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Optimal location of landfills and transfer stations for municipal solid waste in developing countries using non-linear programming

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ABSTRACT

The design of Municipal Solid Waste Management Systems (MSWMS) is one of the fields where optimization techniques have been used in different places. The aim of this study was to develop a mathematical model for the optimization of MSW Transportation System, in order to assist waste management institutions, and local governments to minimise waste transportation time and cost. Non-linear Mixed Integer mathematical model, with the objective function to minimize the time and cost of waste transportation, was developed and solved using Microsoft Excel solver. The developed model was applied to a case study city situated in South Africa. The application of this model to this case study has provided an approximate decrease in total transportation cost per week of 2.04%. The novelty of this research lies in the simplification of the existing mathematical model and the development of a new approach to solve the non-linear model.

1. Introduction

It is known that by 2000 BC, at least three cultures (Babylonian, Egyptian and Indian) had a decent knowledge of mathematics and made use of mathematical models to improve their every-day life [1]. Since then, many techniques and algorithms have been developed and applied in various fields such as control systems, electrical engineering, mechanics, economics, finances, and operations research.

Mathematical programming, in conjunction with computer software, has been extensively applied throughout the world to optimize the siting of waste facilities. However, most of these studies have been conducted in developed countries, leaving developing countries with a gap in the application of mathematical programming for the optimal design and operations of MSWMS.

Collection and transportation of MSW is a huge contributing factor to the total cost of waste management in any country. More than sixty percent of the costs of MSWMS in different countries are due to the collection and transportation process including labor costs, the high price of fuel, machinery, and equipment maintenance [2].

Although Waste Transfer Station (WTS), which is a temporary waste storage facility, has been used in different countries to reduce transportation cost, the challenge that municipalities are still facing is the ability to determine when the construction of these facilities becomes beneficial. This brings the issue of trash hold distance, which is the minimum distance between the waste generation nodes and the landfill, from which the construction of WTS becomes economically profitable. In the same analogy, identifying the best location for LF is also a critical factor as it contributes to the optimal design and operations of MSWMS. Fig. 1 displays standard waste management process from collection to landfilling.

Different cities face different kinds of problems regarding waste management; and waste management is even a major problem in developing countries which do not always have enforcement institutions at the municipality level to regulate waste management systems [3]. The same authors have also found that these developing cities usually find themselves in a state where the density of the population keeps on growing and the local governments do not usually have sufficient waste management data. On the other hand, refuse collection represents a significant expenditure for cities in developing countries, often comprising from 20% to 50% of local government revenues just for coverage of recurrent expenditures [4].

However, MSW does not only have economic impacts on the population, but it also presents considerable social and environmental impacts. A study conducted by Boadi and Kuitunen [5] found that poor handling and disposal of waste are major causes of environmental pollution which create room for reproduction of pathogenic organisms and spread of infectious diseases. Based on these facts, it can be said that developing countries can positively affect the lifestyle of their people and improve the cleanliness of their cities by implementing effective and optimal MSWMSs.

An overview of the Optimization Modelling applications, by Singh [6], revealed that researchers around the world have developed techniques

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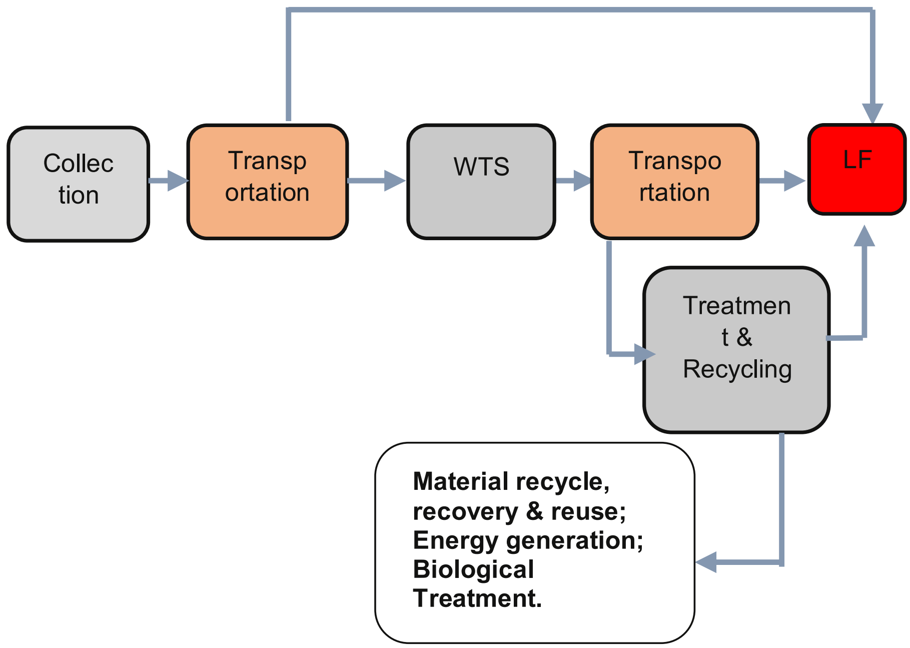

Fig. 1. Standard waste management process.

to solve optimization problems in different fields. According to this study, more than 16 papers have been published on the application of optimization techniques and simulation to improve the effective use of resources in the waste management sector, and these studies have produced positive results. According to the same study, rapid urbanization poses a serious environmental threat with respect to solid waste management (SWM), a problem that is particularly severe in developing countries. Considering the type of the problem being addressed, mathematical programming application in waste management can be categorised in three groups, transportation problems, facility location problems, and a mixed of transportation, facility location and resource utilisation problems. The following are different studies that have applied optimization techniques to solve various problems in waste management.

Talebbeydokhti et al. [2] conducted a study on the optimization of solid waste collection and transportation using TransCAD software, which is a Geographical Information System (GIS) based software for the solid waste routing and transportation optimization. The research included a case study of the city of Marvdasht, in Iran. The study resulted in an optimised system that showed a decrease in the distance travelled and waste transportation time of 15.7% and 29.43% respectively.

While most studies on WMS optimization have focused on the applications of conventional, heuristic and Meta-heuristic optimization techniques to improve the efficiency of MSWMS, the study conducted by Hannan et al. [7] made use of computer software to optimize the collection and routing of solid waste systems. In this research, the Particle Swam Optimization (PSO) algorithm and Capacitated Vehicle Routing Problem (CVRP) were used together with the implementation of smart bins to optimize the collection and routing in a MSW collection system. The research suggested that smart bins could be used to send information about waste levels and weight in the bins to a PSO-based CVRP system that coordinates the scheduling of waste collection vehicles. In this way, the system can only send vehicles to full bins to optimize the routing in the system.

Another study in the same analogy is a study conducted by Nguyen-Trong et al. [8], which proposed a model for the optimization of MSW collection. The methodology followed in this research was also twofold. It first applied the conventional Vehicle Routing Problem (VRP) to optimize the collection, and then integrated a GIS system with agent-based modelling to provide the analysis with a dynamic consideration. The result was a system that reduced the distance and travel time of waste collection vehicles.

Most of the studies mentioned thus far focused on optimizing the time and cost aspects of the systems. However, the study conducted by Bányai et al. [9] considered the environmental aspect as the objective function of the optimization model. In the study, the authors made use of binary bat algorithms, which are a representation of PSO to optimize municipal waste collection routing by minimizing the garbage trucks' energy consumption. The other interesting factor is that the authors have considered the waste collection system as a Cyber-Physical System (CPS), identifying all Industry 4.0 technologies such as RFID, big data tools, cloud and fog computing that have a great impact on the optimization of waste collection systems.

When it comes to identifying the right position for a facility, the task is not as easy as it might sound; different critical factors have to be addressed to ensure that a facility is placed at the optimal position to get the most benefits out of it. Facility location problems in waste management are complicated because locating waste handling facilities involves not only economic factors, but also environmental and social factors. Because waste generation and management is a reality for all the countries on the planet, a myriad of studies have been conducted for the optimization of waste facility location in different cities of the world. Some of these studies include: *a Model for Optimal Operation and Design of Solid Waste Transfer Stations*, University of California [10], where optimization techniques have been used to optimize the WMS of cities. In another study, Chatzouridis and Komilis [11] made use of the non-linear integer programming and the GIS system to optimise the collection of municipal solid waste. The result of this research was a model that minimises the cost per unit of MSW collection. A case study was conducted in the Hellenic region, in Greece, and the minimum collection cost of €42.4 t^{-1} was determined for the city. The novelty of this study is that, the model developed can be applied in a situation where the number and the locations of candidate transfer stations are not determined.

Yadav et al. [12] conducted another study for the optimization of municipal solid collection and transfer. In this study, a non-linear integer programming in conjunction with a GIS were used as a methodology to develop a model for the optimal siting of waste transfer stations.

Again in another study conducted by Lee et al. [13], mathematical modelling was used to develop a MSW system for the city of Hong Kong. In this model, integer linear programming and a mixed-integer programming were used with the objective function to improve the utilisation of solid waste infrastructures.

Among various studies that have used multi – objective optimization techniques is the study conducted by Habibi et al. [14]. The objective of this study was to improve the MSWMS of the city of Tehran by optimizing the site-selection and capacity allocation of waste facilities. Although many similarities exist between this research and other studies conducted in the same field, the novelty of this study was the fact that it has combined the three factors of sustainability in waste management, which include; the cost, the environment and the social aspect. With the use of robust and multi-objective optimization methods, this study has proposed a model that can minimize the cost of MSW management system, its greenhouse gas emission, and visual pollution due to construction of MSW handling and processing facilities.

One study that stood out in terms of mathematical programming application in SWM is the study conducted by Li et al. [15]. In this study, the authors used the Scenario-based, Fuzzy stochastic Quadratic Programming (SFQP) method to identify an optimal waste management policy under uncertainties. The developed method has been applied for long-term waste management operation and planning; and the solution obtained was reasonable and applicable for identifying desired waste flaws allocation plans and for making compromises between system cost, degree of satisfaction and constraints violation risk.

The articles mentioned above, and many other scientific articles have shown that LP techniques have been used extensively to solve different problems in waste management and positive results have been found. The only drawback of these techniques is that they require an advanced level of mathematics to model the problems and a considerable understanding of logic and computer programming to build algorithms to solve the models.

Based on this literature review, it has been found that in transportation problems, VRP and CVRP accompanied with GIS are the most used techniques; while multi-objectives integer programming and non-linear programming are the most used techniques to optimize MSWMS in facility location problems; and finally, in optimization problems with a combination of transportation, facility location and resource utilization, the mixed-integer and the non-linear programming techniques are the most used techniques. Following these conclusions, a non-linear technique has been used in this study to develop a model for the optimization of MSWMS in the context of developing cities.

The motivation for this study is based on the fact that, although so many studies have been conducted in this field, there is still a need for the application of LP to optimize MSWMS, especially in the developing countries. Furthermore, this paper intended to contribute in this field by developing a new approach for the solving LP problems.

The remaining part of the study is divided into the following sections: Section 2 describes the problem statement; Section 3 highlights the aim of the study; while Section 4 describes the methodology followed in the study. Section 5 is about the development and application of the mathematical model within the context of a case study. And finally, Section 6 provides a conclusion of the study.

2. Problem statement

Different challenges are faced by developing countries in terms of waste management, which constitute the main problem that triggered the interest of this research, following is a list of the problems:

i. High cost of waste collection and disposal.
ii. Decrease of available air space for landfills. According to a report released by the South African Cities Network [16], the available air space for landfilling is decreasing by 1.17 million m^3 per year, raising the need for the development of new methods and means for better planning.
iii. Low rate of recycling in developing countries. While developed countries such as Germany recycle more than 65% of their waste, most developing countries such as South Africa only recycle about 10% of its waste [17,18], a problem that can be addressed by developing a system that will favour methods with a high recycling rate.

3. Research aim

The aim of this research was to minimise waste transportation time and costs by developing and solving a mathematical model and determining the optimal location and capacity of waste management facilities (WTS and LF). With that being said, the research intended to assist local governments and other waste management institutions to improve their waste management operations and enhance the life of the people. The novelty of the study was the simplified mathematical models for time and cost minimization and a new approach to solve the mathematical model using Solver add-in for Microsoft excel.

4. Methodology

The methodology followed in this study is twofold. The first part consisted of the mathematical model development and application to the case study. Based on the format of the objective function as well as of the constraints' equations, the data type requirement on the decision variables and the input parameters of the problem on hand, a Mixed-Integer Non-Linear Programming (MINLP) technique was used to develop and solve the model. As highlighted in the introduction, Non-linear programming has been applied around the world in several studies such as the study conducted by Chatzouridis and Komilis [11] to optimize the transportation of MSW, and the results have been satisfactory. MINLP models take the following standard form:

Objective function: $Minimize\ or\ maximize\ Z = f(x, y)$
Subject to: $g(x, y) \leq 0$
Where: $x \in X\ and\ y \in Y;\ X = \{\ x | x \in R^n,\ x^L \leq x \leq x^U,\ Bx \leq b\}$
$Y = \{\ y | y \in \{0, 1\}^n,\ Ay \leq a\}$
$f(x, y)\ and\ g(x, y)$ *are assumed to be convex and bounded over* X.
$f(x, y)\ and\ g(x, y)$ *are commonly linear in* y.

To solve the developed model, three linear programming solvers have been explored to determine the best software for this research; namely the CPlex Optimization studio, Microsoft Excel Solver Add-in and GLPK solver. Although in the study conducted by Meindl and Templ [19], the CPlex optimization studio was found to be the best among commercial and open source linear programming problem solvers in terms of the run time and capability to solve various problems types, modelling the problem is time intensive and requires expertise and experience in computer programming. For nonlinear problems with a relatively small-scale case study as the one in this study, Microsoft Excel with the Solver Add-in was found to be the best fit. This software provides a simple and quicker modelling platform and a great flexibility in changing the initial solution during optimization, which is a best practice in nonlinear optimization problems.

The second part of the study consisted of the determination of technique to be used to verify the developed model. Model verification is the process of ensuring that a model performs as intended. As described by Hillston [20], there are several methods of performing model verification; these methods include, but are not limited to, anti-bugging, simplified model, continuity testing and consistency testing. For the purpose of this research, the simplified method was selected for its applicability to this model.

5. Mathematical model development and optimization process

A mathematical model is an abstract representation that uses mathematical equations to describe the behaviour of a system [21]. These

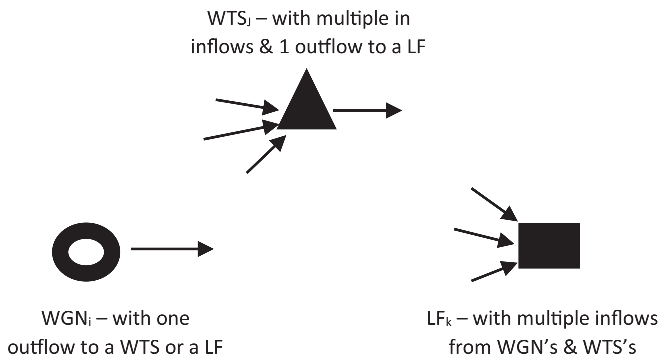

Fig. 2. Material flow in a MSW collection - transfer – disposal system.

equations are mostly solved using some numerical and non-numerical methods to find the best possible solution under certain conditions. These best solutions can be the minimum or the maximum of some parameter(s) of interest commonly referred to as objective function. Based on the existence of constraints, the physical structure of the problem, the nature of the equations involved, the permissible values of the decision variables, deterministic nature of the variables and the number of objective functions, the mathematical model developed here is considered to be constrained, optimal control, non-linear, mixed-Integer, deterministic and single-objective.

5.1. Mathematical formulation

Mathematical models developed in this section are similar to the ones developed by Chatzouridis and Komilis [11,22], with the difference that, these models are simplified by not including the time from the garage to the waste generation nodes and collection time within these generation nodes because these parameters do not affect the optimal location of WTS and LF's. These models are developed to minimize the time and cost of waste collection from the municipalities which are considered as Waste Generation Nodes (WGN) to their final destination (LF – Sink nodes), by making use of a transfer station (intermediate nodes) if necessary. Fig. 2 graphically describes the material flow of a typical WMS.

5.1.1. Decision variables

The decision (adjustable) variables X_i, Y_j, Z_k, X_{i-j}, Y_{i-k}, Z_{j-k} used in this model are binary, which are variables that can only take two values, for example true or false, Yes or No, or in our case zero and one. These variables determine the existence of WTS and or LF's, as well as if a waste generation node *i* should send its waste to a particular WTS *j* or to a particular LF *k*. A value of one means that the facility or the relationship between two facilities exists and a value of zero means the opposite.

5.1.2. Time optimization model

In this section a time optimization model has been developed, and all its characteristics are also described in subsequent section.

i. **Objective function:**

The objective function of the time optimization model is presented in Eq. (1). The objective of this model is to minimise the total waste collection and disposal time per week. This objective function is defined as a non-linear function simply because it involves a non-linear factor, which is the result of determining the distance from to the WGN to the LF via the WTS. To get this distance, one needs to only consider waste that is sent to the transfer station (introducing the x decision variables), and then determine which amount of that waste will be sent to different landfills (introducing the z decisions variables).

$$\begin{aligned} Min\,Time \;=\; & \sum_{i=1}^{N}\sum_{j=1}^{T}\left[\frac{WQ_i}{WCV cap}\left(2\frac{d_{i-j}}{V_{WCV}} + QT_j\right)\right]x_{i-j} \\ & + \sum_{i=1}^{N}\sum_{k=1}^{L}\left[\frac{WQ_i}{WCV cap}\left(2\frac{d_{i-k}}{V_{WCV}} + QT_k\right)\right]y_{i-k} \\ & + \sum_{j=1}^{T}\sum_{k=1}^{L}\left[\frac{\sum_{i=1}^{N} WQ_i.\,x_{i-j}}{Trcap}\left(\frac{2.d_{j-k}}{V_{Tr}} + QT_k\right)\right. \\ & \left. + \frac{\sum_{i=1}^{N} WQ_i.\,x_{i-j}}{Trcap} \times QT_j\right] z_{j-k} \end{aligned} \tag{1}$$

ii. **Constraints**

This optimization model is subject to the following constraints:

a. $$\sum_{j=1}^{T} x_{i-j} + \sum_{k=1}^{L} y_{i-k} = 1 \; for\ all \;\; i = 1 \; to \; N; \tag{2}$$

The constraint expressed as Eq. (2) states that all waste generation nodes (Ward) can only send their waste to one destination, either a transfer station or a landfill.

b. $$\sum_{k=1}^{L} z_{j-k} = 1 \; for\ all \; j = 1 \; to \; T \tag{3}$$

This constraint states that all transfer stations can only transfer waste to one landfill.

c. $$\sum_{i=1}^{N} WQ_i.\, x_{i-j} \;\le Max\ j; for\ all\ j = 1\ to\ T. \tag{4}$$

Eq. (4) ensures that the total waste quantity transferred to a transfer station is less than or equal to its maximum capacity.

d. $$\sum_{i=1}^{N} WQi.x_{i-k} + \sum_{j=1}^{T} x_{j-k} \times \left(\sum_{i=1}^{N} WQi\ .\ x_{i-j}\right) \le Max\ k; for\ all\ k = 1\ to\ L. \tag{5}$$

This last constraint is to ensure that the total waste quantity transferred to a particular landfill is less than or equal to its maximum capacity.

iii. **Model discussion**

The time optimization model presented in this section was used to minimize the total time to transport waste from the WGN to WTS or directly to landfills (LF). This total time includes the time to move waste from the WGN to WTS, from WTS to LF and from WGN to LF.

- The waste generation rate *(WQ)* is calculated per week, by dividing the annual waste quantity by fifty-two, which is the rounded number of weeks in a year. In the study conducted by Komilis, this variable was calculated per collection day, but this method is not applicable when waste is not collected on the same day for all the wards (districts) as is the case for most of cities in developing countries.
- The other element is the haul capacity of the waste collection vehicle (WCV_{cap}) and the semi-trailer trucks $(Tr._{cap})$; this is the maximum amount of waste that these vehicles can carry on a single trip; expressed in *kg* per trip.
- Travel time: this parameter models the time it takes the vehicles to travel between two system nodes, and it is multiplied by two to count for the return trip. For *WCV* travelling from the WGN to the WTS, this time is calculated by dividing the distance between the two nodes (d_{i-j}) by the vehicle average speed (V_{wcv}). Similarly, for the trucks travelling from the WTS to the LF, the time is calculated by dividing the distance between these two nodes (d_{j-k}) by the average speed of the truck (V_{tr}). Finally, the time to travel from the WGN to the landfill is calculated by dividing the distance between the WGN and the LF (d_{i-k}) by the average speed of the WCV (V_{wcv}).

Table 1
Waste generation in Ekurhuleni region A.

AREA	Res. waste (t/a)	Non-res waste (t/a)	Total waste (t/a)	*WQI* (t/wk)
BEDFORD VIEW (WGN 1)	8233	5377	13,610	261.730
BENONI (WGN 2)	8271	9397	17,668	339.769
BOKSBURG (WGN 3)	27,662	106,550	134,212	2 581
GERMISTON (WGN 4)	25,174	93,867	119,041	2 289.250
KEMPTON PARK (WGN 5)	7069	45,933	53,002	1 019.269

Source: [24].

- The last element included in the model is the queue time; this parameter, expressed in hours, models the total time that waste vehicles spend at the waste facilities loading, unloading and waiting. QT_j is the queue time at the transfer station, and QT_k is the queue time at LF k.

5.1.3. Cost optimization model

Like the time optimization model, the cost model tends to minimize the total cost of waste transportation from the WGN's to the LF. This total cost is made of different waste transportation costs including vehicles capital, operating and maintenance costs, labor costs, the costs to operate transfer stations, capital, operational and maintenance costs of semi-trailer trucks; and the tipping fees at landfill, or the cost to operate landfills in case the latter are owned by the municipality.

i. **Objective function**

The objective function for the cost model is to minimise the total weekly cost of waste transportation and is described in Eq. (6). Every term of this function is explained in detail under model discussion.

ii. **Constraints**

All constraints as defined in the time optimization model are also applicable for the cost mode.

iii. **Model discussion**

The cost optimization model takes into consideration many more parameters that are not considered in the time model, this is what makes the cost model more reliable as compared to the time optimization model. Different parameters considered in the objective function of the model are described below:

- WCV_{cost}: This is the daily cost of operating the WCV. This cost is calculated by dividing the total time it takes to transport waste from the WGN to WTS or to the LF, including the queue time at the WTS and LF, by the total allowable daily operating time of the WCV. The result is then multiplied by the WCV daily cost (R/day).
- Daily operating cost of the semi-trailer trucks: Analogically, this cost is found by multiplying the truck daily cost to the quotient of the division between the total time required to transfer waste from the *WTS* to the *LF* and the total allowable daily operating time of the trucks (R/day).
- Vehicle haul cost: This cost includes the depreciated capital, maintenance cost, fuel cost, and other miscellaneous cost related with the operation of *WCV* and semi-trailer trucks (R/kg/km).
- TS unit cost (WTS_{UC}): This is the cost of operating the WTS facility (in R/kg).
- *LF* unit cost (LF_{UC}): This factor models the operating cost of the LF or the fee that a municipality will have to pay in case the *LF* is owned by a private contractor. It is calculated by multiplying the waste quantity disposed at the landfill by the tipping fee rate.
- The last factor included in the model is the labor cost, this is the cost incurred in the payment of employees working in the waste collection and transfer vehicles. It is simply calculated by multiplying the man hour (in R/h) by the total number of operators in the vehicles (Ev) or by the total number of employees in the truck (Et).

Table 2
GPS coordinates of WGN's, WTS's and LF's.

WGN's	Latitude	Longitude
Bedfordbview	−26.170941	28.12963
Benoni	−26.15109	28.36957
Boksburg	−26.23259	28.24097
Germiston	−26.231648	28.17078
Kempton Park	−26.09441	28.22927
WTS's		
Isando WTS	−26.145652	28.200279
Sebenza WTS	−26.125752	28.180143
Highveld WTS	−26.094721	28.235101
LF's		
Rooikraal	−26.307903	28.260556
Simmer And Jack	−26.22587	28.17078
Weltevreden	−26.210864	28.356737
Chloorkop LF's	−26.044194	28.171059

5.2. Application of the model to the case study city

For the purpose of this study, only the cost optimization model has been applied to the case study.

The case study city for the application of the developed model was the city of Ekurhuleni, in the East of the Gauteng province, in South Africa. Within this city, only region A was considered. This region is made of five wards (districts), namely; Bedfordview, Benoni, Boksburg, Germiston as well as Kempton Park. Fig. 3 shows a map of the city of Ekurhuleni, with the delimitation of region A and the positions of the five LF's and the three WTS's that service this region.

These five LF's and the three WTS's are considered as the candidates facilities to service different districts of the region. Tables 1, 2, 3 and 4 provide different aspects of the waste management system of region A, and the data from these tables constituted the inputs for the optimization process. Google Maps was used to determine the location of these facilities. For every location and facility, the coordinates of its central point was considered. These GPS coordinates are the inputs for the determination of distances between two points.

$$\text{Cost/wk} = \sum_{i=1}^{N}\sum_{j=1}^{T} \frac{2\left(\frac{WQi}{WCV_{cap}}\right).\frac{d_{i-j}}{\text{Vwcv}} + QTj \times \frac{WQi}{WCV_{cap}}}{OT_{WCV}} \times WCV_{cost} \times x_{i-j}$$

$$+ \sum_{i=1}^{N}\sum_{k=1}^{L} \frac{2\left(\frac{WQi}{WCV_{cap}}\right).\frac{d_{i-k}}{\text{Vwcv}} + QT_k \times \frac{WQi}{WCV_{cap}}}{OT_{WCV}} \times WCV_{cost} \times y_{i-k}$$

$$+ \sum_{j=1}^{T}\sum_{k=1}^{L} \frac{2\left(\frac{\sum_1^N WQi.x_{i-j}}{Tr.cap} \cdot \frac{d_{j-k}}{\text{Vtr}} + \frac{\sum_1^N WQi.x_{i-j}}{Tr.cap}.QT_k\right) + \left(\frac{\sum_1^N WQi.x_{i-j}}{Tr.cap}.QT_j\right)}{OT_{Tr}}$$

$$.Tr_{cost}.z_{j-k}$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{T} \left(WTS_{UC}\ .\ WQ_i\ .\ x_{i-j}\right)$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{T} 2.\frac{WQi}{WCV\,cap}.d_{i-j}.WCV_{HC}.WCV\,cap.x_{i-j}$$

$$+ \sum_{i=1}^{N}\sum_{k=1}^{L} 2.\frac{WQi}{WCV\,cap}.d_{i-k}.WCV_{HC}.WCV\,cap.y_{i-k}$$

Fig. 3. City of Ekurhuleni map.
Source: Modified from [23].

Table 3
Characteristics of waste collection and transfer vehicles.

PARAMETERS	STD VALUE	UNITS	SOURCE:
WCV VOLUME CAP	19.1	cubic meter	[25]
WCV COMPACTION RATE	596.842	Kg/m3	[25]
WCV MASS CAPACITY	11.4	ton	Calculated
WCV AVERAGE SPEED	70	km/h	Observed
SEMI-TRAILER TRUCK VOLUME CAP	56	Cubic meter	[23]
SEMI-TRAILER TRUCK MASS CAP	31.169	ton	calculated
QT @ WTS	0.5	h/trip	Observed
QT @ LANDFILLS	0.5	h/trip	Observed
SEMI-TRAILER TRUCK AVERAGE SPEED	90	km/h	Observed

$$
\begin{aligned}
&+\sum_{j=1}^{T}\sum_{k=1}^{L} 2.d_{j-k}\left(\sum_{i=1}^{N} WQi.x_{i-j}\right).Tr_{HC}.z_{j-k}\\
&+\sum_{i=1}^{N}\sum_{j=1}^{T}\left(2.\frac{WQi}{WCV_{cap}}.\frac{d_{i-j}}{V_{wcv}}+QT_J.\frac{WQi}{WCV_{cap}}\right).MHr.Ev.x_{i-j}\\
&+\sum_{i=1}^{N}\sum_{k=1}^{L}\left(2.\frac{WQi}{WCV_{cap}}.\frac{d_{i-k}}{V_{wcv}}+QT_k.\frac{WQi}{WCV_{cap}}\right).MHr.Ev.y_{i-k}\\
&+\sum_{j=1}^{T}\sum_{k=1}^{L}\left(2.\frac{\sum_1^N WQi.x_{i-j}}{Tr_{cap}}.\frac{d_{j-k}}{V_{Tr}}\right.\\
&\left.+\frac{\sum_1^N WQi.x_{i-j}}{Tr_{cap}}.QT_k+\frac{\sum_1^N WQi.x_{i-j}}{WCV_{cap}}.QT_j\right).MHr.Et.z_{j-k}\\
&+\sum_{i=1}^{N}\sum_{k=1}^{L} LF_{UC}.WQi.x_{i-k}+\sum_{j=1}^{T}\sum_{k=1}^{L} LF_{UC}.\left(\sum_{i=1}^{N} WQi.x_{i-j}\right).z_{j-k}\\
&+\sum_{j=1}^{T} y_i.MinCostj
\end{aligned}
\tag{6}
$$

$$
d = 2R.\arcsin\left(\sqrt{sin^2\left(\frac{1}{2}(lat\ 2 - lat\ 1)\right) + \cos(lat1).\cos(lat2).sin\left(\frac{long2 - long1}{2}\right)^2}\right) \tag{7}
$$

where R is the radius of the Earth

The scientific formula described in Eq. (7) was used to calculate the distances between any two given points.

All the data described in these tables consisted the input parameters for the application of the cost optimization model in the case study city. Before moving to the solution process, below are the assumptions that have been taken toward optimization process:

I. It is first assumed that there is no limitation in the capacity of LF and WTS; the capacity of these facilities is then considered at a later stage of the solution.
II. LF unit cost (LF_{UC}) is ignored because it is assumed that this cost is the same for all the LF; hence, this cost will not affect the choice of one LF over another.

Table 4
Cost data used in the model.

Description	Amount	Units	Source
$WCVcap$	8	Ton	[11]
$Vwcv$	70	km/h	[11]
QTJ	0.5	H	[11]
$OTwcv$	10	H	[11]
$WCVcost$	37.8	€/day	[11]
$di-j$	10.48	km	GIS
$di-k$	14.69	km	GIS
$dj-k$	8.94	km	GIS
QTk	0.5	h	[11]
$Trcap$	20	ton	[11]
VTr	70	km/h	[11]
$Trcost$	82.2	€/day	[11]
$WTSUC$	2.2	€/ton	[11]
$WCVHC$	0.057	€/ton km	[11]
$TrHC$	0.028	€/ton/km	[11]
MH	9	€/h	[11]
Ev	3.00	Employee	Observed
Et	1.00	Employee	Observed
OTk	10.00	H	[11]

Table 5
MS Excel set up for the solver solution.

	Linear Variable			Quadratic Variable
	x	y	z	$x.z$
Decision Variable	0	0		0
Objective Fct Coef	?	?		?
Objective Fct (Cost)	**R0.00**			
Constraints				
	LHS	SIGN	RHS	
1 Destination For All WGN	0	=	1	
WTS Maximum Capacity	0	<=	3000	
LF Maximum Capacity	0	<=	3500	

III. All WTS are already built and belong to the municipality, hence the capital cost of constructing these WTS (WTS min cost) is ignored.

Similar to the time optimization model, the cost model is a non-linear and convex problem. The convexity of the problem is determined by the fact all the constraints of the problem are either linear or convex, and the objective function is also a convex function (concavity curves upward). Given the complexity of this cost model, a new step by step method has been found to be very useful to solve it.

The following solution steps have been developed for the solution of the cost model:

Step 1: Manually select the LF and WTS closest to each WGN.
Step 2: Formulate sub-problems for each WGN considering only their closest LF and WTS.
Step 3: Solve the sub-problems and determine the local best solution.
Step 4: Check that all constraints and optimality conditions are satisfied.
Step 5: If there is no violation, aggregate the solution to the final solution of the main problem. For any violation of the maximum constraints, change the LF and or WTS of the WGN with the smallest difference between its closest LF (WTS) and its second closest LF (WTS) and repeat the process. Table 5 describes the MS Excel set up for the solver solution to the cost model sub-problem. The decision variables in this model represent x, y and z binary decision variables. The coefficients are from the model, and the objective function is calculated by performing the sum-product of the coefficients and the decision variables. As shown in this Table, the one destination for all WGN as well as the maximum capacity of the LF and WTS are the constraints considered in this model.

Table 6
Initial solution to the problem.

WGN	Destination	Cost
BEDFORDVIEW	Simmer and Jack LF	€934.311
BENONI	Weltevreden LF	€1168.055
BOKSBURG	Simmer and Jack LF	€9103.206
GERMISTON	Simmer and Jack	€4731.271
KEMPTON PARK	Chloorkop LFS	€3798.119

The initial solution to this problem is presented in Table 6, all WGNs transfer their waste directly to the nearest LF, and the total cost for this initial solution is estimated as **€20,642.**

After finding this initial solution, the next step was to check that all constraints and optimality conditions are satisfied. The solver report having already indicated that all the constraints and optimality condition for each sub-problem have been satisfied, the only constraint left, was the maximum capacity constraint of every LF and WTS.

As it can be seen in Table 7, there is a maximum capacity constraint violation for the Simmer and Jack LF, where the LF is receiving 1 653 tonnes more than its weekly capacity. To resolve this problem, three candidates WGN were considered, namely Bedfordview, Germiston and Boksburg. Among these three candidates, Boksburg happened to have the smallest difference between its closest LF and its second closest LF, which is Rooikraal. The reason behind choosing the candidate with the smallest difference is that the smallest difference will result in the smallest increase in the total cost. Since there is a violation, step 5 is skipped and the next step is step 6, where the closest LF for Boksburg is changed to Rooikraal and the solution is recalculated. After making this change, a new solution was found, which satisfied all the constraints. This solution is presented in Table 8.

To finalise this section, a comparison with the current system set up was performed to determine the saving or loss that this new solution will have on the overall system.

Table 9 provides the costs associated with the current waste destination for each WGN within Region A of the city of Ekurhuleni.

As shown in Tables 8 and 9, the difference in total cost between the current system set-up and the new optimized system is estimated as €429 per week. This is the estimated saving the municipality will make if the proposed optimal system is implemented. Fig. 4 shows the architecture of the proposed MSWMS for region A, indicating the flow of waste from WGN's to different LF's.

5.3. Model verification

As stated in the methodology, the simplified method has been chosen to verify the accuracy of the developed model. This method consists of reducing the model to its simplest form, in a way that manual operations can be performed on the model in order to understand its behaviour. Because the time model is the core model from which the cost model emanates, the verification is performed on it. To perform this verification, the following assumptions were considered:

i. Only one area (Bedfordview) generates waste;
ii. Only one transfer station exists in the municipality, Isando;
iii. Only one Landfill exists in the system, Rooikral;

• Manual Solution

By simplifying the model with the given assumptions, the model could be written in the form of Eq. (8).

$$\text{Minimize Time} = \left[\frac{WQ}{WCVcap}\left(2\frac{d_{b-i}}{V_{WCV}} + QT_j\right)\right].x + \left[\frac{WQ}{WCVcap}\left(2\frac{d_{b-r}}{V_{WCV}} + QT_k\right)\right].y + \frac{WQ.\,x}{Trcap}\left(\frac{2.d_{i-r}}{V_{Tr}} + QT_k\right).z + \frac{WQ.\,x}{Trcap} \times QT_j \quad (8)$$

Table 7
Constraints verification.

LF / WTS	Waste Received	Max Cap	Available Air Space	Comment
Simmer and Jack	**5 153**	**3500**	**−1 653**	**Violation**
Weltevreden	339	3500	3 160.23	No violation
Chloorkop LF	1 019	3500	2 480.73	No violation
Rooikraal LF	2 602	3500	897.73	No violation
Candidates Nodes to be Changed				
Node	**Closest distance**	**second closest distance**	**Difference**	**Comment**
Bedfordview	7.36	14.69	7.33	
Germiston	0.64	12.23	11.59	
Boksburg	7.04	8.6	1.56	Smallest difference

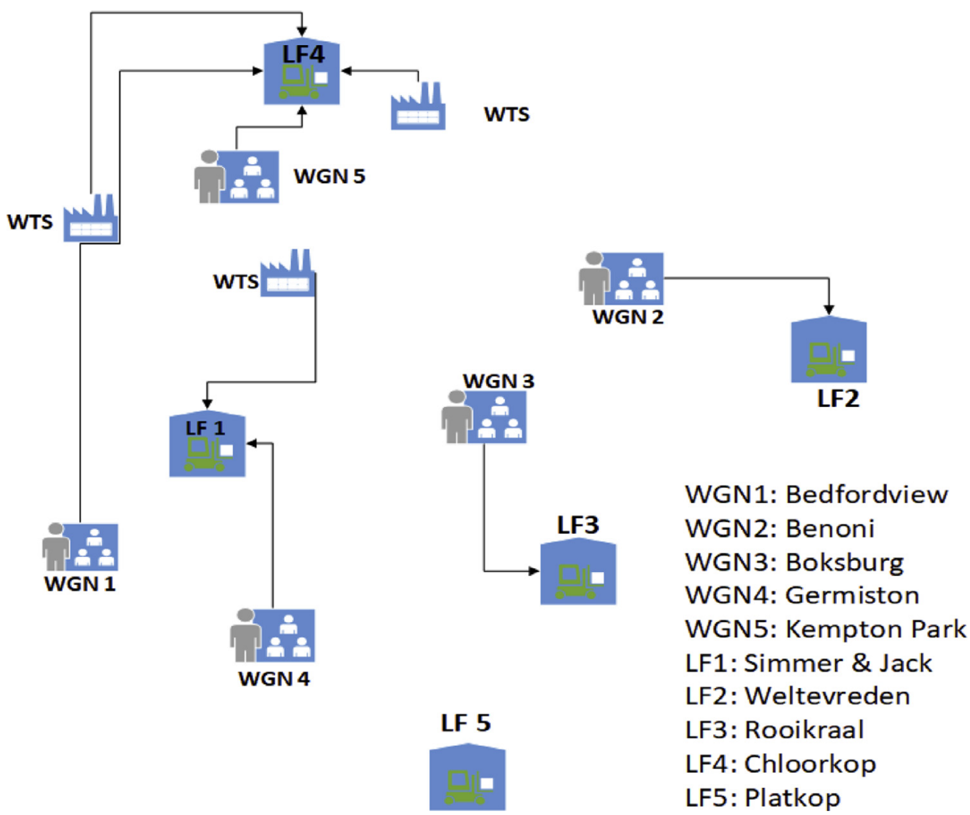

Fig. 4. Architecture of the proposed MSWMS.

Table 8
New solution.

WGN	Destination Lf	Cost
BEDFORDVIEW	Simmer and Jack	€934
BENONI	Weltevreden	€1168
BOKSBURG	Rooikraal	€10,011
GERMISTON	Simmer and Jack	€4731
KEMPTON PARK	Chloorkop LFS	€3798
TOTAL COST PER WEEK		**€20,642**

Table 9
Current system waste destination and costs.

WGN	Destination LF	Cost
BEDFORDVIEW	Chloorkop	€1363
BENONI	Weltevreden	€1168
BOKSBURG	Rooikraal	€10,011
GERMISTON	Simmer and Jack	€4731
KEMPTON PARK	Chloorkop LFS	€3798
TOTAL COST PER WEEK		**€21,072**

To solve this Equation manually, two scenarios were considered; first, the straight to landfill scenario, and second, by making use of the transfer station.

i. **Straight to the landfill scenario**

In considering this scenario, the decision variables x and z are set to zero and y decision variable is set to one. Therefore, the following solution can be observed:

$$\text{Time} = \frac{2,617,30.,77}{,114,00}\left(2\frac{20.06}{70} + 0.5\right)$$

$$= 24.638 \ \boldsymbol{hours}.$$

This is the total time it will take to transport all waste generated in Bedfordview to the Rooikral landfill.

ii. **Via the Isando transfer station.**

In making use of the Isando transfer station, the following is the solution to the model:

$$\text{Time} = \frac{2,617,30.,77}{,114,00}\left(2\frac{7.59}{70} + 0.5\right) + \left[\frac{2,617,30.,77}{,311,69}\left(\frac{2X30.65}{90} + 0.5\right) + \frac{2,617,30.,77}{,311,69}\ X\ 0.5\right]$$

= 30.574 ***hours***.

As shown in this manual solution, the best solution is for the waste to be transported straight to the landfill.

• **Solving the model with Solver add-in in MS Excel**

By setting the given assumptions in the Excel solver, the following solutions have been observed:

The optimal solution obtained suggested that all the x and z decision variables are set to zero. Only the y decision variable, which determines the relationship between the generation nodes and the landfill, was set to one. This optimal solution yields a total estimated time of ***24.638 h***. This indicates that, given this scenario, the optimal solution is that Bedfordview should send its waste directly to the landfill, which is also the optimal solution found using the manual calculation.

6. Conclusion

Through this study, a mathematical model for the optimal design of the MSW transportation system was developed. The research was conducted with the aim to assist waste management institutions, and local governments to minimize waste transportation time and cost.

The developed model was characterised as a Mixed-integer, Nonlinear model. The model was solved using Solver Add-in in MS Excel. Region A of the City of Ekurhuleni was considered as the case study city, where the cost optimization model was applied. The results of this study were a reduction in waste transportation cost from an estimated amount of €21,072 to €20,642 per week, which yields an approximate saving of €429 per week or €21,128 per year, leading to 2.04% decrease in waste transportation costs.

Although these results provide good insight and a starting point for the planning of future WMS, it has to be noted here that, the results presented here are based on assumptions on some parameters of which data was not accessible during this study. Further study is therefore recommended to determine with higher confidence levels the optimal systems for the case study city.

Different studies that might constitute future work include: A study that will analyse all the city's waste management constraints and parameters considering the social and environmental factors – which were beyond the scope of this research project – to determine the optimal solution and plan for the future. Considering social and environmental factors will constitute a holistic approach that can be used to practically plan for the future MSWMS and improve the lives of the inhabitants. Secondly, A study that will consider other parameters such as on-the-road distance, road traffic, uncertainty in the rate of waste generation, the environmental and economic impact of the increased rate of recycling that comes with the use of a transfer stations, as well as the breakdown of waste vehicles. The development of a stochastic model that will consider some or all these parameters will have a substantial impact on the improvement of the current work.

Declaration of Competing Interest

There is no conflict interest.

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