr = [logX1,logX2,logX3,logX4,logX5];
figure
plot(ctrs(:,1),ctrs(:,2),'kx','...'
   'MarkerSize',12,'LineWidth',2)
hold on
plot(ctrs(:,2),ctrs(:,3),'ko','...'
   'MarkerSize',12,'LineWidth',2)
plot(ctrs(:,3),ctrs(:,4),'bo','...'
   'MarkerSize',12,'LineWidth',2)
plot(ctrs(:,4),ctrs(:,2),'yx','...'
   'MarkerSize',12,'LineWidth',2)
plot(ctrs(:,5),ctrs(:,1),'mp','...'
   'MarkerSize',12,'LineWidth',2)
legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','Cluster 5',...'
   'Centroids','Location','NW')
hold off
figure
subplot(2,1,1)
scatter(y(cluster1,1),y(cluster1,1),10,'r+');
hold on
scatter(y(cluster2,1),y(cluster2,1),10,'bo');
scatter(y(cluster3,1),y(cluster3,1),10,'g*');
scatter(y(cluster4,1),y(cluster4,1),10,'yx');
scatter(y(cluster5,1),y(cluster5,1),10,'mp');
legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','Cluster 5',...
    'Centroids','Location','NW')
hold off

P = posterior(gm,y);
subplot(2,1,2)
scatter(y(cluster1,1),y(cluster1,1),10,P(cluster1,1),'+')
hold on
scatter(y(cluster2,1),y(cluster2,1),10,P(cluster2,1),'o')
scatter(y(cluster3,1),y(cluster3,1),10,P(cluster3,1),'**')
scatter(y(cluster4,1),y(cluster4,1),10,P(cluster4,1),'x')
scatter(y(cluster5,1),y(cluster5,1),10,P(cluster5,1),'p')
legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','Cluster 5',...
    'Centroids','Location','NW')
hold off

clrmap = jet(80); colormap(clrmap(9:72,:))
ylabel(colorbar,'Component 1 Posterior Probability')
[~,order] = sort(P(:,1));
figure
plot(1:size(y,1),P(order,1),'r-',1:size(y,1),P(order,2),'b-',...
     1:size(y,1),P(order,3),'g-',1:size(y,1),P(order,4),'y-',...
     1:size(y,1),P(order,5),'m-');
legend(['Cluster 1 Score' 'Cluster 2 Score' 'Cluster 3 Score'...
    'Cluster 4 Score' 'Cluster 5 Score'],'location','NW');
ylabel('Cluster Membership Score');
xlabel('Point Ranking');
### Appendix 6: Realisations of expected mean recharge and residual mean (m)

<table>
<thead>
<tr>
<th>Realisations of expected mean recharge</th>
<th>Residual mean of observed well heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.759</td>
</tr>
<tr>
<td>100</td>
<td>0.552</td>
</tr>
<tr>
<td>300</td>
<td>0.365</td>
</tr>
<tr>
<td>500</td>
<td>0.349</td>
</tr>
<tr>
<td>700</td>
<td>0.304</td>
</tr>
<tr>
<td>800</td>
<td>0.302</td>
</tr>
<tr>
<td>1000</td>
<td>0.302</td>
</tr>
</tbody>
</table>

### Appendix 7: Drainage regions from WMA found in study area

<table>
<thead>
<tr>
<th>Area</th>
<th>Tertiary Drainage</th>
<th>Quarterly Catchments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOGALAKWENA</td>
<td>A62</td>
<td>A62A, B, C, D, E, F, G, H, J, A61A</td>
</tr>
<tr>
<td>MOGALAKWENA</td>
<td>A63A</td>
<td>A63A, B, C, D</td>
</tr>
<tr>
<td>SAND</td>
<td>A71</td>
<td>A71A, B, A71E, F, G</td>
</tr>
</tbody>
</table>