Load Balancing in Distribution Feeder through Reconfiguration

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Abstract – The electrical distribution system is to ensure that an adequate supply is available to meet the estimated load of the consumers in both the near and more distant future. This must of course, be done for minimum possible cost consistent with satisfactory reliability and quality of the supply. In order to avoid excessive voltage drop and minimize technical loss, it may be economical to install apparatus to balance or partially balance the loads. It is believed that the technology to achieve an automatic load balancing lends itself readily for the implementation of different types of algorithms for automatically rearranging the connection of consumers on the low voltage and of a feeder for optimal performance.

I. INTRODUCTION

Between 30 and 40% of the total investments in the electrical sector goes to distribution systems, but nevertheless, they have not received the technological impact in the same manner as the generation and the transmission systems. Many of the distribution networks work have minimum monitoring systems, mainly with local and manual control, sectionalizing switches and voltage regulators; and without adequate computation support for the system's operators.

Nevertheless, there is an increasing trend to automate distribution systems to improve their reliability, efficiency and service quality. Automation is possible due to advance power electronics equipment, to its increasing cost reduction and due to its joint use with computing system and telecommunication technologies. It is possible to install a feeder operation technology on each feeder for the purpose of monitoring, processing and instituting control action remotely. With the aid of this technology it is possible to monitor relevant quantities of connection to the feeder, make necessary computations and implement algorithms that will result in control actions that give optimal efficiency and voltage drop performance of the feeder.

Ordinarily, the consumption of consumers connected to a feeder fluctuates, thus leading to the fluctuation of the total load connected to each phase of the feeder. This in turn implies that the degree of unbalance of the feeder also keeps varying. The worse the degree of unbalance the poorer is the efficiency and voltage drop on the feeder [2]. It has been well establish [2] that load balancing alone can lead to tremendous saving in investment costs while giving good performance to the feeder.

To improve the system reliability in ordinary modern distribution systems, some sectionalizing switches usually sectionalize the feeders. Furthermore, in some modern distribution system with distribution automation functions, tie breakers are also needed for feeder reconfiguration. Normally the sectionalizing switches are closed. On the contrary, the tie breakers are usually open [1]. The operation of these breakers makes the current and voltage unbalances worse, and most of the time the operator can open and close any switch to keep the current in the getaway of a feeder from the substation as nearly balanced as possible to avoid the unintentional relay tripping due to a large current in the neutral line. The neutral current is usually caused by the unbalance of the loads. The conventional trial and error approach is unable to find the optimal phase arrangement to balance the load, and then the current in every feeder segment [2].

In this paper the mathematical model for the phase balancing and the loss reduction in a low voltage distribution system is formulated as a constrained optimization problem that is solved with the dynamic leapfrog method.

II. PHASE BALANCING

In general, distribution loads show different characteristics according to their corresponding distribution lines and line sections, and therefore, load levels for each time period can be regarded as non-identical. In the case of a distribution system with some overloaded and some lightly loaded branches, there is the need to reconfigure the system such that loads are transferred from heavily loaded to less loaded feeders. Here the maximum load current the feeder conductor can take may be taken as the reference. Nonetheless, the transfer of load must be such that a certain predefined objective is satisfied. In this case, the objective is for the ensuing network to have minimum real power loss. Consequently, phase balancing may be redefined as the rearrangement of the network such as to minimize the total real power losses arising from line branches. Mathematically, the total power loss may be expressed as follows [3], [4]:

$$\sum_{i=1}^{n} r_i \frac{P_i^2 + Q_i^2}{\left|V_i\right|^2} \qquad (1)$$

where r_i, P_i, Q_i, respectively, is resistance, real power, and reactive power of branch i, and n is the total number of branches in the system. The aim of this study is to minimize the power loss represented by equation (1) subject to the following constraints:

1. The voltage unbalance coefficient (*UB*) is regulated to be below 2% [6]. The *UB* coefficient is given by [6]:

$$UB = \sqrt{\frac{1-\sqrt{3-6\lambda}}{1+\sqrt{3-6\lambda}}} \times 100 \tag{2}$$

with:

$$\lambda = \frac{V_{12}^4 + V_{23}^4 + V_{31}^4}{\left(V_{12}^2 + V_{23}^2 + V_{31}^2\right)^2} \tag{3}$$

and V_{12}, V_{23} and V_{31} are the line voltages (rms values).

2. The voltage magnitude of each node of each branch must lie within a permissible range. Here a branch can be a transformer, a line section or a tie line with a sectionalizing switch.

$$\boldsymbol{V}_j^{\min} \le \left|\boldsymbol{V}_j\right| \le \boldsymbol{V}_j^{\max} \tag{4}$$

The equation (5) shows the relation per phase between no-load voltage ($\boldsymbol{V}_{oj}$), internal impedance ($\boldsymbol{Z}_j$) and load current ($\boldsymbol{I}_j$), where $\boldsymbol{V}_j$, $\boldsymbol{I}_j$ and are complex phasors and j =1,2,3:

$$\boldsymbol{V}_j = \boldsymbol{V}_{oj} - \boldsymbol{Z}_j \cdot \boldsymbol{I}_j \tag{5}$$

Given the above dependency between voltage and load current, this study will focus on the currents.

3. Due to some practical considerations, there could be a constraint on the number of switch-on/switch–off operations involved in the network reconfiguration.

III. PROPOSED MODEL

Given a distribution system as shown in Figure 1, a network with 3 phases with a known structure, the problem consists of finding a condition of balancing; the unbalanced load creates losses in the network. The mathematical model can be expressed as:

$$\boldsymbol{I}_{ph1k} = \sum_{i=1}^{3} sw_{k1i}\boldsymbol{I}_{ki} + \boldsymbol{I}_{ph1(k+1)} \tag{6}$$

$$\boldsymbol{I}_{ph2k} = \sum_{i=1}^{3} sw_{k2i}\boldsymbol{I}_{ki} + \boldsymbol{I}_{ph2(k+1)} \tag{7}$$

$$\boldsymbol{I}_{ph3k} = \sum_{i=1}^{3} sw_{k3i}\boldsymbol{I}_{ki} + \boldsymbol{I}_{ph3(k+1)} \tag{8}$$

where $\boldsymbol{I}_{ph1k}$, $\boldsymbol{I}_{ph2k}$ and $\boldsymbol{I}_{ph3k}$ represent the currents (phasors) per phase 1, 2 & 3 after the k point of connection, sw_{k11} … sw_{k33} are different switches (the value of "1" means the switch is closed and "0" means it is open), and $\boldsymbol{I}_{k1}$, $\boldsymbol{I}_{k2}$ and $\boldsymbol{I}_{k3}$ represent different load currents (phasors) connected to the distribution system at point k of connection (see Figure 1); a load-connection is done via a switching matrix, that is achieved with triacs or anti-parallel-connected thyristors.

The proposed switching technique is zero current switch-off and zero voltage switch-on. Figure 2 shows the transition of current I_{k1} from phase one to phase three. As can be noticed, the switch controller firstly receives the decision of changing the connection of the load I_{k1} waits until the current extinguishes and then switches on the switch sw_{k13}.

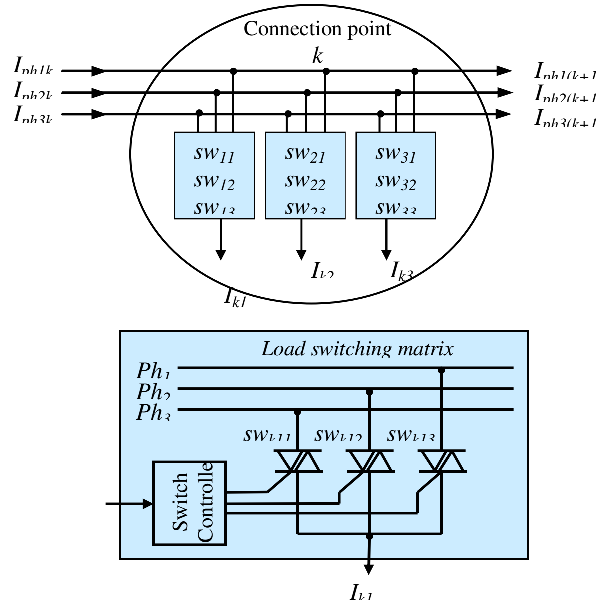

Fig. 1 Distribution network

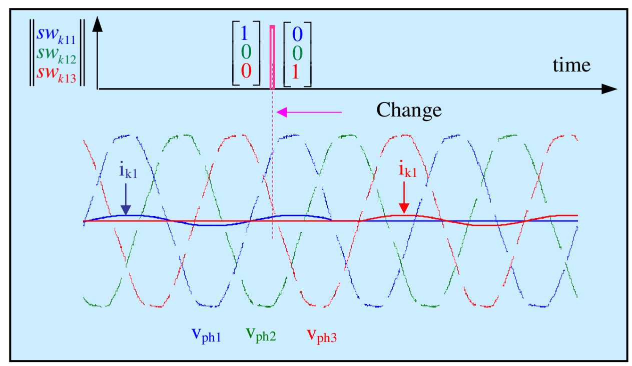

Fig. 2 Re-arrangement of current I_{k1} from phase one to three

The dead time created is very small and cannot be bigger than 17 msec-below the concern of the regulation [6].

The constraint of only allowing one breaker in each of equations (6) to (8) to be closed, we can write the following set of constraints:

$$\sum_{i=1}^{3} sw_{k1i} - 1 = 0 \tag{9}$$

$$\sum_{i=1}^{3} sw_{k2i} - 1 = 0 \tag{10}$$

$$\sum_{i=1}^{3} sw_{k3i} - 1 = 0 \tag{11}$$

in the general form:

$$sw_{k1i} + sw_{k2i} + sw_{k3i} - 1 = 0 \tag{12}$$

where i vary from 1 to 3.

To minimize the power loss eq. (1), the neutral current should be minimized. Therefore, the objective of this new algorithm is to minimize the difference of the amplitude of the phase currents ($\hat{I}_{phik}$):

$$\text{Minimize} \begin{vmatrix} \hat{I}_{ph1k} - \hat{I}_{ph2k} \\ \hat{I}_{ph1k} - \hat{I}_{ph3k} \\ \hat{I}_{ph2k} - \hat{I}_{ph3k} \end{vmatrix} \tag{13}$$

The Least Squares objective function proposed for this study is:

$$J = (I_{ph1k} - I_{ph2k})^2 + (I_{ph1k} - I_{ph3k})^2 + (I_{ph2k} - I_{ph3k})^2 \tag{14}$$

Now, the task is to minimize eq. (13) subject to constraints (9) – (11). There will be a need to derive the expression for the gradient vector. Let n, be the number of different variables. The gradient will equal:

$$\frac{\partial J}{sw_{kw1i}} = 2(I_{ph1k} - I_{ph2k})I_{ki} + 2(I_{ph1k} - I_{ph3k})I_{ki} \tag{15}$$

$$\frac{\partial J}{sw_{kw2i}} = -2(I_{ph1k} - I_{ph2k})I_{ki} + 2(I_{ph1k} - I_{ph3k})I_{ki} \tag{16}$$

$$\frac{\partial J}{sw_{kw13}} = -2(I_{ph1k} - I_{ph2k})I_{ki} + 2(I_{ph1k} - I_{ph3k})I_{ki} \tag{17}$$

The variation of the load currents is random and thereof the minimization of the objective function J should be converted into a physical problem such as Brownian movement.

IV. Numerical Solvers

A. Gauss-Newton Method

To solve the system of non-linear equations in (13), the system is linearized around some working point $\mathbf{x}_k$ by using a Taylor series expansion:

$$\mathbf{J}_{xx}(\mathbf{x}_k)\Delta\mathbf{x}_k + \mathbf{J}_x(\mathbf{x}_k) = \mathbf{0} \tag{18}$$

where $\mathbf{J}_{xx}(\mathbf{x}_k)$ is the Hessian matrix of the objective function J. The state correction vector $\Delta\mathbf{x}_k$ can then be calculated from the following system of linear equations:

$$\mathbf{J}_{xx}(\mathbf{x}_k)\Delta\mathbf{x}_k = -\mathbf{J}_x(\mathbf{x}_k) \tag{19}$$

An iterative procedure can be used and better iterates of the state vector can be calculated by [7,8]:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}_k \tag{20}$$

The next step is to derive the objective function J for the gradient vector and the Hessian matrix (**H**). Let n be the number of state variables. The gradient vector (**D**) is:

$$\mathbf{J}_x = \left[\frac{\partial J}{\partial x_1}, \frac{\partial J}{\partial x_2}, \cdots, \frac{\partial J}{\partial x_n}\right]^T = \mathbf{H}^T\mathbf{D}\mathbf{W}^2\hat{\mathbf{e}} \tag{21}$$

where $\mathbf{H}$ is the Hessian matrix (for *i=1,2…m* and *j=1,2…n* follows), $\hat{\mathbf{e}}$ is an m dimensional column vector with all its entries equal to one.

$$[\mathbf{H}]_{ij} = \frac{\partial \tilde{r}_{s_i}}{\partial x_j} = \frac{\partial\left\{\dfrac{z_i - h_i(\mathbf{x})}{\sigma_i\omega_i E_s}\right\}}{\partial x_j} = -\frac{1}{\sigma_i\omega_i E_s}\frac{\partial h_i(\mathbf{x})}{\partial x_j} \tag{22}$$

and the elements of the diagonal matrices are:

$$[\mathbf{D}]_{ii} = \frac{\partial \rho}{\partial \tilde{r}_{s_i}} = \begin{cases} \tilde{r}_{s_i} & \text{if } \left|\tilde{r}_{s_i}\right| \le \beta \\ sign(\tilde{r}_{s_i})\cdot\beta & \text{if } \left|\tilde{r}_{s_i}\right| > \beta \end{cases} \tag{23}$$

$$[\mathbf{W}]_{ii} = \omega_i \tag{24}$$

The Hessian matrix is given by:

$$\mathbf{J}_{xx} = \alpha_k \sum_{i=1}^{m}\left(\omega_i^2 \frac{\partial \rho}{\partial \tilde{r}_{s_i}}\mathbf{G}_i\right) + \mathbf{H}^T\tilde{\mathbf{D}}\mathbf{W}^2\mathbf{H} \tag{25}$$

with:

$$[\mathbf{G}_i(\mathbf{x})]_{jk} = \frac{\partial^2 \tilde{r}_{s_i}}{\partial x_j \partial x_k} = \frac{\partial^2\left\{\dfrac{z_i - h_i(\mathbf{x})}{\sigma_i\omega_i E_s}\right\}}{\partial x_j \partial x_k} = -\frac{1}{\sigma_i\omega_i E_s}\frac{\partial^2 h_i(\mathbf{x})}{\partial x_j \partial x_k} \tag{26}$$

where for *i=1,2…m* and *j,k=1,2…n* and the elements of the diagonal matrix $\tilde{\mathbf{D}}$ are:

$$[\tilde{\mathbf{D}}]_{ii} = \frac{\partial^2 \rho}{\partial \tilde{r}_{s_i}^2} = \begin{cases} 1 & \text{if } \left|\tilde{r}_{s_i}\right| \le \beta \\ 0 & \text{if } \left|\tilde{r}_{s_i}\right| > \beta \end{cases} \tag{27}$$

Where respectively σ_i, ω_i, E_s are the weight of the Hessian matrix, the measurement of load current that is classified as leverage point and the scaling factor for the residuals; β is the break-even point of the non quadratic function.

The parameter α_k in (25) determines the amount of second order information is use in constructing the Hessian. If the equation (25) is used for the Hessian matrix, then the full Newton method is used. But the calculation of the second order derivative matrix $\mathbf{G}_i(\mathbf{x})$ does not converge all the times.

The Gauss-Newton method approximates the Hessian matrix by setting the second order derivative term $\mathbf{G}_i(\mathbf{x})$ equal to zero. The Hessian then becomes:

$$\mathbf{J}_{xx} = \mathbf{H}^T\tilde{\mathbf{D}}\mathbf{W}^2\mathbf{H} \tag{28}$$

This method has fast local convergence on mildly non-linear problems, but it may also be non-convergent for very non-linear problems or problems that have large residuals [8].

According to the above findings, the Gauss-Newton method does not converge for this particular application and another method has been evaluated (dynamic leap-frog).

B. Dynamic Leap-frog Method

This method [5] differs conceptually from the other gradient methods, like the conjugate gradient method. It considers the analogous physical dynamic problem of the motion of a particle of unit mass in an n-dimensional conservative force field. The potential energy of the particle at point $x(t)$ at time t is represented by the function $J(x)$ to be minimised. This method requires the solution of the particle's equations of motion, subject to its initial position and velocity.

Using the *leap-frog* (Euler forward - Euler backward) method, an approximation to the associated trajectory is calculated. In such a conservative force field the total energy of a particle is conserved. The total energy consists of the kinetic and potential energy. By monitoring the kinetic energy, an interfering strategy is adopted such that the potential energy is systematically reduced. The particle is thus forced to follow a trajectory to the local minimum in the potential energy.

The characteristics of this method can be listed as follows:

- Uses only gradient information.
- No explicit lines searches are performed.
- Very robust: it handles discontinuities and steep valleys in functions and gradients.
- Not as efficient on smooth and near quadratic functions when compared to classical methods.
- Algorithm seeks a low local minimum and can thus be used in a methodology for global optimization.

B.1 Basic dynamic model

Assume that a particle of unit mass is moving in an n-dimensional conservative force field with a potential energy at x given by $J(x)$. The force on the particle is then given by:

$$a = \ddot{x} = -\nabla J(x) \tag{29}$$

The kinetic energy associated with the particle is:

$$T(x) = \left\| \dot{x}(t) \right\|^2 \tag{30}$$

Where $\| \dot{x}(t) \|$ is the velocity ($v(t)$) at time t. Then at any time instant t, because of the conservation of energy:

$$T(t) + J(t) = constant \tag{31}$$

Note that for any changes ΔJ and ΔT along the trajectory it follows that $\Delta J = -\Delta T$ and therefore as long as T increases J decreases.

The method can be stated as follows [5]:

- Compute the dynamic trajectory by solving the Initial Value Problem (IVP):

$$\ddot{x}(t) = -\nabla J\left(x(t)\right) \tag{32}$$

with $\mathbf{x}(0)$ and $\mathbf{v}(0)$ given. To solve this IVP, we do numerical integration of (30) using the *leap-frog* method. With initial starting point $\mathbf{x}_0$ and time step Δt, set $v_o = a_o \cdot \Delta t/2$ and compute in each iteration:

$$x_{k+1} = x_k + v_k \cdot \Delta t \tag{33}$$

$$a_{k+1} = \nabla J_k \tag{34}$$

$$v_{k+1} = v_k + a_{k+1} \cdot \Delta t \tag{35}$$

- Monitor the kinetic energy. As long as it increases, the potential energy decreases.
- When the kinetic energy decreases, apply some interfering strategy. A typical interfering strategy would be:

if $\| v_{k+1} \| \geq \| v_k \|$
continue
else
set: $v_k = (v_{k+1} + v_k)/4$ and $x_k = (x_{k+1} + x_k)/2$
compute the new v_{k+1} and continue

- The starting value of Δt is dependent on the magnitude of a specified maximum step size δ, and the initial gradient $\nabla J(x_0)$. A good rule to use when choosing δ is $\delta \leq \sqrt{n} \cdot (\text{range of the variables})$, where n is the number of variables and the range is the difference between the typical maximum and minimum values of the state variables. The initial value of Δt can be calculated by (24) to (26). During the iteration process the value of Δt is internally adjusted by the algorithm:

$$\Delta t = \sqrt{\frac{\delta}{5 \cdot \left\| \nabla J\left(x_0\right) \right\|}} \tag{36}$$

To prevent oscillations of the trajectory at maximum step size δ, the time step is halved with the switching of gradient direction (i.e. $a_{k+1} \cdot a_k < 0$).

B.2 Algorithm flow chart

A flow chart of the unconstrained dynamic leapfrog method is shown in figure 3.

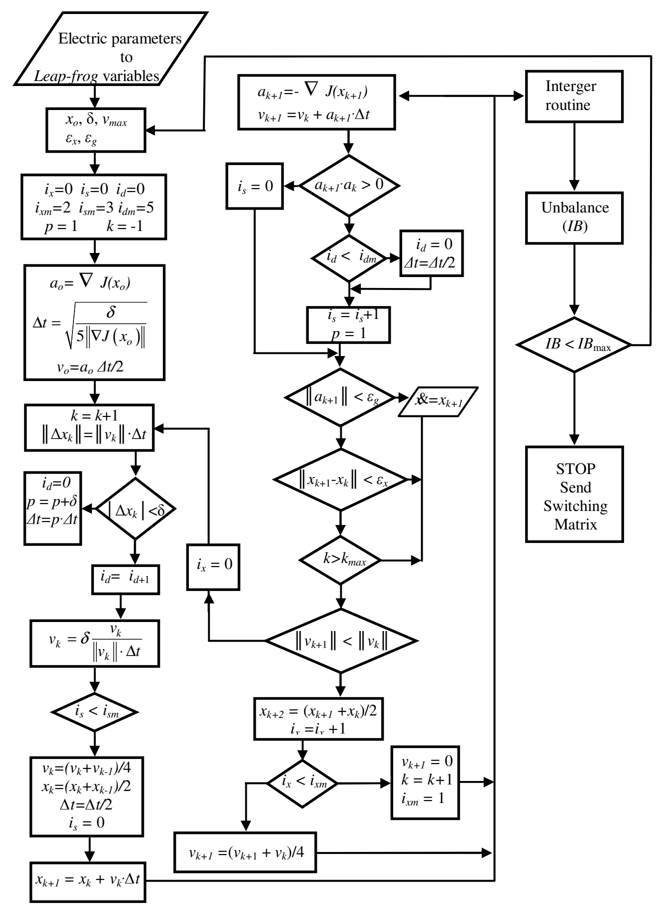

Fig. 3 Flow chart of dynamic *leap-frog* method

The following additional variables are used in the flow chart:

- Convergence ε_g and ε_x for the gradient and state variable, respectively
- Counters i_x, i_s and i_d with maximum values i_{xm}, i_{sm} and i_{dm} , respectively
- p is used as adjustment of Δt.
- k is the iteration counter and k_{max} is the maximum number of allowed iterations.
- β is the unbalance coefficient with maximum admitted value of $\beta_{\max}$.

After the dynamic leap-frog finishes the computing, the program converts back the results in the new electrical parameters: the new switching matrix and phase-currents.

SIMULATION RESULTS

The solution of this problem consists of keeping the load balanced and to reduce the power loss in the network. In order to check the proposed automatic load balancing, a number of 15 loads (house-holds) were considered and have been grouped in five connection points. To make the method easer, the power factor of each load was taken as 1.

The current balancing coefficient (*IB*) si not regulated [6], therefore this paper propose it as:

$$IB = \sqrt{\frac{I_M - I_m}{I_M + I_m}} \qquad (37)$$

where I_M is the maximum current in the three-phase system and I_m is the minimum current, and the neutral current will be equal:

$$I_n = I_{ph1} + I_{ph2} + I_{ph3} \qquad (38)$$

From the results shown, it can be seen that the Leapfrog method converged in situations where Gauss-Newton failed to converge. Also in some situations where Gauss-Newton gave completely unusable results, the Leapfrog method gave a better and more usable solution of the automatic load balancing.

Next the algorithm has been tested for a situation of 15 house-hold loads with five connection points, where $k = 1$ is close to the distribution transformer; it is generally accepted that the power factor of a house-hold is close to unity. Table I presents the initial switching matrix $\|sw_{kij}\|$ where $k = 1,\ldots,5$ $i,j = 1,2,3$ and the load current matrix $\|\boldsymbol{I}_{ki}\|$. Initially, loads $\boldsymbol{I}_{k1}$ are connected to phase 1 ($\boldsymbol{I}_{ph1}$), loads $\boldsymbol{I}_{k2}$ are connected to phase 2 ($\boldsymbol{I}_{ph1}$) and $\boldsymbol{I}_{k3}$ to phase 3 ($\boldsymbol{I}_{ph1}$).

If the distribution transformer has a no-load voltage of 230 V_{ph} and internal impedance per phase of 0.038Ω and 0.3 mH, then the initial 60% unbalance in current is converted into 0.6% unbalance in voltage.

Table II shows the feeder' load distribution after automatic balance algorithm determined the new switching matrix. Let us consider the situation changes. Table III shows the new load currents distribution and the associated switching matrix as calculate by the expert system.

TABLE I - STUDY CASE: BEFORE BALANCING

	k = 1		k = 2		k = 3		k = 4		k = 5	
	$\|sw_{1ij}\|$	$\|I_{1i}\|$	$\|sw_{2ij}\|$	$\|I_{2i}\|$	$\|sw_{3ij}\|$	$\|I_{3i}\|$	$\|sw_{4ij}\|$	$\|I_{4i}\|$	$\|sw_{5ij}\|$	$\|I_{5i}\|$
$I_{k1}\angle 0^0$	1 0 0	10	1 0 0	12	1 0 0	8	1 0 0	3	1 0 0	5
$I_{k2}\angle -120^0$	0 1 0	15	0 1 0	14	0 1 0	18	0 1 0	14	0 1 0	17
$I_{k2}\angle +120^0$	0 0 1	18	0 0 1	10	0 0 1	11	0 0 1	17	0 0 1	20
$I_{ph1}\angle 0^0$	38		28		16		8		5	
$I_{ph2}\angle -120^0$	81		63		49		31		17	
$I_{ph2}\angle +120^0$	76		58		48		37		20	
I_n	$40.7\angle -173.9^0$		$32.8\angle -172.4^0$		$32.5\angle +178.6^0$		$26.5\angle +168.7^0$		$13.8\angle +167.5^0$	
$V_{ph1}\angle 0^0$	226.1									
$V_{ph2}\angle -120^0$	221.8									
$V_{ph2}\angle +120^0$	222.3									

TABLE II - STUDY CASE: NEW SWITCHING MATRIX

	k = 1		k = 2		k = 3		k = 4		k = 5	
	$\|sw_{1ij}\|$	$\|I_{1i}\|$	$\|sw_{2ij}\|$	$\|I_{2i}\|$	$\|sw_{3ij}\|$	$\|I_{3i}\|$	$\|sw_{4ij}\|$	$\|I_{4i}\|$	$\|sw_{5ij}\|$	$\|I_{5i}\|$
$I_{k1}\angle 0^0$	1 0 0	10	1 0 0	12	0 1 0	8	0 0 1	3	1 0 0	5
$I_{k2}\angle -120^0$	0 1 0	15	0 0 1	14	1 0 0	18	0 1 0	14	0 1 0	17
$I_{k2}\angle +120^0$	0 0 1	18	0 0 1	10	0 0 1	11	1 0 0	17	0 0 1	20
$I_{ph1}\angle 0^0$	62		52		40		22		5	
$I_{ph2}\angle -120^0$	64		49		39		31		17	
$I_{ph2}\angle +120^0$	66		48		34		23		20	
I_n	$3.5\angle +150^0$		$3.6\angle -14^0$		$5.6\angle -51.5^0$		$8.5\angle -125.8^0$		$13.8\angle +167.5^0$	
$V_{ph1}\angle 0^0$	226.1									
$V_{ph2}\angle -120^0$	221.8									
$V_{ph2}\angle +120^0$	222.3									

TABLE III - STUDY CASE: ANOTHER SET OF LOAD CURRENTS

	$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$	
	$\|sw_{1ij}\|$	$\|I_{1i}\|$	$\|sw_{2ij}\|$	$\|I_{2i}\|$	$\|sw_{3ij}\|$	$\|I_{3i}\|$	$\|sw_{4ij}\|$	$\|I_{4i}\|$	$\|sw_{5ij}\|$	$\|I_{5i}\|$
$I_{k1}\angle 0^0$	0 1 0	15	1 0 0	9	1 0 0	8	0 0 1	3	1 0 0	15
$I_{k2}\angle -120^0$	1 0 0	20	0 0 1	20	0 0 1	13	0 1 0	14	0 1 0	7
$I_{k2}\angle +120^0$	0 0 1	10	0 1 0	10	0 1 0	11	0 1 0	17	0 0 1	20
$I_{ph1}\angle 0^0$	68		38		39		31		15	
$I_{ph2}\angle -120^0$	63		48		38		27		7	
$I_{ph2}\angle +120^0$	63		53		33		20		20	
I_n	$5\angle 0^0$		$6\angle +120^0$		$5.57\angle -51.05^0$		$9.64\angle -38.9^0$		$11.35\angle +82.4^0$	
$V_{ph1}\angle 0^0$	223.1									
$V_{ph2}\angle -120^0$	223.6									
$V_{ph2}\angle +120^0$	223.6									

As a result of applying the proposed automatic balancing method (Table II & III), one can notice the improvement in current and voltage balancing.

VII. CONCLUSIONS

In this paper a load balancing in low voltage distribution feeders and the dynamic leapfrog method is presented with a set of simulation. The only disadvantage about the leapfrog is the slowly-ness to obtain convergence. With the two tables presented the results have been shown before and after the automatic balancing to demonstrate the effectiveness of the *leap-frog* algorithm.

The Gauss-Newton and Dynamic Leapfrog methods were tested in solving the problem. In the cases where the Gauss-Newton method failed to converge, the dynamic method guaranteed convergence. Thus, in general we can use the Gauss-Newton method to solve the load balancing problem and when it fails to produce results, switch to the Dynamic method. Although the dynamic method is slow compared to Gauss-Newton to obtain convergence, it can be used in situations where convergence must be obtained. Also, research can be done on using a combination of Gauss-Newton and the Dynamic method: start the load balancing process with the Dynamic method, and when it comes close to the solution, switch to Gauss-Newton to speed up the convergence. For the situation presented in tables I to III the computing time was below one second; if the number of loads is increased to one hundred, then the computing time could raise up to few seconds. This computing time is not very high considering the relatively slow dynamic of the house hold loads.

Analyzing the algorithm and the simulations, it results that for more loads the balancing is improved much better, the neutral current becomes very low and the transformer losses due to unbalancing decrease significantly, which was the primary aim of this study.

The system seems to be a little bit costly, but it can perform other functions necessary for distribution systems management.

The study will further focus on the effect of very short dips (approximately 17 msec) upon various types of loads and the methods to minimize this effect.

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