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Chirped singular solitons for Chen-Lee-Liu equation in optical fibers and PCF

Houria Triki [a], Yamina Hamaizi [a], Qin Zhou [b,*], Anjan Biswas [c,d,e], Malik Zaka Ullah [d], Seithuti P. Moshokoa [e], Milivoj Belic [f]

[a] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria
[b] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, PR China
[c] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal AL-35762, USA
[d] Operator Theory and Applications Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, PO Box-80203, Jeddah 21589, Saudi Arabia
[e] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[f] Science Program, Texas A & M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This paper obtains chirped singular optical soliton solutions that is modeled by Chen-Lee-Liu equation in the context of optical fibers and PCF. A special complex envelope traveling-wave method is applied to find a nonlinear equation with a fifth-degree nonlinear term describing the dynamics of field amplitude in the nonlinear media. It is shown that the chirp associated to the obtained solutions is directly proportional to the intensity of the wave and its amplitude is controlled by self steepening term. Parametric conditions for the existence of the chirped singular structures are also presented.

1. Introduction

Studies of envelope soliton solutions of the nonlinear Schrödinger (NLS) equation have been of great value in understanding widely different nonlinear physical phenomena in nonlinear optics and plasma physics as well as nonlinear dispersive water waves [1,2]. Indeed, exact solutions allow one to calculate certain important physical quantities analytically as well as serving as diagnostics for simulations [3]. These soliton structures exist in a variety of forms and shapes such as bright and dark soliton solutions, W-shaped and M-shaped soliton solutions, and so on. In addition, singular-type soliton solutions have recently become a subject of intense research activity. Such structures were analytically investigated within the framework of a variety of higher-order NLS equations by means of many significant methods [4–6].

However, much of the recent works on this topic were chiefly focused on the case of unchirped singular soliton solutions. Recently, it has been demonstrated that higher-order NLS models incorporating self-steepening and self-frequency shift terms can support bright, dark and double-kink solitons with nonlinear chirp [7–12]. Up to now, to our knowledge no theoretical exploration of singular solitons with nonlinear chirp for a given NLS-type equation has yet been done.

In this paper, we investigate the properties of nonlinearly chirped singular solitons in a derivative NLS equation with group velocity dispersion and self-steepening term and find the parametric conditions of their existence. It should be noted chirped pulses have many application in pulse compression or amplification and thus they are particularly useful in the

* Corresponding author.
E-mail addresses: qinzhou@whu.edu.cn, qzh@whu.edu.cn, qinzhouwhu@gmail.com (Q. Zhou).

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design of fiber-optic amplifiers, optical pulse compressors, and solitary-wave-based communications links [7,13,14]. The obtained results clearly show how self-steepening effect induces a nontrivial chirping, which depends on the wave intensity.

2. Governing model

We consider the evolution of a slowly varying envelope q as modeled by a family of the CLL equation of the form:

$$iq_t + aq_{xx} + ib|q|^2 q_x = 0, \tag{1}$$

which collapses to the case of regular CLL equation when $a = b = 1$ [15].

In the optical fiber setting, the term involving the parameter b is usually associated with the self-steepening phenomena [16], while the term related to a is group velocity dispersion. In such contexts, the coordinates t and x denote propagation distance and retarded time, but represent slow time and spatial coordinate traveling with the group velocity in hydrodynamics, respectively [16].

The DNLS equation (1) without the inclusion of the usual cubic nonlinear term is a widely used model with important physical applications. It is of interest to find exact chirped soliton solutions of this equation. To start with, we seek traveling-wave solutions of Eq. (1) of the form

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where $g(s)$ is the unknown envelope function (assumed to be real), with

$$s = x - vt, \tag{3}$$

and the phase $\phi(x, t)$ given by

$$\phi(x, t) = -\kappa x + \omega t + \theta(s). \tag{4}$$

where $\theta(s)$ is an extra phase function depending upon the traveling variable s. Also, κ is the frequency of the soliton, ω is the wave number of the soliton, and v is the soliton velocity. The corresponding chirp is given by

$$\delta\omega(x, t) = -\frac{\partial}{\partial x}[-\kappa x + \omega t + \theta(s)] = \kappa - \theta'(s). \tag{5}$$

Substituting Eq. (2) into Eq. (1), the real and imaginary parts of the resulting equation respectively read

$$-\omega g + v\theta' g + ag'' - ag\theta'^2 - a\kappa^2 g + 2a\kappa\theta' g - b\theta' g^3 + b\kappa g^3 = 0, \tag{6}$$

and

$$a(g\theta'' + 2g'\theta') - vg' - 2a\kappa g' + bg^2 g' = 0, \tag{7}$$

where primes denote differentiations with respect to s.

To solve the above equations, we make the ansatz

$$\theta' = \alpha g^2 + \beta, \tag{8}$$

where α and β denote the constant and nonlinear chirp parameters, respectively. The substitution of ansatz Eq. (8) into Eq. (7) yields two algebraic equations that define the chirp parameters:

$$\alpha = -\frac{b}{4a}, \ \ \beta = \kappa + \frac{v}{2a}, \tag{9}$$

Accordingly, the resultant chirp consisting of linear and nonlinear contributions can be obtained as

$$\delta\omega(x, t) = \frac{b}{4a}g^2 - \frac{v}{2a}. \tag{10}$$

which shows that the chirping is directly proportional to the intensity of wave [with $I = |q|^2 = g^2$]. It is worth remarking that the self-steepening and GVD parameters can be used to control the amplitude of chirping.

On substituting Eq. (8) along with Eq. (9) in Eq. (6), one obtains

$$g'' + \delta g + \sigma g^3 + \gamma g^5 = 0, \tag{11}$$

where

$$\delta = \frac{v^2}{4a^2} + \frac{v\kappa}{a} - \frac{\omega}{a}, \tag{12a}$$

$$\sigma = -\frac{bv}{2a^2}, \tag{12b}$$

$$\gamma = \frac{3b^2}{16a^2}. \tag{12c}$$

Eq. (11) is an elliptic equation with a fifth-degree nonlinear term describing the dynamics of field amplitude in the physical system. This equation can be mapped into φ^6 field equation, which is well known to admit bright soliton, dark soliton, kink, and double kink solutions [7,17,18]. In the following we are interested to find the exact singular soliton solution for this equation to construct the analytical chirped solutions of Eq. (1).

3. Chirped singular solitons

We now study Eq. (11) for different parameter conditions and obtained exact singular soliton solutions in the most general case, when all the coefficients in Eq. (11) have nonzero values. We also find the chirping function associated with each of these soliton solutions.

3.1. Type-I

For $\gamma = \frac{3\sigma^2}{16\delta}$, one obtains an exact solution of Eq. (11) of the form

$$g(s) = \pm p\sqrt{1 \pm \coth(\mu s)}, \tag{13}$$

where p and μ are real parameters defined by the relations

$$p^2 = -\frac{2\delta}{\sigma}, \ \ \mu^2 = -\delta, \tag{14}$$

provided that $\delta < 0$ and $\sigma > 0$, which implies $\omega > v\left(\frac{v}{4a} + \kappa\right)$ and $bv < 0$.

By substituting the solution Eq. (13) into Eq. (2), we get a first chirped singular soliton solution for the CLL equation (1) of the form

$$q(x,t) = \pm p\sqrt{1 \pm \coth[\mu(x - vt)]}e^{i[-\kappa x + \omega t + \theta(s)]}, \tag{15}$$

The corresponding chirping takes the form

$$\delta\omega(x,t) = \frac{bp^2}{4a}\{1 \pm \coth[\mu(x - vt)]\} - \frac{v}{2a}. \tag{16}$$

where the soliton parameters p and μ are given by Eq. (14) while the velocity v of the soliton is an arbitrary constant.

3.2. Type-II

For $\gamma > \left|\frac{3\sigma^2}{16\delta}\right|$, one can find a singular-type soliton solution of Eq. (11) of the form

$$g(s) = \frac{A}{\sqrt{1 + r\sinh(\mu s)}}, \tag{17}$$

where A, μ and r are real parameters defined by the expressions

$$A^2 = -\frac{4\delta}{\sigma}, \ \ \mu^2 = -4\delta, \ \ r^2 = \frac{16\delta\gamma}{3\sigma^2} - 1. \tag{18}$$

provided that $\delta < 0$ and $\sigma > 0$.

Hence, the second chirped singular soliton solution of Eq. (1) is given by

$$q(x,t) = \frac{A}{\sqrt{1 + r\sinh[\mu(x - vt)]}}e^{i[-\kappa x + \omega t + \theta(s)]}, \tag{19}$$

for which the chirping will be

$$\delta\omega(x,t) = \frac{b}{4a}\left(\frac{A^2}{1 + r\sinh[\mu(x - vt)]}\right) - \frac{v}{2a}. \tag{20}$$

where the soliton parameters A, μ and r are given by Eq. (18) while the velocity of the soliton v is an arbitrary constant.

3.3. Type-III

For $\delta < 0$, Eq. (11) exhibits the other kind of exact soliton solution which is given as

$$g(s) = \frac{P\mathrm{csch}(Qs)}{\sqrt{1 - R\coth^2(Qs)}}, \tag{21}$$

where

$$Q^2 = -\delta, \; P^2 = \frac{2\delta(1+R)}{\sigma}, \; \gamma = \frac{3\sigma^2 R}{4\delta(1+R)^2} \tag{22}$$

with $\sigma < 0$, $\gamma > 0$, $R < -1$, and s is given by Eq. (3).

Thus, the nonlinearly chirped singular soliton solution for Eq. (1) is of the form

$$q(x,t) = \frac{P\mathrm{csch}[Q(x-vt)]}{\sqrt{1-R\coth^2[Q(x-vt)]}} e^{i[-\kappa x+\omega t+\theta(s)]}, \tag{23}$$

The corresponding chirping takes the form

$$\delta\omega(x,t) = \frac{bP^2}{4a}\left\{\frac{\mathrm{csch}^2[Q(x-vt)]}{1-R\coth^2[Q(x-vt)]}\right\} - \frac{v}{2a}. \tag{24}$$

where the soliton parameters P and Q are given by Eq. (22) while v is an arbitrary constant.

3.4. Type-IV

For $\gamma < \left|\frac{3\sigma^2}{16\delta}\right|$, the solution of Eq. (11) is of the following form:

$$g(s) = \left(\frac{A}{D+\sinh^2(\mu s)}\right)^{1/2}, \tag{25}$$

where A and μ are given by the expressions:

$$A = -\frac{2\delta(2D-1)}{\sigma}, \; \mu^2 = -\delta, \tag{26}$$

with

$$2D-1 = \pm\left[1-\frac{16\delta\gamma}{3\sigma^2}\right]^{-1/2}. \tag{27}$$

provided that $\delta < 0$ in order to ensure the parameter μ to be real.

For this case, the exact chirped singular soliton solution of Eq. (1) can be written as

$$q(x,t) = \left(\frac{A}{D+\sinh^2[\mu(x-vt)]}\right)^{1/2} e^{i[-\kappa x+\omega t+\theta(s)]}, \tag{28}$$

and the chirping is given by

$$\delta\omega(x,t) = \frac{b}{4a}\left(\frac{A}{D+\sinh^2[\mu(x-vt)]}\right) - \frac{v}{2a}. \tag{29}$$

where the soliton parameters A and μ are given by Eq. (26) while v is an arbitrary constant.

4. Conclusions

In this paper, a broad class of exact singular soliton solutions with nonlinear chirp to the Chen-Lee-Liu equation has been found by means of an appropriate complex envelope traveling-wave method. It is shown that these chirped structures have nontrivial phase chirping which varies as a function of the intensity of the wave and can be controlled by self steepening term. These solutions exist provided that certain relations between the parameters of group velocity dispersion and self-steepening nonliearity are fulfilled.

Future research may include a systematic study of the self-similar propagation of chirped soliton pulses within the framework of the generalized Chen-Lee-Liu equation with distributed coefficients.

Conflict of interest

The authors declare that there is no conflict of interest.

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