Optik 157 (2018) 1235–1240

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical soliton perturbation with Gerdjikov–Ivanov equation by modified simple equation method

Anjan Biswas[a,b,c], Yakup Yildirim[d], Emrullah Yasar[d], Houria Triki[e], Ali Saleh Alshomrani[b], Malik Zaka Ullah[b], Qin Zhou[f,*], Seithuti P. Moshokoa[c], Milivoj Belic[g]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey
[e] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P. O. Box 12, 23000 Annaba, Algeria
[f] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[g] Science Program, Texas A & M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

Article history:
Received 16 November 2017
Accepted 19 December 2017

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Perturbation
Modified simple equation method

ABSTRACT

The modified simple equation scheme is employed to secure dark and singular optical soliton solution to the perturbed Gerdjikov–Ivanov equation that models pulse dynamics in optical fibers and PCF. The solvability conditions for this algorithm is also presented.

1. Introduction

There are several nonlinear evolution equations that govern soliton dynamics in a variety of waveguides [1–15]. While the most commonly studied form is the nonlinear Schrödinger's equation (NLSE), this paper will study the Gerdjikov–Ivanov (GI) equation that has gained popularity since its first appearance a few years ago. Lots of work has already been done with this model, in the past, since its first appearance. The conservation laws have been retrieved for this model [1]. GI equation has also been studied by the aid of semi-inverse variational principle [2]. The bi-Hamiltonian structure of this equation has been established and bifurcation analysis has also been carried out for this equation [4,6]. Additional integration algorithms have been applied to extract solitons and other solutions to this model. They are trial equation method, $\exp(\phi(\xi))$ method, $\tan(\phi/2)$ method and traveling wave hypothesis [3,7,8,11]. Later, Darboux transform has also been applied to secure soliton

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2017.12.101
0030-4026/

and other solutions to the GI equation [13]. This paper will study the model with modified simple equation approach. Thus, dark and singular soliton solutions will be revealed with this integration scheme and the soliton existence criteria will be presented as constraint conditions. The details follow in the subsequent sections and subsections.

1.1. Governing model

The GI equation in its dimensionless form is written as:

$$iq_t + aq_{xx} + b|q|^4 q + icq^2 q_x^* = 0 \tag{1}$$

Here, in (1), $q(x, t)$ represents complex-valued wave profile with x and t being the independent spatial and temporal variables respectively. The first term is linear temporal evolution. The coefficient of a stands for group velocity dispersion and b represents the coefficient of quintic nonlinearity. The last term is a form of nonlinear dispersion. The parameters a, b and c are all real-valued constants.

In presence of perturbation terms, the GI equation that is discussed in this paper extends to:

$$iq_t + aq_{xx} + b|q|^4 q + icq^2 q_x^* = i\left[\alpha q_x + \lambda\left(|q|^{2m} q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where α is the inter-modal dispersion, λ accounts for self-steepening with short pulses and μ is the higher-order dispersion coefficient and finally m is the full nonlinearity parameter.

2. Recapitulation of modified simple equation method

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step-1: We use the transformation

$$u(x, t) = U(\xi), \quad \xi = x - ct, \tag{4}$$

where c is a constant to be determined, to reduce Eq. (3) to the following ODE:

$$Q(U, U', U'', U''', \ldots) = 0 \tag{5}$$

where Q is a polynomial in $U(\xi)$ and its total derivatives, while $' = d/d\xi$.

Step-2: We suppose that Eq. (5) has the formal solution

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{6}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (6) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step-4: We substitute (6) into (5), then we calculate all the necessary derivatives U',U'', ... of the unknown function $U(\xi)$ and we account the function $U(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi)$, $j = 0,1,2, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (3).

3. Soliton solutions

In order to solve Eq. (2) by the modified simple equation method, we use the following wave transformation

$$q(x, t) = U(\xi)e^{i\phi(x,t)}, \quad q^*(x, t) = U(\xi)e^{-i\phi(x,t)} \tag{7}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.

Substituting Eq. (7) into Eq. (2) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v = -2a\kappa - \alpha + cU^2 - ((2m+1)\lambda + 2m\mu)U^{2m} \tag{8}$$

while the real part gives

$$aU'' - \left(\omega + a\kappa^2 + \alpha\kappa\right) U - \kappa\lambda U^{2m+1} - c\kappa U^3 + bU^5 = 0 \tag{9}$$

Eq. (8) gives the velocity of the soliton while Eq. (9) can be integrated to determine the soliton profile.

By using transformation $U = V^{\frac{1}{2}}$, Eq. (9) becomes

$$a\left(-\left(V'\right)^2 + 2VV''\right) - 4V^2\left(\omega + \alpha\kappa + a\kappa^2\right) - 4\lambda\kappa V^{m+2} + 4\left(-c\kappa V^3 + bV^4\right) = 0 \tag{10}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (10), then we get $N = 1$. Consequently, we reach

$$V\left(\xi\right) = a_0 + a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right) \tag{11}$$

Case-1:

Substituting Eq. (11) into Eq. (10) along with $m = 1$ and then setting the coefficients of ψ^{-j}, $j = 0,1,2,3,4$ to zero, then we obtain a set of algebraic equations as follows:

ψ^{-4} coeff.:

$$a_1^2\left(\psi'\right)^4\left(4ba_1^2 + 3a\right) = 0, \tag{12}$$

ψ^{-3} coeff.:

$$-4a_1\left(\psi'\right)^2\left(\left(\kappa ca_1^2 + \kappa\lambda a_1^2 - 4ba_0a_1^2 - aa_0\right)\psi' + aa_1\psi''\right) = 0, \tag{13}$$

ψ^{-2} coeff.:

$$\begin{aligned}&-a_1\left(\left(4\kappa^2 aa_1 + 12\kappa ca_0a_1 + 12\kappa\lambda a_0a_1 - 24ba_0^2a_1 + 4\kappa\alpha a_1 + 4\omega a_1\right)\left(\psi'\right)^2\right.\\ &\left.\quad +6aa_0\psi'\psi'' - 2aa_1\psi'\psi''' + aa_1\left(\psi''\right)^2\right) = 0,\end{aligned} \tag{14}$$

ψ^{-1} coeff.:

$$-2a_0a_1\left(\left(4\kappa^2 a + 6\kappa ca_0 + 6\kappa\lambda a_0 - 8ba_0^2 + 4\kappa\alpha + 4\omega\right)\psi' - a\psi'''\right) = 0, \tag{15}$$

ψ^{0} coeff.:

$$-4a_0^2\left(\kappa^2 a + \kappa ca_0 + \kappa\lambda a_0 - ba_0^2 + \kappa\alpha + \omega\right) = 0. \tag{16}$$

Solving this system, we obtain

$$a_0 = \frac{3\kappa\left(\lambda + c\right)}{4b}, \qquad a_1 = \pm\frac{1}{2}\sqrt{-\frac{3a}{b}}, \tag{17}$$

$$\omega = -\frac{\kappa\left(16\kappa ab + 3\kappa c^2 + 6\kappa c\lambda + 3\kappa\lambda^2 + 16\alpha b\right)}{16b}, \tag{18}$$

and

$$\psi'' = \pm\frac{\kappa\left(\lambda + c\right)\sqrt{3}}{2\sqrt{-ab}}\psi', \tag{19}$$

$$\psi''' = -\frac{3\kappa^2(\lambda + c)^2}{4ab}\psi'. \tag{20}$$

From Eqs. (19) and (20), we can deduce that

$$\psi' = \pm\frac{2\sqrt{-ab}}{\kappa\left(\lambda + c\right)\sqrt{3}}c_1 e^{\pm\frac{\kappa(\lambda+c)\sqrt{3}}{2\sqrt{-ab}}\xi}, \tag{21}$$

and

$$\psi = -\frac{4ab}{3\kappa^2(\lambda + c)^2}c_1 e^{\pm\frac{\kappa(\lambda+c)\sqrt{3}}{2\sqrt{-ab}}\xi} + c_2, \tag{22}$$

where c_1 and c_2 are constants of integration. Substituting Eqs. (21) and (22) into Eq. (11), we obtain following exact solution to Eq. (2)

$$q(x,t)=\left\{\frac{3\kappa(\lambda+c)}{4b}\pm\frac{1}{2}\sqrt{-\frac{3a}{b}}\left(\frac{\pm\frac{2\sqrt{-ab}}{\kappa(\lambda+c)\sqrt{3}}c_1 e^{\pm\frac{\kappa(\lambda+c)\sqrt{3}}{2\sqrt{-ab}}(x-vt)}}{-\frac{4ab}{3\kappa^2(\lambda+c)^2}c_1 e^{\pm\frac{\kappa(\lambda+c)\sqrt{3}}{2\sqrt{-ab}}(x-vt)}+c_2}\right)\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (18). If we set

$$c_1=-\frac{3\kappa^2(\lambda+c)^2}{4ab}e^{\pm\frac{\kappa(\lambda+c)\sqrt{3}}{2\sqrt{-ab}}\xi_0},\quad c_2=\pm 1$$

we obtain:

$$q(x,t)=\left\{\frac{3\kappa(\lambda+c)}{8b}\left(1\pm\tanh\left[\frac{\kappa(\lambda+c)\sqrt{3}}{4\sqrt{-ab}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{23}$$

$$q(x,t)=\left\{\frac{3\kappa(\lambda+c)}{8b}\left(1\pm\coth\left[\frac{\kappa(\lambda+c)\sqrt{3}}{4\sqrt{-ab}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{24}$$

where Eqs. (23) and (24) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$ab<0.$$

Case-2:

Substituting Eq. (11) into Eq. (10) along with $m=2$ and then setting the coefficients of $\psi^{-j}, j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations as follows:

ψ^{-4} coeff.:

$$-a_1^2\left(\psi'\right)^4\left(4\kappa\lambda a_1^2-4ba_1^2-3a\right)=0, \tag{25}$$

ψ^{-3} coeff.:

$$-4a_1\left(\psi'\right)^2\left(\left(4\kappa\lambda a_0a_1^2+\kappa ca_1^2-4ba_0a_1^2-aa_0\right)\psi'+aa_1\psi''\right)=0, \tag{26}$$

ψ^{-2} coeff.:

$$\begin{aligned}-a_1\Big(&\left(24\kappa\lambda a_0^2a_1+4\kappa^2aa_1+12\kappa ca_0a_1-24ba_0^2a_1+4\kappa\alpha a_1+4\omega a_1\right)\left(\psi'\right)^2\\&+6aa_0\psi'\psi''-2aa_1\psi'\psi'''+aa_1\left(\psi''\right)^2\Big)=0,\end{aligned} \tag{27}$$

ψ^{-1} coeff.:

$$-2a_0a_1\left(\left(8\kappa\lambda a_0^2+4\kappa^2a+6\kappa ca_0-8ba_0^2+4\kappa\alpha+4\omega\right)\psi'-a\psi'''\right)=0, \tag{28}$$

ψ^{0} coeff.:

$$-4a_0^2\left(\kappa\lambda a_0^2+\kappa^2a+\kappa ca_0-ba_0^2+\kappa\alpha+\omega\right)=0. \tag{29}$$

Solving this system, we obtain

$$a_0=-\frac{3c\kappa}{4(\kappa\lambda-b)},\quad a_1=\pm\frac{1}{2}\sqrt{\frac{3a}{\kappa\lambda-b}}, \tag{30}$$

$$\omega=-\frac{\kappa\left(16\kappa^2a\lambda-16\kappa ab+16\kappa\alpha\lambda-3\kappa c^2-16\alpha b\right)}{16(\kappa\lambda-b)}, \tag{31}$$

and

$$\psi''=\pm\frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}\psi', \tag{32}$$

$$\psi'''=\frac{3\kappa^2c^2}{4(\kappa\lambda-b)a}\psi'. \tag{33}$$

From Eqs.(32) and (33), we can deduce that

$$\psi' = \pm \frac{2\sqrt{(\kappa\lambda - b)a}}{\kappa c\sqrt{3}} c_1 e^{\pm \frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}\xi}, \tag{34}$$

and

$$\psi = \frac{4(\kappa\lambda - b)a}{3\kappa^2 c^2} c_1 e^{\pm \frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}\xi} + c_2, \tag{35}$$

where c_1 and c_2 are constants of integration. Substituting Eqs. (34) and (35) into Eq. (11), we obtain following exact solution to Eq. (2)

$$q(x,t) = \left\{ -\frac{3c\kappa}{4(\kappa\lambda - b)} \pm \frac{1}{2}\sqrt{\frac{3a}{\kappa\lambda - b}} \left(\frac{\pm \frac{2\sqrt{(\kappa\lambda-b)a}}{\kappa c\sqrt{3}} c_1 e^{\pm \frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}(x-vt)}}{\frac{4(\kappa\lambda-b)a}{3\kappa^2c^2} c_1 e^{\pm \frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}(x-vt)} + c_2} \right) \right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (31). If we set

$$c_1 = \frac{3\kappa^2 c^2}{4(\kappa\lambda - b)a} e^{\pm \frac{\kappa c\sqrt{3}}{2\sqrt{(\kappa\lambda-b)a}}\xi_0}, \quad c_2 = \pm 1$$

we obtain:

$$q(x,t) = \left\{ -\frac{3c\kappa}{8(\kappa\lambda - b)} \left(1 \pm \tanh\left[\frac{\kappa c\sqrt{3}}{4\sqrt{(\kappa\lambda - b)a}} \left(x - vt + \xi_0\right) \right] \right) \right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{36}$$

$$q(x,t) = \left\{ -\frac{3c\kappa}{8(\kappa\lambda - b)} \left(1 \pm \coth\left[\frac{\kappa c\sqrt{3}}{4\sqrt{(\kappa\lambda - b)a}} \left(x - vt + \xi_0\right) \right] \right) \right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{37}$$

where Eqs. (36) and (37) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$(\kappa\lambda - b)a > 0.$$

4. Conclusions

This paper retrieves dark and singular optical soliton solutions to the perturbed GI equation where the perturbation terms appear with full nonlinearity. The modified simple equation algorithm is implemented to secure these soliton solutions that are derivable for specific values of the full nonlinearity parameter m. Evidently, one drawback of this algorithm is that, this integration scheme fails to obtain bright soliton solution to the model. Thus, a complete spectrum of soliton solutions are not available with this scheme. Nevertheless, this scheme has been successfully applied in the past to study optical solitons for NLSE [12].

Acknowledgements

The work of the seventh author (QZ) was supported by the National Science Foundation for Young Scientists of Wuhan Donghu University under the grant number 2017dhzk001. The eigth author (SPM) would like to thank the research support provided by the Department of Mathematics and Statistics at Tshwane University of Technology and the support from the South African National Foundation under Grant number 92052 IRF1202210126. The research work of nineth author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP 8-028-1-001. The authors also declare that there is no conflict of interest.

References

[1] A. Biswas, Y. Yildirim, E. Yasar, M.M. Babatin, Conservation laws for Gerdjikov–Ivanov equation in fiber optics and PCF, Optik 148 (2017) 209–214.

[2] A. Biswas, R.T. Alqahtani, Chirp-free bright optical solitons for perturbed Gerdjikov–Ivanov equation by semi-inverse variational principle, Optik 147 (2017) 72–76.

[3] Q. Zhou, A. Biswas, Optical solitons in parity-time-symmetric mixed linear and nonlinear lattice with non-Kerr law nonlinearity, Superlattices Microstruct. 109 (2017) 588–598.

[4] E. Fan, Integrable evolution systems based on Gerdjikov–Ivanov equations, bi-Hamiltonian structure, finite-dimensional integrable systems and *N*-fold Darboux transformation, Journal of Mathematical Physics 41 (2000) 7769.

[5] X. Guo, Two expanding integrable systems of the GI soliton hierarchy and a generalized GI hierarchy with self-consistent sources as well as its extension form, Commun. Nonlinear Sci. Numer. Simul. 14 (12) (2009) 4065–4070.

[6] B. He, Q. Meng, Bifurcations and new exact travelling wave solutions for the Gerdjikov–Ivanov equation, Commun. Nonlinear Sci. Numer. Simul. 15 (7) (2010) 1783–1790.

[7] N. Kadkhoda, H. Jafari, Analytical solutions of the Gerdjikov–Ivanov equation by using exp $(-\phi(\xi))$-expansion method, Optik 139 (2017) 72–76.

[8] J. Manafian, M. Lakestani, Optical soliton solutions for the Gerdjikov–Ivanov model via $\tan(\phi/2)$-expansion method, Optik 127 (20) (2016) 9603–9620.

[9] H. Nie, J. Zhu, X. Geng, Trace formula and new form of *N*-soliton to the Gerdjikov–Ivanov equation, Anal. Math. Phys. (2017), http://dx.doi.org/10.1007/s13324-017-0179-3 (in press).

[10] C. Rogers, K.W. Chow, Localized pulses for the quintic derivative nonlinear Schrödinger equation on a continuous-wave background, Phys. Rev. E 86 (2012) 037601.

[11] H. Triki, R.T. Alqahtani, Q. Zhou, A. Biswas, New envelope solitons for Gerdjikov–Ivanov model, Superlatt. Microstruct. 111 (2017) 326–334.

[12] E. Yasar, Y. Yildirim, Q. Zhou, S.P. Moshokoa, M.Z. Ullah, H. Triki, A. Biswas, M. Belic, Perturbed dark and singular optical solitons in polarization preserving fibers by modified simple equation method, Superlatt. Microstruct. 111 (2017) 487–498.

[13] H. Yilmaz, Exact solutions of the Gerdjikov–Ivanov equation using Darboux transformations, J. Nonlinear Math. Phys. 22 (1) (2015) 32–46.

[14] J.B. Zhang, Y.Y. Gongye, S.T. Chen, Soliton solutions to the coupled Gerdjikov–Ivanov equation with rogue-wave-like phenomena, Chin. Phys. Lett. 34 (9) (2017) 090201.

[15] L.X. Zheng, L.X. Yong, Z.L. Ying, Z.J. Liang, Exact solutions of Gerdjikov–Ivanov equation, Acta Phys. Sin. 51 (4) (2008) 2031–2034.