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ABSTRACT

This paper serves as a sequel to previously published results on bright, dark and singular solitons in birefringent fibers with spatio-temporal dispersion, during 2014. The second form of singular soliton solutions is retrieved in this paper for birefringent nano-fibers with Kerr and parabolic laws of nonlinearity. There are constraint conditions that evolve with the solution structure.

1. Introduction

Optical solitons is one of the fastest growing areas of research in the field of nonlinear optics and telecommunications engineering. There are several overwhelming results that are constantly pouring into several journals from all across the globe [1–32]. These results are gradually leading to advances and successful achievements in telecommunications. The modern day engineering marvels such as Internet industry, face-book, and twitter are possible only with soliton communication technology.

Therefore this area of research, namely optical solitons is being further probed into, constantly, from all avenues and all angles. One aspect of soliton propagation through nonlinear optical fibers is that results from vector solitons are less visible. This includes solitons from birefringent fibers, cascaded system, thirring solitons, DWDM systems and others. This paper is going to address solitons in birefringent fibers. Very recently, bright, dark and singular soliton solutions in birefringent fibers with Kerr law and parabolic law nonlinearity, in presence of Hamiltonian perturbation terms, are reported [1]. This paper is going to study the same model with a second form of singular soliton solution for both of these nonlinear media. There are several constraint conditions that will be listed for these solitons to exist.

For this model, as reported in 2014 [1], the governing nonlinear Schrödinger's equation will be considered with spatio-temporal dispersion (STD) in addition to group velocity dispersion (GVD). The inclusion of STD only makes the model

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well-posed as pointed out during 2012 [5,8]. Another aspect for considering STD, in addition to GVD, is the reduction of Internet bottleneck.

The mathematical analysis of singular soliton solutions will be carried out in the rest of the paper. The study will be split into subsequent two sections with Kerr law and parabolic law nonlinearity.

2. Kerr Law

This section addresses the bright, dark as well as singular solitons in optical fibers with Kerr law nonlinearity. The dimensionless form of the coupled NLSE with STD and Kerr law nonlinearity is given by [1,3]

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left(c_1|q|^2 + d_1|r|^2\right) q + i\left\{\alpha_1 q_x + \lambda_1\left(|q|^2 q\right)_x + \nu_1\left(|q|^2\right)_x q + \theta_1|q|^2 q_x + \gamma_1 q_{xxx}\right\} = 0 \tag{1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left(c_2|r|^2 + d_1|q|^2\right) r + i\left\{\alpha_2 r_x + \lambda_2\left(|r|^2 r\right)_x + \nu_2\left(|r|^2\right)_x r + \theta_2|r|^2 r_x + \gamma_2 r_{xxx}\right\} = 0 \tag{2}$$

In Eqs. (1) and (2) $q(x, t)$ and $r(x, t)$ are complex valued functions that represents the soliton profiles for the two components in birefringent fibers. For l = 1; 2, a_l represents the GVD and b_l are STD terms along the two components. Then, c_l and d_l represents the self-phase modulation (SPM) and cross-phase modulation (XPM) terms respectively. In the perturbation terms α_l represents the inter-modal dispersion, λ_l is the self-steepening term, ν_l and θ_l are nonlinear dispersions and finally γ_l is the third order dispersion that must be taken into account in case the GVD is small [1,3,7].

The aim of this section is to obtain an exact bright, dark and singular 1-soliton solution to this coupled set of equations. The ansatz method is used. In order to set up the starting point, the solitons are written in the phase-amplitude format as [1,16,17]

$$q(x, t) = A_1 coth^{p_1}\left[B(x - vt)\right] e^{i(-\kappa_1 + \omega_1 t + \sigma_1)} = A_1 coth^{p_1} \tau e^{i\Phi_1(x,t)} \tag{3}$$

and

$$r(x, t) = A_2 coth^{p_2}\left[B(x - vt)\right] e^{i(-\kappa_2 + \omega_2 t + \sigma_2)} = A_2 coth^{p_2} \tau e^{i\Phi_2(x,t)} \tag{4}$$

where A_l and B are free parameters, while $\Phi_l(x, t)$ are phase components, defined as

$$\Phi_l(x, t) = -\kappa_l + \omega_l t + \sigma_l \tag{5}$$

For l = 1; 2. Also, τ is given by

$$\tau = B(x - vt) \tag{6}$$

Here, κ_l represents frequency of solitons for each of the two components, ω_l are the wave numbers, while σ_l are the phase constants. Substituting Eqs. (3) and (4) into Eqs. (1) and (2) and decomposing into real and imaginary parts lead to

$$\{\omega_l(b_l\kappa_l - 1) - a_l\kappa_l^2 + \alpha_l\kappa_l - \gamma_l\kappa_l^3 - 4p_l^2(a_l + 3\gamma_l\kappa_l)B^2 + 2p_l(p_l - 2)vb_lB^2\}coth^2\tau + d_lA_{\bar{l}}^2 coth^{2p_{\bar{l}}+2}\tau + (\lambda_l\kappa_l + \theta_l\kappa_l + c_l)A_l^2 coth^{2p_l+2}\tau + p_l\{(p_l + 1)(a_l + 3\gamma_l\kappa_l) - (p_l - 3)vb_l\} B^2 coth^4\tau + p_l(p_l - 1)(a_l + 3\gamma_l\kappa_l - vb_l)B^2 = 0 \tag{7}$$

and

$$\left\{v(1 - b_l\kappa_l) - 2a_l\kappa_l + b_l\omega_l + \alpha_l - \gamma_l\left(3\kappa_l^2 + 2p_l^2B^2\right)\right\} coth^2\tau + \gamma_l\left(p_l^2 + 3p_l + 2\right) B^4 coth^4\tau + \left(3\lambda_l + 2\nu_l + \theta_l\right) A_l^2 coth^{2p_l+2}\tau + \gamma_l(p_l - 1)(p_l - 2)B^2 = 0, \tag{8}$$

respectively. Application of balancing principle to Eqs. (7) and (8) reveals

$$p_l = 1 \tag{9}$$

for l = 1; 2 and $\bar{l} = 3 - l$.

From the imaginary part Eq. (8), upon setting the coefficients of the linearly independent function to zero yields the speed of the soliton as

$$v = \frac{2a_l\kappa_l - b_l\omega_l - \alpha_l + \gamma_l\left(3\kappa_l^2 + 2B^2\right)}{b_l\kappa_l - 1}, \tag{10}$$

for

$$b_l\kappa_l \neq 1 \tag{11}$$

Now, equating the two components of speed gives the free parameter

$$B = \left[\frac{(b_1\kappa_1 - 1)\left(2a_2\kappa_2 - b_2\omega_2 - \alpha_2 + 3\gamma_2\kappa_2^2\right) - (b_2\kappa_2 - 1)\left(2a_1\kappa_1 - b_1\omega_1 - \alpha_1 + 3\gamma_1\kappa_1^2\right)}{2\left\{\gamma_1(b_2\kappa_2 - 1) - \gamma_2(b_1\kappa_1 - 1)\right\}}\right]^{\frac{1}{2}} \tag{12}$$

subject to the constraint:

$$\gamma_1\left(b_2\kappa_2-1\right)-\gamma_2\left(b_1\kappa_1-1\right)\times\times\left\{\left(b_1\kappa_1-1\right)\left(2a_2\kappa_2-b_2\omega_2-\alpha_2+3\gamma_2\kappa_2^2\right)-\left(b_2\kappa_2-1\right)\left(2a_1\kappa_1-b_1\omega_1-\alpha_1+3\gamma_1\kappa_1^2\right)\right\}>0 \quad (13)$$

The coefficient of second linearly independent function gives the free parameter

$$A_l=\left[\frac{3\gamma_l\{(b_l\kappa_l-1)(2a_{\bar{l}}\kappa_{\bar{l}}-b_{\bar{l}}\omega_{\bar{l}}-\alpha_{\bar{l}}+3\gamma_{\bar{l}}\kappa_{\bar{l}}^2)-(b_{\bar{l}}\kappa_{\bar{l}}-1)(2a_l\kappa_l-b_l\omega_l-\alpha_l+3\gamma_l\kappa_l^2)\}}{(3\lambda_l+2\nu_l+\theta_l)\{\gamma_{\bar{l}}(b_l\kappa_l-1)-\gamma_l(b_{\bar{l}}\kappa_{\bar{l}}-1)\}}\right]^{\frac{1}{2}} \quad (14)$$

provided

$$\gamma_l\left\{\left(b_l\kappa_l-1\right)\left(2a_{\bar{l}}\kappa_{\bar{l}}-b_{\bar{l}}\omega_{\bar{l}}-\alpha_{\bar{l}}+3\gamma_{\bar{l}}\kappa_{\bar{l}}^2\right)-\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)\left(2a_l\kappa_l-b_l\omega_l-\alpha_l+3\gamma_l\kappa_l^2\right)\right\}\times\left[\left(3\lambda_l+2\nu_l+\theta_l\right)\right.$$
$$\left.\left\{\gamma_{\bar{l}}\left(b_l\kappa_l-1\right)-\gamma_l\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)\right\}>0\right] \quad (15)$$

Next, the real part Eq. (7) gives, in a similar manner,

$$v=\frac{2\left(a_l+3\gamma_l\kappa_l\right)B^2+\left(\lambda_l\kappa_l+\theta_l\kappa_l+c_l\right)A_l^2+d_lA_{\bar{l}}^2}{2b_lB^2} \quad (16)$$

and the wave number

$$\omega_l=\frac{a_l\kappa_l^2-\alpha_l\kappa_l+\gamma_l\kappa_l^3-\left(\lambda_l\kappa_l+\theta_l\kappa_l+c_l\right)A_l^2-d_lA_{\bar{l}}^2}{b_l\kappa_l-1} \quad (17)$$

with the constraint given by Eq. (11).

Finally, substituting the free parameter A_l from Eq. (14) into Eq. (17) leads to the coupled system of equation for the frequencies of the two components:

$$\begin{bmatrix}P^{(1)} & Q^{(1)}\\ P^{(2)} & Q^{(2)}\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}=\begin{bmatrix}R^{(1)}\\ R^{(2)}\end{bmatrix} \quad (18)$$

where

$$P^{(l)}=1-\frac{3b_l\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)}{\left(b_l\kappa_l-1\right)\left\{\gamma_l\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)-\gamma_{\bar{l}}\left(b_l\kappa_l-1\right)\right\}}\left[\frac{d_l\gamma_{\bar{l}}}{3\lambda_{\bar{l}}+2\nu_{\bar{l}}+\theta_{\bar{l}}}+\frac{\gamma_l\left(\lambda_l\kappa_l+\theta_l\kappa_l+c_l\right)}{3\lambda_l+2\nu_l+\theta_l}\right] \quad (19)$$

$$Q^{(l)}=\frac{3b_{\bar{l}}}{\left\{\gamma_l\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)-\gamma_{\bar{l}}\left(b_l\kappa_l-1\right)\right\}}\left[\frac{d_l\gamma_{\bar{l}}}{3\lambda_{\bar{l}}+2\nu_{\bar{l}}+\theta_{\bar{l}}}+\frac{\gamma_l\left(\lambda_l\kappa_l+\theta_l\kappa_l+c_l\right)}{3\lambda_l+2\nu_l+\theta_l}\right] \quad (20)$$

and

$$R^{(l)}=\frac{a_l\kappa_l^2-\alpha_l\kappa_l+\gamma_l\kappa_l^3}{b_l\kappa_l-1}+3\left\{\left(2a_{\bar{l}}\kappa_{\bar{l}}-\alpha_{\bar{l}}+3\gamma_{\bar{l}}\kappa_{\bar{l}}^2\right)\left(b_l\kappa_l-1\right)-\left(2a_l\kappa_l-\alpha_l+3\gamma_l\kappa_l^2\right)\left(b_{\bar{l}}\kappa_{\bar{l}}-1\right)\right\}$$
$$\times\times\left[\frac{d_l\gamma_{\bar{l}}}{3\lambda_{\bar{l}}+2\nu_{\bar{l}}+\theta_{\bar{l}}}+\frac{\gamma_l\left(\lambda_l\kappa_l+\theta_l\kappa_l+c_l\right)}{3\lambda_l+2\nu_l+\theta_l}\right] \quad (21)$$

for $l=1; 2$ and $\bar{l}=3-l$. Inverting the matrix in (18) leads to the frequencies of the two components:

$$\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}=\frac{1}{P^{(1)}Q^{(2)}-P^{(2)}Q^{(1)}}\begin{bmatrix}Q^{(2)}R^{(1)}-Q^{(1)}R^{(2)}\\ P^{(1)}R^{(2)}-P^{(2)}R^{(1)}\end{bmatrix} \quad (22)$$

as long as

$$\begin{bmatrix}P^{(1)} & Q^{(1)}\\ P^{(2)} & Q^{(2)}\end{bmatrix}\neq 0 \quad (23)$$

Hence, singular 1-soliton solutions to (1) and (2) are given by

$$q(x,t)=A_1 coth\left[B(x-vt)\right]e^{i(-\kappa_1+\omega_1 t+\sigma_1)} \quad (24)$$

$$r(x,t)=A_2 coth\left[B(x-vt)\right]e^{i(-\kappa_2+\omega_2 t+\sigma_2)} \quad (25)$$

with all parameters as indicated.

3. Parabolic law

For parabolic law nonlinearity, the governing equations in birefringent nano-fibers are given by the following coupled system [1,3,6]:

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left(c_1|q|^2 + d_1|r|^2\right) q + \left(\xi_1|q|^4 + \eta_1|q|^2|r|^2 + \zeta_1|q|^4\right) + i\left\{\alpha_1 q_x + \lambda_1\left(|q|^2 q\right)_x + \nu_1\left(|q|^2\right)_x q + \theta_1|q|^2 q_x + \gamma_1 q_{xxx}\right\} = 0 \tag{26}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left(c_2|r|^2 + d_2|q|^2\right) r + \left(\xi_2|r|^4 + \eta_2|r|^2|q|^2 + \zeta_2|q|^4\right) + i\left\{\alpha_2 r_x + \lambda_2\left(|r|^2 r\right)_x + \nu_2\left(|r|^2\right)_x r + \theta_2|r|^2 r_x + \gamma_2 r_{xxx}\right\} = 0 \tag{27}$$

In order to seek singular soliton solutions of second type, for birefringent fibers with parabolic law, the starting hypothesis is taken to be [16,17,19,20]:

$$q(x,t) = \left\{A_1 + B_1 coth\left[B(x - vt)\right]\right\}^{p_1} e^{i(-\kappa_1 + \omega_1 t + \sigma_1)} = (A_1 + B_1 coth\tau)^{p_1} e^{i\Phi_1(x,t)} \tag{28}$$

and

$$r(x,t) = \left\{A_2 + B_2 coth\left[B(x - vt)\right]\right\}^{p_2} e^{i(-\kappa_2 + \omega_2 t + \sigma_2)} = (A_2 + B_2 coth\tau)^{p_2} e^{i\Phi_2(x,t)} \tag{29}$$

Once again, A_l and B_l are free parameters. Substituting (28) and (29) into (26) and (27), and decomposing into real and imaginary parts gives a pair of relations. The imaginary part leads to

$$\begin{aligned}&\gamma_l B^2(p_l-1)(p_l-2)(B_l^2-A_l^2)^2 + 2(2p_l^2-3p_l+1)\gamma_l A_l B^2(B_l^2-A_l^2)(A_l+B_l coth\tau) +\\ &+[B_l^2\{(b_l\kappa_l-1)\nu - 2a_l\kappa_l + b_l\omega_l + \alpha_l - 3\gamma_l^2\} + 2\gamma_l B^2 p_1^2(3A_l^2 - B_l^2)](A_l+B_l coth\tau)^2 -\\ &-2\gamma_l B^2 A_l(2p_l^2+3p_l+1)(A_l+B_l coth\tau)^3 + \gamma_l B^2(p_l^2-9p_l+14)(A_l+B_l coth\tau)^4 +\\ &(3\lambda_l+2\nu_l+\theta_l)(A_l+B_l coth\tau)^{2p_l+2} = 0\end{aligned} \tag{30}$$

From the linearly independent functions, it shows that third-order dispersion must vanish, namely

$$\gamma_l = 0 \tag{31}$$

and also

$$A_l = B_l \tag{32}$$

Eq. (31) shows that singular solitons of second type, in birefringent fibers, with parabolic law nonlinearity, will exist, provided third order dispersion vanishes, which is contrary to Kerr law nonlinearity. This observation is being made for the first time in this paper.

Now, the governing Eqs. (26) and (27), by virtue of Eq. (31), reduces to

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left(c_1|q|^2 + d_1|r|^2\right) q + \left(\xi_1|q|^4 + \eta_1|q|^2|r|^2 + \zeta_1|q|^4\right) + i\left\{\alpha_1 q_x + \lambda_1\left(|q|^2 q\right)_x + \nu_1\left(|q|^2\right)_x q + \theta_1|q|^2 q_x\right\} = 0 \tag{33}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left(c_2|r|^2 + d_2|q|^2\right) r + \left(\xi_2|r|^4 + \eta_2|r|^2|q|^2 + \zeta_2|q|^4\right) + i\left\{\alpha_2 r_x + \lambda_2\left(|r|^2 r\right)_x + \nu_2\left(|r|^2\right)_x r + \theta_2|r|^2 r_x\right\} = 0 \tag{34}$$

respectively. The remaining linearly independent functions, from Eq. (30), lead to the constraint

$$3\lambda_l + 2\nu_l + \theta_l = 0 \tag{35}$$

and the speed of the soliton

$$v = \frac{2a_l\kappa_l - b_l\omega_l - \alpha_l}{b_l\kappa_l - 1} \tag{36}$$

for $l = 1; 2$. Now, equating the two expressions for the speed of the soliton from Eq. (36), for $l = 1; 2$ implies the constraint:

$$(2a_1\kappa_1 - b_1\omega_1 - \alpha_1)(b_2\kappa_2 - 1) = (2a_2\kappa_2 - b_2\omega_2 - \alpha_2)(b_1\kappa_1 - 1) \tag{37}$$

Next, the real part equation, after setting $\gamma_l = 0$, simplifies to:

$$\begin{aligned}&B^2 p_l(p_l+1)(a_l-b_l\nu)(A_l+B_l coth\tau)^2 - 2B^2 A_l p_l(2p_l+1)(a_l-b_l\nu)(A_l+B_l coth\tau)^3 +\\ &[B_l^2\{\omega_l(b_l\kappa_l-1) - a_l\kappa_l^2 + \alpha_l\kappa_l\} \quad + 2B^2 p_l^2(a_l-b_l\nu)(3A_l^2-B_l^2)](A_l+B_l coth\tau)^2 +\\ &2p_l(2p_l-1)B^2 A_l(a_l-b_l\nu)(B_l^2-A_l^2)(A_l+B_l coth\tau) + B^2 p_l(p_l-1)(a_l-b_l\nu)(B_l^2-A_l^2)^2\\ &+(\lambda_l\kappa_l+\theta_l\kappa_l+c_l)(A_l+B_l coth\tau)^{2p_l+2} + d_l(A_l+B_l coth\tau)^2(A_{\bar{l}}+B_{\bar{l}} coth\tau)^{2p_{\bar{l}}} +\\ &\xi_l(A_l+B_l coth\tau)^{4p_l+2} + \eta_l(A_l+B_l coth\tau)^{2p_l+2}(A_{\bar{l}}+B_{\bar{l}} coth\tau)^{2p_{\bar{l}}} + \zeta_l(A_l+B_l coth\tau)^2\\ &(A_{\bar{l}}+B_{\bar{l}} coth\tau)^{4p_{\bar{l}}} = 0\end{aligned} \tag{38}$$

Balancing principle indicates

$$p_l = \frac{1}{2}, \tag{39}$$

for $l = 1; 2$. From (38), setting the coefficients of linearly independent functions to zero yields the wave number and the free parameter B as

$$\omega_l = \frac{\kappa_l (a_l \kappa_l - \alpha_l) + (b_l v - a_l) B^2}{b_l \kappa_l - 1}, \tag{40}$$

and

$$B = \frac{\left(\lambda_l \kappa_l + \theta_l \kappa_l + c_l\right) A_l}{2 (a_l - b_l v)} \tag{41}$$

respectively, together with the condition

$$\left(\lambda_l \kappa_l + \theta_l \kappa_l + c_l\right) (a_l - b_l v) A_l > 0 \tag{42}$$

Another expression for the wave number, that falls out from the linearly independent function is

$$\omega_l = \frac{3\kappa_l (a_l \kappa_l - \alpha_l) + 4\xi_l A_l^2}{b_l \kappa_l - 1}, \tag{43}$$

The relations (40) and (43) are guaranteed to hold by virtue of (11).

Finally, singular 1-soliton solution, of second type, in birefringent fibers is given by

$$q(x, t) = \sqrt{A_1 \left\{1 + coth\left[B(x - vt)\right]\right\}} e^{i(-\kappa_1 + \omega_1 t + \sigma_1)} \tag{44}$$

and

$$r(x, t) = \sqrt{A_2 \left\{1 + coth\left[B(x - vt)\right]\right\}} e^{i(-\kappa_2 + \omega_2 t + \sigma_2)} \tag{45}$$

4. Conclusion

This paper filled in the gap on the study of solitons in birefringent fibers with perturbation terms and spatio-temporal dispersion. The singular soliton solutions, of second type, are obtained in this paper by the aid of ansatz method. The two types of nonlinear media that are studied in this paper are Kerr law and parabolic law. The results come with their corresponding constraints that must hold for the soliton solutions to exist.

These results are encouraging towards future goals. The solutions of this paper can serve as a foundation stone for analytical studies of optical rogons. An additional prospect of this work is to extend to DWDM systems. This is just a tip of the iceberg.

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