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Optical solitons for Gerdjikov–Ivanov model by extended trial equation scheme

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ABSTRACT

This paper studies the dynamics of soliton propagation through optical fibers and PCF that are governed by Gerdjikov–Ivanov equation. The extended trial function approach is the integration scheme that is employed in this paper. There are bright and singular optical soliton solutions retrieved. The existence condition for these solitons is also presented.

1. Introduction

The dynamics of optical solitons to various forms of waveguides are modeled by a number of nonlinear evolution equations. Of course, the most visible one is the nonlinear Schrödinger's equation. There are however various other models that are lesser known and less visible. These are Chen–Lee–Liu equation, Sasa–Satsuma equation, Manakov model, Thirring solitons, Gabitov–Turitsyn equation, complex Ginzburg–Landau equation, Lakshmanan–Porsezian–Daniel model and several others. This paper will study the dynamics of soliton evolution to one such model that had lately gained popularity. This is the Gerdjikov–Ivanov equation. This equation has been studied by a variety of authors and some of the latest results are clearly visible all across [1–20]. This paper is going to implement another well-known and yet popular integration scheme that will retrieve soliton and other solutions to the model. This is the extended trial equation method and it extracts bright

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and singular soliton solutions. This integration methodology has been successfully applied to study magneto-optic waveguides and nematicons in liquid crystals [4,6]. In addition to soliton solutions, this integration scheme as a byproduct, obtains a few other solutions such as singular periodic waves are also recovered and they are also discussed in this work. The rest of the paper discusses the model, the integration scheme and then the application of the algorithm.

2. The model

We start by considering the propagation of an optical pulse propagation through a waveguide that obeys the Gerdjikov–Ivanov equation which is modeled as [1]:

$$iq_t + aq_{xx} + ibq^2 q_x^* + c|q|^4 q = 0, \tag{1}$$

where $q = q(x, t)$ is the complex wave function, q^* is the complex conjugate of q, while a, b, and c are real-valued constants.

In this case when $a = 1$, $b = -1$ and $c = \frac{1}{2}$, Eq. (1) collapses to the case of regular Gerdjikov–Ivanov equation. In the context of optical fiber physics, the term involving the parameter b is usually associated with the "self-steepening" phenomena, while terms involving a and c are associated with group velocity dispersion and quintic nonlinearity respectively. In such contexts, the coordinates t and x denote propagation distance and retarded time. They, however, represent slow time and spatial coordinate traveling with the group velocity as pointed out earlier [18].

As we are interested in solutions with nonlinear chirp of Eq. (1), we start from the representation of the complex field $q(x, t)$ in the form [1,18]

$$q(x,t) = \rho\left(\xi\right) e^{i[\chi(\xi) - \omega t]}, \tag{2}$$

where $\xi = x - vt$, $\rho\left(\xi\right)$ is the amplitude function, and $\chi\left(\xi\right)$ is the phase function. Also, v is the wave velocity, and ω is the frequency of the wave oscillation.

The substitution of this expression into Eq. (1) and the separation of the real and imaginary parts lead to a pair of coupled equations in ρ and χ:

$$\omega\rho + v\chi'\rho + a\rho'' - a\rho\chi'^2 + b\chi'\rho^3 + c\rho^5 = 0, \tag{3}$$

and

$$a\rho\chi'' + 2a\rho'\chi' - v\rho' + b\rho^2\rho' = 0, \tag{4}$$

where $\rho' = d\rho/d\xi$, $\rho'' = d^2\rho/d\xi^2$, $\chi' = d\chi/d\xi$, and $\chi'' = d^2\chi/d\xi^2$.

Multiplying both sides of (4) by ρ and integrating gives

$$\chi' = -\frac{b}{4a}\rho^2 + \frac{v}{2a} + \frac{A}{a\rho^2}, \tag{5}$$

where A is an integration constant.

The corresponding chirp defined by the relation $\delta\omega = -\frac{\partial}{\partial x}\left[\chi\left(\xi\right) - \omega t\right] = -\chi'\left(\xi\right)$ can be written as

$$\delta\omega(x,t) = \frac{b}{4a}\rho^2 - \frac{v}{2a} - \frac{A}{a\rho^2}, \tag{6}$$

which shows a nontrivial dependence of the chirping on the wave intensity [with $I = \rho^2$]. It is worth remarking that this chirp depends on GVD and self-steepening parameters which implies that its amplitude can be controlled by varying these parameters.

Now, substituting (5) into (3) gives

$$\rho'' + \left(\frac{16ac - 5b^2}{16a^2}\right)\rho^5 + \frac{bv}{2a^2}\rho^3 + \left(\frac{v^2 + 4a\omega + 6bA}{4a^2}\right)\rho - \frac{A^2}{a^2\rho^3} = 0. \tag{7}$$

Furthermore, multiplying (7) by ρ' and integrating yields

$$\rho'^2 + \left(\frac{16ac - 5b^2}{48a^2}\right)\rho^6 + \frac{bv}{4a^2}\rho^4 + \left(\frac{v^2 + 4a\omega + 6bA}{4a^2}\right)\rho^2 + \frac{A^2}{a^2\rho^2} + 2B = 0. \tag{8}$$

where B is the second integration constant.

Eq. (8) is a nonlinear differential equation describing the evolution of the wave amplitude in a nonlinear medium that is governed by the quintic derivative NLS model given in Eq. (1). It is relevant to mention that the integration constants A and B in Eqs. (5) and in (8) can be fixed with the initial conditions $\chi'\left(\xi = 0\right)$ and $\rho\left(\xi = 0\right)$, which are related to the initial chirp and amplitude of the pulse.

3. Soliton solutions

In what follows, we present various chirped soliton solutions of the model (1), for different parameter conditions. Before discussing exact solutions to Eq. (8), let us rewrite it in a more simplified form. It is of interest to introduce the following change of variable $F=\rho^2$. Applying this transformation to Eq. (8) yields

$$F'^2 = c_0 + c_1F + c_2F^2 + c_3F^3 + c_4F^4, \tag{9}$$

where

$$c_0 = -\frac{4A^2}{a^2}, \quad c_1 = -8B, \quad c_2 = -\frac{v^2+4a\omega+6bA}{a^2}, \quad c_3 = -\frac{bv}{a^2}, \quad c_4 = \frac{5b^2-16ac}{12a^2}. \tag{10}$$

It is worth observing that the coefficients c_i (with $i=0, a, 2$) contain the integration constants A and B, thus implying that they have effects on the shape of propagating envelopes.

3.1. Extended trial equation scheme

To start off with the extended trial equation scheme [9–12], the following assumption for the solution structure of (9) is made:

$$F = \sum_{i=0}^{\varsigma} \tau_i \Omega^i, \tag{11}$$

where

$$(\Omega')^2 = \Delta(\Omega) = \frac{\Phi(\Omega)}{\Upsilon(\Omega)} = \frac{\mu_\varrho \Omega^\varrho + \cdots + \mu_1\Omega + \mu_0}{\chi_\sigma \Omega^\sigma + \cdots + \chi_1\Omega + \chi_0}. \tag{12}$$

Here $\tau_0, \ldots, \tau_\varsigma$; $\mu_0, \ldots, \mu_\varrho$ and $\chi_0, \ldots, \chi_\sigma$ are constants to be determined later. From (11) and (12), the term F'^2 is derived as

$$F'^2 = \frac{\Phi(\Omega)}{\Upsilon(\Omega)}\left(\sum_{i=0}^{\varsigma} i\tau_i\Omega^{i-1}\right)^2, \tag{13}$$

where $\Phi(\Omega)$ and $\Upsilon(\Omega)$ are polynomials of Ω. Eq. (12) can be reduced to

$$\pm(\xi-\xi_0) = \int \frac{d\Omega}{\sqrt{\Delta(\Omega)}} = \int \sqrt{\frac{\Upsilon(\Omega)}{\Phi(\Omega)}}d\Omega. \tag{14}$$

Balancing F'^2 with F^4 in Eq. (9) gives

$$\varrho = \sigma + 2\varsigma + 2. \tag{15}$$

When $\varrho=4$, $\sigma=0$ and $\varsigma=1$ in Eq. (15), then extended trial equation scheme admits the use of the finite expansion

$$F = \tau_0 + \tau_1\Omega, \tag{16}$$

where τ_0 and τ_1 are constants to be determined later such that $\tau_1 \neq 0$, and Ω satisfies Eq. (12). Substituting (16) into (9), collecting the coefficients of Ω, and solving the resulting system we have the following sets of solutions

$$\begin{aligned}
&\mu_2 = \mu_2, \quad \mu_4 = \mu_4, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1,\\
&\mu_0 = \frac{\mu_2\tau_0^2}{\tau_1^2} + \frac{\mu_4\left[-48A^2 - 96a^2\tau_0B + 24bv\tau_0^3 + \tau_0^4\left(80ac-25b^2\right)\right]}{\tau_1^4\left(5b^2-16ac\right)},\\
&\mu_1 = \frac{2\mu_2\tau_0}{\tau_1} + \frac{4\mu_4\left[-24a^2B + 9bv\tau_0^2 + \tau_0^3\left(32ac-10b^2\right)\right]}{\tau_1^3\left(5b^2-16ac\right)},\\
&\mu_3 = \frac{4\mu_4\left[-3bv + \tau_0\left(5b^2-16ac\right)\right]}{\tau_1\left(5b^2-16ac\right)}, \quad \chi_0 = \frac{12a^2\mu_4}{\tau_1^2\left(5b^2-16ac\right)},\\
&\omega = \frac{\mu_2\tau_1^2\left(16ac-5b^2\right) - 6\mu_4\left[2\left(6bA+v^2\right) + 6bv\tau_0 + \tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}.
\end{aligned} \tag{17}$$

Substituting these set of solutions into Eqs. (12) and (14) gives

$$\pm(\xi-\xi_0)=\mho\int\frac{d\Psi}{\sqrt{\Delta(\Omega)}}, \tag{18}$$

where

$$\Delta(\Omega)=\Omega^4+\frac{\mu_3}{\mu_4}\Omega^3+\frac{\mu_2}{\mu_4}\Omega^2+\frac{\mu_1}{\mu_4}\Omega+\frac{\mu_0}{\mu_4},\quad \mho=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{19}$$

As a consequence, the following traveling wave solutions are obtained:

For $\Delta(\Omega)=(\Omega-\zeta_1)^4$,

$$\begin{aligned} q(x,t) &= \left\{\tau_0+\tau_1\zeta_1\pm\frac{\tau_1\mho}{\xi-\xi_0}\right\}^{1/2} \\ &\times\exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right]. \end{aligned} \tag{20}$$

If $\Delta(\Omega)=(\Omega-\zeta_1)^3(\Omega-\zeta_2)$ and $\zeta_2>\zeta_1$,

$$\begin{aligned} q(x,t) &= \left\{\tau_0+\tau_1\zeta_1+\frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\left(\xi-\xi_0\right)\right]^2}\right\}^{1/2} \\ &\times\exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right]. \end{aligned} \tag{21}$$

However, when $\Delta(\Omega)=(\Omega-\zeta_1)^2(\Omega-\zeta_2)^2$,

$$\begin{aligned} q(x,t) &= \left\{\tau_0+\tau_1\zeta_2+\frac{\tau_1(\zeta_2-\zeta_1)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\}^{1/2} \\ &\times\exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{22}$$

and

$$\begin{aligned} q(x,t) &= \left\{\tau_0+\tau_1\zeta_1+\frac{\tau_1(\zeta_1-\zeta_2)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\}^{1/2} \\ &\times\exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right]. \end{aligned} \tag{23}$$

Whenever $\Delta(\Omega)=(\Omega-\zeta_1)^2(\Omega-\zeta_2)(\Omega-\zeta_3)$ and $\zeta_1>\zeta_2>\zeta_3$,

$$\begin{aligned} q(x,t) &= \left\{\tau_0+\tau_1\zeta_1-\frac{2\tau_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{2\zeta_1-\zeta_2-\zeta_3+(\zeta_3-\zeta_2)\cosh\left[\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\mho}\xi\right]}\right\}^{1/2} \\ &\times\exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right]. \end{aligned} \tag{24}$$

Finally, if $\Delta(\Omega) = (\Omega - \zeta_1)(\Omega - \zeta_2)(\Omega - \zeta_3)(\Omega - \zeta_4)$ and $\zeta_1 > \zeta_2 > \zeta_3 > \zeta_4$,

$$q(x,t) = \left\{ \tau_0 + \tau_1\zeta_2 + \frac{\tau_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_4-\zeta_2+(\zeta_1-\zeta_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\mho}\left(\xi-\xi_0\right),m\right]} \right\}^{1/2} \times \exp\left[i\left\{ \chi\left(\xi\right) - \left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right) - 6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4} \right) t \right\} \right], \tag{25}$$

where

$$m^2 = \frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}. \tag{26}$$

The chirping corresponding to Eqs. (20)–(25) can be obtained readily as

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_1 \pm \frac{\tau_1\mho}{\xi-\xi_0}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_1 \pm \frac{\tau_1\mho}{\xi-\xi_0}\right\}^{-1}, \tag{27}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_1 + \frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\left(\xi-\xi_0\right)\right]^2}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_1 + \frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\left(\xi-\xi_0\right)\right]^2}\right\}^{-1}, \tag{28}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_2 + \frac{\tau_1(\zeta_2-\zeta_1)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_2 + \frac{\tau_1(\zeta_2-\zeta_1)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\}^{-1}, \tag{29}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_1 + \frac{\tau_1(\zeta_1-\zeta_2)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_1 + \frac{\tau_1(\zeta_1-\zeta_2)}{\exp\left[\frac{\zeta_1-\zeta_2}{\mho}\left(\xi-\xi_0\right)\right]-1}\right\}^{-1}, \tag{30}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_1 - \frac{2\tau_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{2\zeta_1-\zeta_2-\zeta_3+(\zeta_3-\zeta_2)\cosh\left[\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\mho}\xi\right]}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_1 - \frac{2\tau_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{2\zeta_1-\zeta_2-\zeta_3+(\zeta_3-\zeta_2)\cosh\left[\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\mho}\xi\right]}\right\}^{-1}, \tag{31}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\tau_0+\tau_1\zeta_2 + \frac{\tau_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_4-\zeta_2+(\zeta_1-\zeta_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\mho}\left(\xi-\xi_0\right),m\right]}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\tau_0+\tau_1\zeta_2 + \frac{\tau_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_4-\zeta_2+(\zeta_1-\zeta_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\mho}\left(\xi-\xi_0\right),m\right]}\right\}^{-1}. \tag{32}$$

It is important to note that ζ_j for $j = 1, \ldots, 4$ are the roots of

$$\Delta(\Omega) = 0. \tag{33}$$

For $\tau_0 = -\tau_1\zeta_1$ and $\xi_0 = 0$, the solutions (20)–(24) can be reduced to plane wave solutions

$$\begin{aligned} q(x,t) &= \sqrt{\pm\frac{\tau_1\mho}{\xi}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{34}$$

$$\begin{aligned} q(x,t) &= \sqrt{\frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\xi\right]^2}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{35}$$

singular soliton solutions

$$\begin{aligned} q(x,t) &= \sqrt{\frac{\tau_1(\zeta_2-\zeta_1)}{2}\left(1\mp\coth\left[\left(\frac{\zeta_1-\zeta_2}{2\mho}\right)\xi\right]\right)} \\ &\times \exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{36}$$

and bright soliton solution

$$\begin{aligned} q(x,t) &= \frac{K}{\sqrt{M+\cosh\left(L\xi\right)}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{37}$$

where

$$\begin{aligned} K &= \sqrt{\frac{2\tau_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{\zeta_3-\zeta_2}}, \\ L &= \frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\mho}, \\ M &= \frac{2\zeta_1-\zeta_2-\zeta_3}{\zeta_3-\zeta_2}. \end{aligned} \tag{38}$$

Here, K is the amplitude of the soliton, while L is the inverse width of the soliton. These solitons exist for $\tau_1 < 0$. The chirping corresponding to Eqs. (34)–(37) can be derived as

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\pm\frac{\tau_1\mho}{\xi}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\pm\frac{\tau_1\mho}{\xi}\right\}^{-1}, \tag{39}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\xi\right]^2}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{4\tau_1\mho^2(\zeta_2-\zeta_1)}{4\mho^2-\left[(\zeta_1-\zeta_2)\xi\right]^2}\right\}^{-1}, \tag{40}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{\tau_1(\zeta_2-\zeta_1)}{2}\left(1\mp\coth\left[\left(\frac{\zeta_1-\zeta_2}{2\mho}\right)\xi\right]\right)\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{\tau_1(\zeta_2-\zeta_1)}{2}\left(1\mp\coth\left[\left(\frac{\zeta_1-\zeta_2}{2\mho}\right)\xi\right]\right)\right\}^{-1}, \tag{41}$$

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{K^2}{M+\cosh\left(L\xi\right)}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{K^2}{M+\cosh\left(L\xi\right)}\right\}^{-1}. \tag{42}$$

On the other hand, if $\tau_0 = -\tau_1\zeta_2$ and $\xi_0 = 0$, Jacobi elliptic function solutions (25) can be transformed to

$$\begin{aligned} q(x,t) &= \frac{K_1}{\sqrt{M_1+\mathrm{sn}^2\left[L_j\xi,\ \frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}\right]}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right)-\left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{43}$$

where

$$K_1 = \sqrt{\frac{\tau_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_1-\zeta_4}},$$
$$M_1 = \frac{\zeta_4-\zeta_2}{\zeta_1-\zeta_4}, \tag{44}$$
$$L_j = \frac{(-1)^j\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\mho} \quad \text{for} \quad j=1,2.$$

The corresponding chirping is given by

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{K_1^2}{M_1+\text{sn}^2\left[L_j\xi,\, \frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}\right]}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{K_1^2}{M_1+\text{sn}^2\left[L_j\xi,\, \frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}\right]}\right\}^{-1}. \tag{45}$$

Remark 1. When the modulus $m \to 1$, singular optical soliton solutions are

$$\begin{aligned} q(x,t) &= \frac{K_1}{\sqrt{M_1+\tanh^2\left(L_j\xi\right)}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right) - \left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{46}$$

where $\zeta_3 = \zeta_4$. The corresponding chirping takes the form

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{K_1^2}{M_1+\tanh^2\left(L_j\xi\right)}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{K_1^2}{M_1+\tanh^2\left(L_j\xi\right)}\right\}^{-1}. \tag{47}$$

Remark 2. However, if $m \to 0$, periodic singular solutions are

$$\begin{aligned} q(x,t) &= \frac{K_1}{\sqrt{M_1+\sin^2\left(L_j\xi\right)}} \\ &\times \exp\left[i\left\{\chi\left(\xi\right) - \left(\frac{\mu_2\tau_1^2\left(16ac-5b^2\right)-6\mu_4\left[2\left(6bA+v^2\right)+6bv\tau_0+\tau_0^2\left(16ac-5b^2\right)\right]}{48a\mu_4}\right)t\right\}\right], \end{aligned} \tag{48}$$

where $\zeta_2 = \zeta_3$. The chirping is given as

$$\delta\omega(x,t) = \frac{b}{4a}\left\{\frac{K_1^2}{M_1+\sin^2\left(L_j\xi\right)}\right\} - \frac{v}{2a} - \frac{A}{a}\left\{\frac{K_1^2}{M_1+\sin^2\left(L_j\xi\right)}\right\}^{-1}. \tag{49}$$

4. Conclusions

This paper obtained bright and singular optical soliton solutions to the Gerdjikov–Ivanov model that appears with quintic nonlinearity. The results of this paper are of great interest to telecommunications engineers. These soliton solutions come with a few restrictions that are listed as constraints in the paper. The results of this paper carry a lot of future hope. Later, these results will be extended to the case of perturbed Gerdjikov–Ivanov equation. In addition, the case of coupled Gerdjikov–Ivanov equation will also be addressed to study solitons with DGD. Those results are currently awaited and will soon be reported.

Conflict of interest

The authors declare that there is no conflict of interest.

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