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A new hyperchaotic system and its circuit implementation

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Abstract

A new hyperchaotic system which has two large positive Lyapunov exponents is presented and physically implemented. Spectral analysis shows that the system in the hyperchaotic mode has an extremely broad frequency bandwidth of high magnitudes, verifying its unusual random nature and indicating its great potential for some relevant engineering applications such as secure communications.

1. Introduction

Hyperchaos characterized with more than one positive Lyapunov exponent (LE), has attracted increasing attention from various scientific and engineering communities [1–3]. It is very important to generate hyperchaos with more complicated dynamics as a model for theoretical research and practical implication. Hyperchaos firstly reported by Rössler [4], and the first circuit implementation of hyperchaos was realized by Matsumoto et al. [5]. Lately, there has been considerable interest in the study of hyperchaos generation by modifying and coupling the existing chaotic systems or hyperchaotic systems. For example, to generate hyperchaos, Nikolov and Clodong [6] modified the Rössler hyperchaotic system [4] and coupled two Chua's circuits in [7]. Cafagna and Grassi [8,9] coupled three Chua's circuits together to obtain a new hyperchaotic attractor. Thamilmaran et al. [10] modified Chua's circuit to present a new hyperchaotic system. Recently, Li et al. [11,12] also obtained a hyperchaotic systems by modifying two existing chaotic systems. However, there exist some common problems in most existing hyperchaotic systems, i.e. the two positive LEs simultaneously appear in a very narrow parameter range and their values are relatively small, especially for the first positive LE [4–12].

Recently, we proposed a 3-D chaotic system [14] and a 4-D chaotic system [15]. Some complicated dynamical behaviors, bifurcation analysis and circuit implementation of the 4-D chaotic system were investigated and controlled [16–18]. In this paper, we propose a new hyperchaotic system, called Qi hyperchaotic system. We will show that the system has

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two very large positive LEs over very wide parameter regions, implying that system orbits extensively expand in some directions but rapidly shrink in some other directions, which significantly increase the system's orbital degree of disorder and randomness, and is very desirable for engineering applications such as secure communications. The electronic circuit realizing the Qi hyperchaotic system is implemented and demonstrated.

2. The new 4-D hyperchaotic system

Consider the following new 4D dynamical system (called Qi hyperchaotic system):

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3, \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3, \\ \dot{x}_3 &= -c x_3 - e x_4 + x_1 x_2, \\ \dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2. \end{aligned} \tag{1}$$

Here, $x_i (i = 1, 2, 3, 4)$ are state variables and a, b, c, d, e, f are positive constant parameters.

Fig. 1 indicates the LE spectrum with $b \in [15.425, 27]$, and $a = 50$ $c = 13$, $d = 8$, $e = 33$, $f = 30$. One can observe that there are two positive LEs over quite a wide range of parameter values. As the parameter b varies, as shown in Fig. 1, the first LE is very large with $l_1 \in [8.3585, 13.4632]$, the second LE is quite large with $l_2 \in [0.1, 3.4781]$, the third LE is approximately zero, and the forth LE is negative with $l_4 < -60$.

The leading LE is less than 2 for most existing hyperchaotic systems, namely $l_1 \in [0.11, 1.7]$ in [4–9,11,12]. The second largest LE is also relatively small with $l_2 \in [0.02, 0.18]$ in [4–8,10–12]. However, the leading LE l_1 of the Qi hyperchaotic system here is notably as large as 13.4632. Similarly, the second largest LE is considerably large and positive, with a value up to 3.4781. The exponential expansions of an attractor with positive LEs are incompatible with motions on a bounded space unless there are many folds. The values of the leading LE and the second LE imply an exponential rate of stretching and folding, i.e. $e^{l_1 t}$ and $e^{l_2 t}$, in two different directions, as the orbit moves. This is a good measure as how chaotic the attractor is [13]. For comparison, suppose a hyperchaotic system with two positive LEs, $l_1 = 0.6$, $l_2 = 0.1$. Then, the rates of stretching and folding of the Qi hyperchaotic system with $l_1 = 13.4632$ and $l_2 = 3.478$, will be n times and m times faster than that of the assumed hyperchaotic system, where

$$n = \left.\frac{e^{13.4632t}}{e^{0.6t}}\right|_{t=1} \approx 2.4 \times 10^5, \quad m = \left.\frac{e^{3.478t}}{e^{0.1t}}\right|_{t=1} \approx 29.3. \tag{2}$$

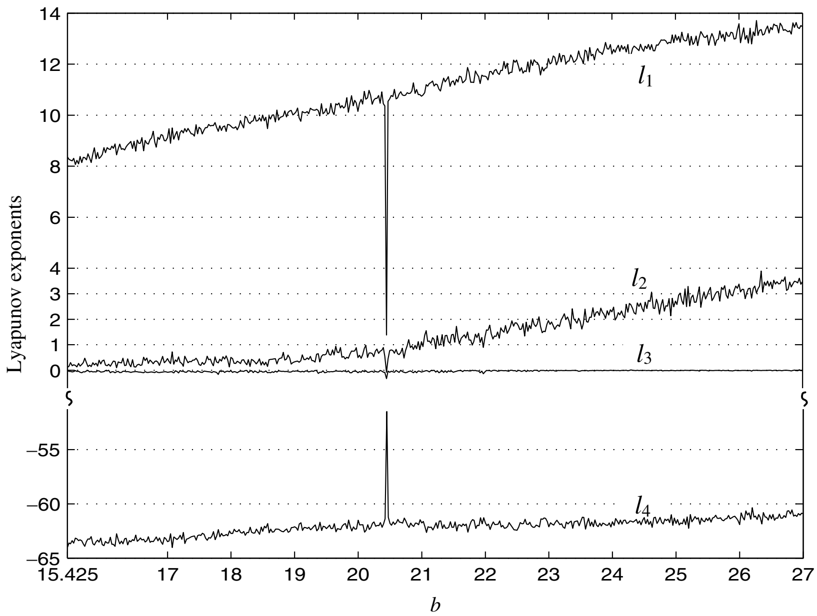
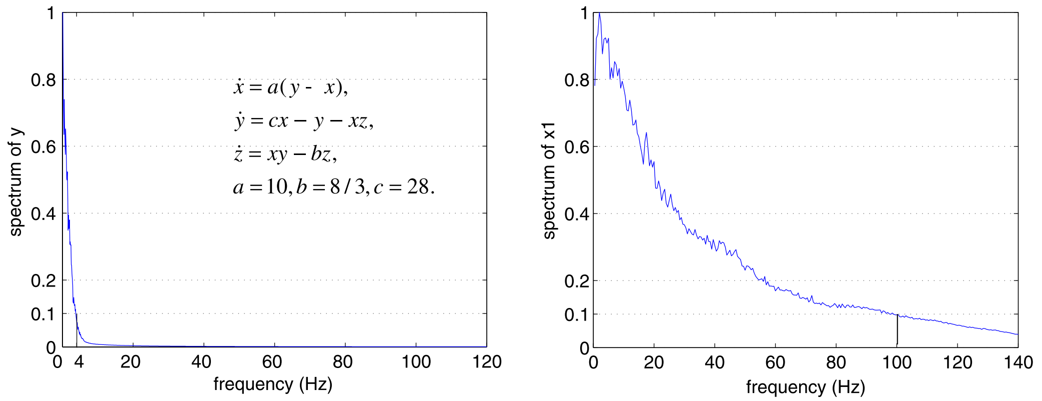


Fig. 1. The Lyapunov exponent spectrum versus b, with $a = 50$ $c = 13$, $d = 8$, $e = 33$, $f = 30$.



(a) The Lorenz system

(b) The Qi hyperchaotic system with $a = 42, b = 26, c = 13, d = 20, e = 50, f = 40$

Fig. 2. The comparison of frequency spectra.

Many proposed chaos-based encryption schemes have been totally or partially broken by different attacks. One reason is that the degree of randomness of simple chaotic or hyperchaotic signals used therein are not high enough, as reflected by the narrow high-magnitude bandwidths of those signals [19–21].

3. Analysis of frequency spectrum

Fig. 2a shows the frequency spectra of the Lorenz system. For comparison, the Runge–Kutta method was used to solve all systems, all with sampling time 0.0002(s), running time 0–500 (s), number of spectral averages 100, and all

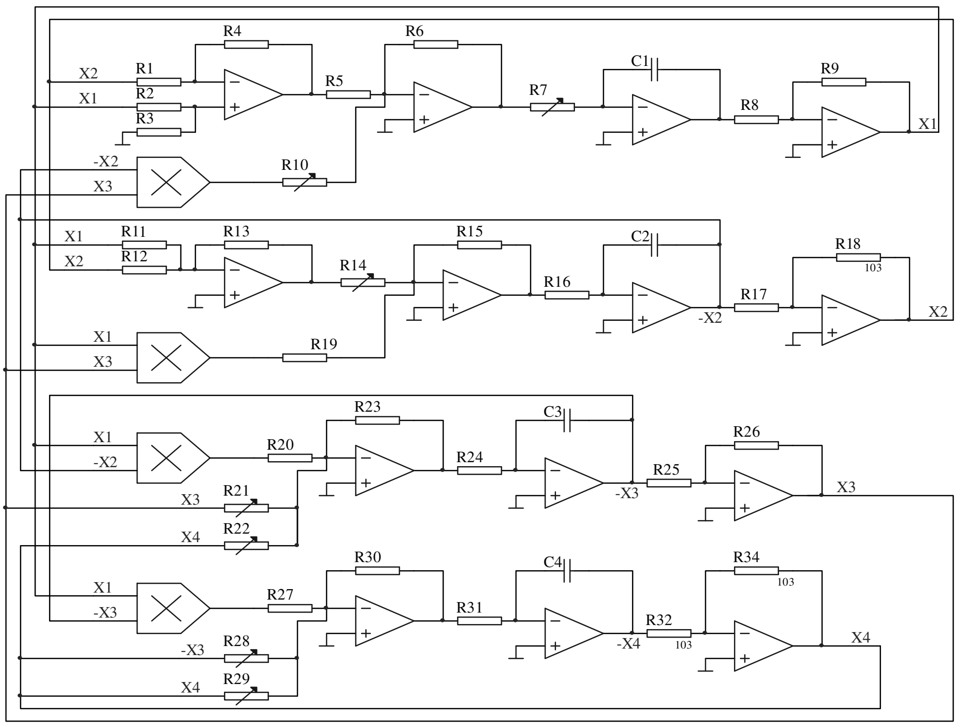
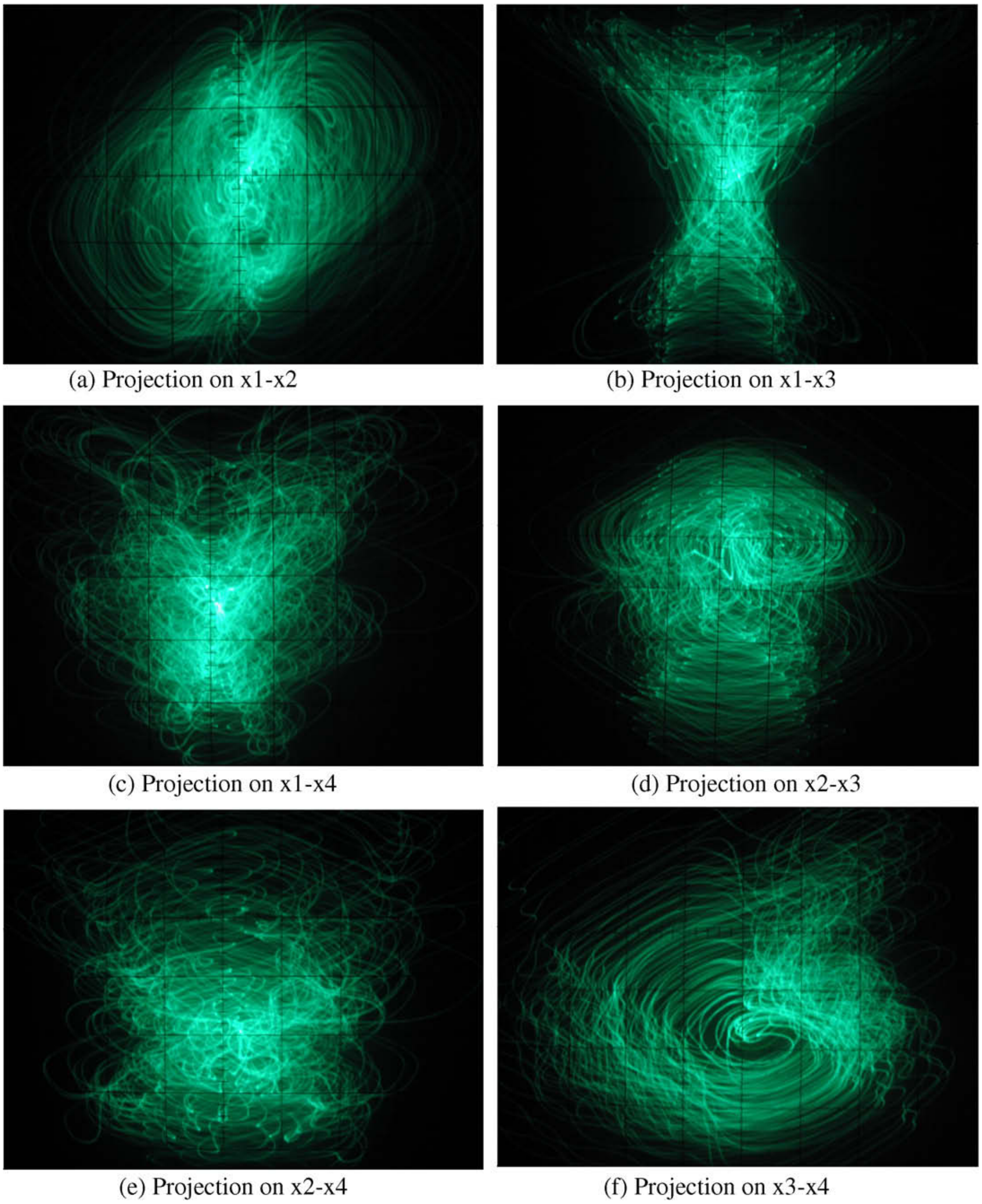


Fig. 3. The diagram of implementation of the Qi hyperchaotic system, where R1, R2, R3, R4, R5, R6, R8, R9, R11, R12, R13, R15, R17, R18, R23, R25, R26, R30, R32, R33 = 10k; R16, R24, R31 = 100k; R7 = 20k; R14 = 4.17k; R21 = 7.69k; R22 = 3.03k; R28 = 3.33k; R29 = 12.5k; R10 = 500Ω; R19, R20, R27 = 100Ω; Ci = 5 (nF), $i = 1,\ldots,4$.

spectra are normalized, where only the frequency ranges with spectral values over 10^{-1} are considered. The high-magnitude bandwidths of the Lorenz system, the Chua circuit and the Rössler hyperchaotic system are all less than 4 Hz numerically. Some hyperchaotic systems also have narrow high-magnitude spectral bandwidths. For example, Li et al. [12] presented a hyperchaotic system with positive LEs $l_1 = 0.5011$, $l_2 = 0.1858$, with the frequency spectral bandwidth less than 4.5 Hz. Therefore, the chaotic signals cannot be used to sufficiently mask the messages in real communication applications. Once intercepted, there is a high possibility that the messages can be extracted.

In practice, the frequency of the analog signal of a chaotic system can be extended to several kHz, even several MHz, therefore 4 Hz is essentially useless. To compare, only the bandwidths of the signals generated numerically by the



(a) Projection on x1-x2 (b) Projection on x1-x3

(c) Projection on x1-x4 (d) Projection on x2-x3

(e) Projection on x2-x4 (f) Projection on x3-x4

Fig. 4. The phase portraits of the Qi hyperchaotic system observed on the oscilloscope, with $a = 50$, $b = 24$, $c = 13$, $d = 8$, $e = 33$, $f = 30$.

system itself are discussed here, without investigating possible electronic techniques that may somewhat improve the bandwidths of such chaotic systems including the new system.

When taking the parameters $a = 50$, $c = 26$,$d = 8$, $e = 33$, $f = 30$ in the Qi hyperchaotic system, there are two positive LEs, $l_1 = 13.4632$, and $l_2 = 3.478$. The frequency spectra of the signals x_1 and x_2, generated numerically from the Qi hyperchaotic system, are shown in Fig. 2b. Note that the high-magnitude bandwidth of signal x_1 is 100 Hz, which is more than 20 times wider than that of all the other systems mentioned above. One can see that the frequency property of the Qi hyperchaotic system is indeed exhibiting very strong randomness. In fact, the strong random property is also confirmed and demonstrated by the following circuit implementation.

4. Circuit implementation

Fix the parameter $b = 24$. Then, the LEs are $[l_1, l_2, l_3, l_4] = [12.3981, 2.2944, -0.0009, -61.6578]$.

As shown in Fig. 3, an electronic circuit has been designed and built to realize system (1) with 4 channels to perform the integration of the four state variables, x_1, x_2, x_3, x_4, respectively.

Fig. 4 shows the experimental results observed on an oscilloscope. It can be seen that: (1) the orbits are more complicated and disordered than the ordinary chaotic and hyperchaotic ones; (2) the attractors have very irregular forms, neither butterfly nor scroll ones, which are visually more complex than ordinary chaotic attractors.

5. Conclusions

In this paper, we have mathematically constructed and electronically built the Qi hyperchaotic system. The Lyapunov exponent spectrum of the Qi hyperchaotic system, particularly its leading and second LEs, shows that the system is a truly hyperchaotic one within a large parameter range, giving the system very strong randomness, a high degree of disorder and extremely rich dynamics. More importantly, the frequency spectral analysis demonstrates that the new system hyperchaos exists over an extremely broad high-magnitude bandwidth, which is very desirable for some engineering applications such as secure communications.

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