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Dispersive optical solitons with differential group delay by extended trial equation method

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ABSTRACT

This paper retrieves dispersive optical soliton solutions to Schrödinger-Hirota equation in birefringent fibesr that models dispersive optical soliton propagation. The extended trial equation method is the integration algorithm employed here. Bright and singular solitons solutions are obtained along with other forms of solutions such as snoidal waves and singular periodic solutions.

1. Introduction

Optical solitons are pulse molecules that propagate through a fiber or metamaterial or PCF or other forms of waveguides. These soliton dynamics are extensively studied using a variety of advanced mathematical techniques [1–15]. Some of the commonly applied mathematical techniques are traveling wave hypothesis, Lie symmetry analysis, inverse scattering transform, method of undetermined coefficients, mapping method, semi-inverse variational principle, Jacobi elliptic function method, just to name a few. In addition to the integtrability aspect, there are several other topics that are studied in the context of optical solitons. These are soliton perturbation theory, conservation laws, quasi-monochromatic solitons, quasi-particle theory to suppress intra-channel soliton collision, four-wave mixing, collision induced timing jitter, ghost pulses and several others.

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Dispersive optical solitons are observed when third-order dispersion (3OD) is taken into account, in addition to group velocity dispersion (GVD). In this case, solitons have a dispersive effect. The corresponding nonlinear Schrödinger's equation (NLSE) with the effect of 3OD included is converted to Schrödinger–Hirota equation (SHE) by the aid of Lie transform. When solitons propagate through an optical fiber, these pulses tend to split with bend, twist and other forms of rough handling of such fibers. This lead to splitting of an optical pulse that gives way to differential group delay (DGD). The cumulative effect of DGD, for long-haul fibers, leads to birefringence. This paper addresses SHE in birefringent fibers by the aid of extended trial equation method. Thus, bright and singular soliton solutions are retrieved. The details are now presented in the subsequent sections after a quick intro to SHE.

2. Governing model

The dimensionless form of vector-coupled Schrödinger-Hirota equation (SHE) is given by [1,3,5]

$$iq_t + i\alpha_1 q_x + a_1 q_{xx} + b_1 q_{xt} + \left(c_1|q|^2 + d_1|r|^2\right) q + i\gamma_1 q_{xxx} + i\left(\sigma_1|q|^2 + \xi_1|r|^2\right) q_x = 0, \tag{1}$$

$$ir_t + i\alpha_2 q_x + a_2 r_{xx} + b_2 q_{xt} + \left(c_2|r|^2 + d_2|q|^2\right) r + i\gamma_2 r_{xxx} + i\left(\sigma_2|r|^2 + \xi_2|q|^2\right) r_x = 0. \tag{2}$$

In (1) and (2), $q(x, t)$ and $r(x, t)$ represent the wave profile of the two pulses, α_j represents the inter-modal dispersion, a_j represents group velocity dispersion GVD, b_j gives the spatio-temporal dispersion, c_j is from self-phase modulation while d_j is due to the cross-phase modulation, then γ_j is the third order dispersion coefficient (3OD) while σ_j and ξ_j are from the nonlinear dispersions [4,12,13]. Here, $j = 1, 2$. Also, the first term in both of these equations represents the temporal evolution of the pulses in birefringent fibers. It needs to be noted that the inclusion of spatio-temporal dispersion was first proposed during 2012 to keep the model well-posed [7,8].

To proceed with the integration of the model (1) and (2), the following soliton solution structure is hypothesized:

$$q(x, t) = P_1(x, t)\exp[i\phi_1(x, t)], \tag{3}$$

$$r(x, t) = P_2(x, t)\exp[i\phi_2(x, t)], \tag{4}$$

where

$$\phi_j(x, t) = -\kappa_j x + \omega_j t + \theta_j, \tag{5}$$

for $j = 1, 2$. Here in (3) and (4), $P_j(x, t)$ represents the solitary wave profile and $\phi_j(x, t)$ is the phase component of the solitons. Also from (5), κ_j is the soliton frequency for the two components, ω_j is the wave number of the two components and finally θ_j represents the centers of phase. Substituting (3)–(5) into (1) and (2) leads to

$$\left\{\omega_j - \kappa_j(\alpha_j + b_j\omega_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\right\} P_j - (c_j + \sigma_j\kappa_j)P_j^3 - (d_j + \kappa_j\xi_j)P_j P_{\bar{j}}^2 - b_j\frac{\partial^2 P_j}{\partial x\partial t} - (a_j + 3\gamma_j\kappa_j)\frac{\partial^2 P_j}{\partial x^2} = 0, \tag{6}$$

$$(1 - b_j\kappa_j)\frac{\partial P_j}{\partial t} - \left(2a_j\kappa_j - \alpha_j - b_j\omega_j + 3\gamma_j\kappa_j^2\right)\frac{\partial P_j}{\partial x} + \sigma_j P_j^2\frac{\partial P_j}{\partial x} + \xi_j P_{\bar{j}}^2\frac{\partial P_j}{\partial x} + \gamma_j\frac{\partial^3 P_j}{\partial x^3} = 0, \tag{7}$$

for real and imaginary part, respectively. Here, the notation $\bar{j} = 3 - j$ was introduced. Under the travelling wave transformations

$$P_j(x, t) = U_j(\eta), \quad \eta = B(x - vt), \tag{8}$$

we have

$$\left\{\omega_j - \kappa_j(\alpha_j + b_j\omega_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\right\} U_j - (c_j + \sigma_j\kappa_j)U_j^3 - (d_j + \kappa_j\xi_j)U_j U_{\bar{j}}^2 + (b_j v - a_j - 3\gamma_j\kappa_j)B^2\frac{d^2U_j}{d\eta^2} = 0, \tag{9}$$

$$-\left\{v(1 - b_j\kappa_j) + 2a_j\kappa_j - \alpha_j - b_j\omega_j + 3\gamma_j\kappa_j^2\right\}\frac{dU_j}{d\eta} + \sigma_j U_j^2\frac{dU_j}{d\eta} + \xi_j U_{\bar{j}}^2\frac{dU_j}{d\eta} + \gamma_j B^2\frac{d^3U_j}{d\eta^3} = 0. \tag{10}$$

Using the balancing principle yields

$$U_{\bar{j}} = U_j, \tag{11}$$

and integrating Eq. (10) with zero constant of integration, then Eqs. (9) and (10) reduce to

$$\left\{\omega_j - \kappa_j(\alpha_j + b_j\omega_j) + a_j\kappa_j^2 + \gamma_j\kappa_j^3\right\} U_j - (c_j + d_j + \sigma_j\kappa_j + \xi_j\kappa_j)U_j^3 + (b_j v - a_j - 3\gamma_j\kappa_j)B^2\frac{d^2U_j}{d\eta^2} = 0, \tag{12}$$

$$-3\left\{v(1 - b_j\kappa_j) + 2a_j\kappa_j - \alpha_j - b_j\omega_j + 3\gamma_j\kappa_j^2\right\} U_j + \left(\sigma_j + \xi_j\right)U_j^3 + 3\gamma_j B^2\frac{d^2U_j}{d\eta^2} = 0. \tag{13}$$

This pair of relations (12) and (13) will now be analyzed for soliton and other solutions by extended trial equation method [6,10,11,15] in the next section.

3. Extended trial equation method

In order to start with the extraction of solutions to (12) and (13), the following assumption for the soliton structure is made:

$$U_j = \sum_{i=0}^{\varsigma} \tau_{ji} \Psi^i, \tag{14}$$

where

$$(\Psi')^2 = \Gamma(\Psi) = \frac{\Phi(\Psi)}{\Upsilon(\Psi)} = \frac{\mu_\epsilon \Psi^\epsilon + \cdots + \mu_1 \Psi + \mu_0}{\chi_\rho \Psi^\rho + \cdots + \chi_1 \Psi + \chi_0}. \tag{15}$$

Employing the relations (14) and (15), the terms $U_j^{''}$ can be derived as

$$U_j^{''} = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)} \left(\sum_{i=0}^{\varsigma} i\tau_{ji} \Psi^{i-1} \right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)} \left(\sum_{i=0}^{\varsigma} i(i-1)\tau_{ji} \Psi^{i-2} \right), \tag{16}$$

where $\Phi(\Psi)$ and $\Upsilon(\Psi)$ are polynomials of Ψ. Eq. (15) can be reduced to the integral form as

$$\pm(\eta - \eta_0) = \int \frac{d\Psi}{\sqrt{\Gamma(\Psi)}} = \int \sqrt{\frac{\Upsilon(\Psi)}{\Phi(\Psi)}} d\Psi. \tag{17}$$

Balancing $U_j^{''}$ with U_j^3 in Eqs. (12) and (13) leads to

$$\epsilon - \rho - 2\varsigma - 2 = 0. \tag{18}$$

When $\epsilon = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (18), then it is obtained that

$$U_j = \tau_{j0} + \tau_{j1} \Psi, \tag{19}$$

$$U_j^{''} = \frac{\tau_{j1}(4\mu_4 \Psi^3 + 3\mu_3 \Psi^2 + 2\mu_2 \Psi + \mu_1)}{2\chi_0}, \tag{20}$$

where $\mu_4 \neq 0$ and $\chi_0 \neq 0$. Substituting (19) and (20) into (12) and (13), collecting the coefficients of Ψ and solving the resulting system, the sets are derived as

$$\begin{aligned}
&\mu_0 = \mu_0, \quad \mu_2 = \mu_2, \quad \chi_0 = \chi_0, \quad \tau_{j0} = \tau_{j0}, \quad \tau_{j1} = \tau_{j1}, \\
&\mu_1 = \frac{2\tau_{j0}}{3\tau_{j1}} \left(3\mu_2 + \frac{2\chi_0 \tau_{j0}^2 (\xi_j + \sigma_j)}{B^2 \gamma_j} \right), \\
&\mu_3 = -\frac{2\chi_0 \tau_{j0} \tau_{j1} (\xi_j + \sigma_j)}{3B^2 \gamma_j}, \\
&\mu_4 = -\frac{\chi_0 \tau_{j1}^2 (\xi_j + \sigma_j)}{6B^2 \gamma_j}, \\
&\nu = \frac{a_j(\xi_j + \sigma_j) - 3\gamma_j(c_j + d_j)}{b_j(\xi_j + \sigma_j)}, \\
&\omega_j = \frac{\kappa_j \Theta_j + b_j \Xi_j (3c_j + 3d_j + 2\kappa_j(\xi_j + \sigma_j))}{b_j \chi_0 (\xi_j + \sigma_j)}, \\
&\alpha_j = \frac{\Theta_j - b_j \left[\chi_0 \tau_{j0}^2 (\xi_j + \sigma_j)^2 + \gamma_j \Pi_j + b_j \Xi_j (3c_j + 3d_j + 2\kappa_j(\xi_j + \sigma_j)) \right]}{b_j \chi_0 (\xi_j + \sigma_j)},
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
&\Xi_j = \chi_0 \tau_{j0}^2 (\xi_j + \sigma_j) + \gamma_j \left(B^2 \mu_2 + \chi_0 \kappa_j^2 \right), \\
&\Theta_j = \chi_0 \left[a_j(\xi_j + \sigma_j) - 3\gamma_j(c_j + d_j) \right], \\
&\Pi_j = (\xi_j + \sigma_j) \left(B^2 \mu_2 - 3\chi_0 \kappa_j^2 \right) - 6\chi_0 \kappa_j (c_j + d_j).
\end{aligned} \tag{22}$$

Substituting the solution set (21) into Eqs. (15) and (17) leads to

$$\pm(\eta-\eta_0)=\Omega\int\frac{d\Psi}{\sqrt{\Gamma(\Psi)}}, \tag{23}$$

where

$$\Gamma(\Psi)=\Psi^4+\frac{\mu_3}{\mu_4}\Psi^3+\frac{\mu_2}{\mu_4}\Psi^2+\frac{\mu_1}{\mu_4}\Psi+\frac{\mu_0}{\mu_4},\quad \Omega=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{24}$$

In view of these results, exact solutions for the vector-coupled SHE are secured in the forms:

For $\Gamma(\Psi)=(\Psi-\delta_1)^4$,

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_1\pm\frac{\tau_{11}\Omega}{B\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)-\eta_0}\right\}\times\exp\left[i\left\{-\kappa_1x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{25}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_1\pm\frac{\tau_{21}\Omega}{B\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)-\eta_0}\right\}\times\exp\left[i\left\{-\kappa_2x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right]. \tag{26}$$

If $\Gamma(\Psi)=(\Psi-\delta_1)^3(\Psi-\delta_2)$ and $\delta_2>\delta_1$,

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_1+\frac{4\tau_{11}\Omega^2(\delta_2-\delta_1)}{4\Omega^2-\left[(\delta_1-\delta_2)\left(B\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)-\eta_0\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa_1x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{27}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_1+\frac{4\tau_{21}\Omega^2(\delta_2-\delta_1)}{4\Omega^2-\left[(\delta_1-\delta_2)\left(B\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)-\eta_0\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa_2x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2\left(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2)\right)}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right]. \tag{28}$$

However, when $\Gamma(\Psi)=(\Psi-\delta_1)^2(\Psi-\delta_2)^2$,

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_2+\frac{\tau_{11}(\delta_2-\delta_1)}{\exp\left[\frac{\delta_1-\delta_2}{\Omega}\left(B\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)-\eta_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_1x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{29}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_2+\frac{\tau_{21}(\delta_2-\delta_1)}{\exp\left[\frac{\delta_1-\delta_2}{\Omega}\left(B\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)-\eta_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_2x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{30}$$

and

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_1+\frac{\tau_{11}(\delta_1-\delta_2)}{\exp\left[\frac{\delta_1-\delta_2}{\Omega}\left(B\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)-\eta_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_1 x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{31}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_1+\frac{\tau_{21}(\delta_1-\delta_2)}{\exp\left[\frac{\delta_1-\delta_2}{\Omega}\left(B\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)-\eta_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right]. \tag{32}$$

Whenever $\Gamma(\Psi)=(\Psi-\delta_1)^2(\Psi-\delta_2)(\Psi-\delta_3)$ and $\delta_1>\delta_2>\delta_3$,

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_1-\frac{2\tau_{11}(\delta_1-\delta_2)(\delta_1-\delta_3)}{2\delta_1-\delta_2-\delta_3+(\delta_3-\delta_2)\cosh\left[Q\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_1 x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{33}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_1-\frac{2\tau_{21}(\delta_1-\delta_2)(\delta_1-\delta_3)}{2\delta_1-\delta_2-\delta_3+(\delta_3-\delta_2)\cosh\left[Q\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{34}$$

where

$$Q=\frac{B\sqrt{(\delta_1-\delta_2)(\delta_1-\delta_3)}}{\Omega}. \tag{35}$$

Finally, if $\Gamma(\Psi)=(\Psi-\delta_1)(\Psi-\delta_2)(\Psi-\delta_3)(\Psi-\delta_4)$ and $\delta_1>\delta_2>\delta_3>\delta_4$,

$$q(x,t)=\left\{\tau_{10}+\tau_{11}\delta_2+\frac{\tau_{11}(\delta_1-\delta_2)(\delta_4-\delta_2)}{\delta_4-\delta_2+(\delta_1-\delta_4)\,\mathrm{sn}^2\left[\frac{Q_j}{B}\left(B\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)-\eta_0\right),m\right]}\right\}\times\exp\left[i\left\{-\kappa_1 x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{36}$$

$$r(x,t)=\left\{\tau_{20}+\tau_{21}\delta_2+\frac{\tau_{21}(\delta_1-\delta_2)(\delta_4-\delta_2)}{\delta_4-\delta_2+(\delta_1-\delta_4)\,\mathrm{sn}^2\left[\frac{Q_j}{B}\left(B\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)-\eta_0\right),m\right]}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{37}$$

where modulus m and Q_j are given by

$$m^2=\frac{(\delta_2-\delta_3)(\delta_1-\delta_4)}{(\delta_1-\delta_3)(\delta_2-\delta_4)}, \tag{38}$$

and

$$Q_j=\frac{(-1)^j B\sqrt{(\delta_1-\delta_3)(\delta_2-\delta_4)}}{2\Omega}\quad\text{for}\quad j=1,2. \tag{39}$$

It needs to mention that δ_j for $j = 1, \ldots, 4$ are the roots of the equation

$$\Gamma(\Psi) = 0. \tag{40}$$

Under the conditions $\tau_{10} = -\tau_{11}\delta_1$, $\tau_{20} = -\tau_{21}\delta_1$ and $\eta_0 = 0$, the solutions (25)–(34) can be reduced to plane wave solutions

$$q(x,t) = \left\{ \pm \frac{\tau_{11}\Omega}{B\left(x - \left\{\frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)} \right\} \times \exp\left[i\left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{41}$$

$$r(x,t) = \left\{ \pm \frac{\tau_{21}\Omega}{B\left(x - \left\{\frac{a_2(\xi_2+\sigma_2) - 3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)} \right\} \times \exp\left[i\left\{ -\kappa_2 x + \left(\frac{\kappa_2\Theta_2 + b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)} \right) t + \theta_2 \right\} \right], \tag{42}$$

$$q(x,t) = \left\{ \frac{4\tau_{11}\Omega^2(\delta_2-\delta_1)}{4\Omega^2 - \left[B(\delta_1-\delta_2)\left(x - \left\{\frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)\right]^2} \right\} \times \exp\left[i\left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{43}$$

$$r(x,t) = \left\{ \frac{4\tau_{21}\Omega^2(\delta_2-\delta_1)}{4\Omega^2 - \left[B(\delta_1-\delta_2)\left(x - \left\{\frac{a_2(\xi_2+\sigma_2) - 3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)\right]^2} \right\} \times \exp\left[i\left\{ -\kappa_2 x + \left(\frac{\kappa_2\Theta_2 + b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)} \right) t + \theta_2 \right\} \right], \tag{44}$$

singular soliton solutions

$$q(x,t) = \left\{ \frac{\tau_{11}(\delta_2-\delta_1)}{2}\left(1 \mp \coth\left[\frac{B(\delta_1-\delta_2)}{2\Omega}\left(x - \left\{\frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)\right]\right)\right\} \times \exp\left[i\left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{45}$$

$$r(x,t) = \left\{ \frac{\tau_{21}(\delta_2-\delta_1)}{2}\left(1 \mp \coth\left[\frac{B(\delta_1-\delta_2)}{2\Omega}\left(x - \left\{\frac{a_2(\xi_2+\sigma_2) - 3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)\right]\right)\right\} \times \exp\left[i\left\{ -\kappa_2 x + \left(\frac{\kappa_2\Theta_2 + b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)} \right) t + \theta_2 \right\} \right], \tag{46}$$

and bright soliton solutions

$$q(x,t) = \left\{ \frac{P}{R + \cosh\left[Q\left(x - \left\{\frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)\right]} \right\} \times \exp\left[i\left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{47}$$

$$r(x,t)=\left\{\frac{\bar{P}}{R+\cosh\left[Q\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{48}$$

where

$$P=\frac{2\tau_{11}(\delta_1-\delta_2)(\delta_1-\delta_3)}{\delta_3-\delta_2}, \tag{49}$$

$$\bar{P}=\frac{2\tau_{21}(\delta_1-\delta_2)(\delta_1-\delta_3)}{\delta_3-\delta_2}, \tag{50}$$

$$R=\frac{2\delta_1-\delta_2-\delta_3}{\delta_3-\delta_2}. \tag{51}$$

It is important to emphasize that the amplitudes of the solitons are given by (49) and (50), while the inverse width of the solitons is given by (35). These solitons are valid for $\tau_{11}<0$ and $\tau_{21}<0$. Moreover, under the conditions $\tau_{10}=-\tau_{11}\delta_2$, $\tau_{20}=-\tau_{21}\delta_2$ and $\eta_0=0$, Jacobi elliptic function solutions (36) and (37) are reduced to

$$q(x,t)=\left\{\frac{P_1}{R_1+\ \mathrm{sn}^2\left[Q_j\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right),\frac{(\delta_2-\delta_3)(\delta_1-\delta_4)}{(\delta_1-\delta_3)(\delta_2-\delta_4)}\right]}\right\}\times\exp\left[i\left\{-\kappa_1 x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{52}$$

$$r(x,t)=\left\{\frac{\bar{P}_1}{R_1+\ \mathrm{sn}^2\left[Q_j\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right),\frac{(\delta_2-\delta_3)(\delta_1-\delta_4)}{(\delta_1-\delta_3)(\delta_2-\delta_4)}\right]}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{53}$$

where

$$P_1=\frac{\tau_{11}(\delta_1-\delta_2)(\delta_4-\delta_2)}{\delta_1-\delta_4}, \tag{54}$$

$$\bar{P}_1=\frac{\tau_{21}(\delta_1-\delta_2)(\delta_4-\delta_2)}{\delta_1-\delta_4}, \tag{55}$$

$$R_1=\frac{\delta_4-\delta_2}{\delta_1-\delta_4}. \tag{56}$$

Remark 1. When the modulus $m\to 1$, singular optical soliton solutions are procured as

$$q(x,t)=\left\{\frac{P_1}{R_1+\tanh^2\left[Q_j\left(x-\left\{\frac{a_1(\xi_1+\sigma_1)-3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_1 x+\left(\frac{\kappa_1\Theta_1+b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)}\right)t+\theta_1\right\}\right], \tag{57}$$

$$r(x,t)=\left\{\frac{\bar{P}_1}{R_1+\tanh^2\left[Q_j\left(x-\left\{\frac{a_2(\xi_2+\sigma_2)-3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_2 x+\left(\frac{\kappa_2\Theta_2+b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)}\right)t+\theta_2\right\}\right], \tag{58}$$

$$q(x,t) = \left\{ \frac{P_1}{R_1 + \tanh^2\left[Q_j \left(x - \left\{ \frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)} \right\} t \right) \right]} \right\} \times \exp\left[i \left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{57}$$

$$r(x,t) = \left\{ \frac{\bar{P}_1}{R_1 + \tanh^2\left[Q_j \left(x - \left\{ \frac{a_2(\xi_2+\sigma_2) - 3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)} \right\} t \right) \right]} \right\} \times \exp\left[i \left\{ -\kappa_2 x + \left(\frac{\kappa_2\Theta_2 + b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)} \right) t + \theta_2 \right\} \right], \tag{58}$$

for $\delta_3 = \delta_4$.

Remark 2. However, if $m \to 0$, periodic singular solutions are obtained as

$$q(x,t) = \left\{ \frac{P_1}{R_1 + \sin^2\left[Q_j \left(x - \left\{ \frac{a_1(\xi_1+\sigma_1) - 3\gamma_1(c_1+d_1)}{b_1(\xi_1+\sigma_1)} \right\} t \right) \right]} \right\} \times \exp\left[i \left\{ -\kappa_1 x + \left(\frac{\kappa_1\Theta_1 + b_1\Xi_1(3c_1+3d_1+2\kappa_1(\xi_1+\sigma_1))}{b_1\chi_0(\xi_1+\sigma_1)} \right) t + \theta_1 \right\} \right], \tag{59}$$

$$r(x,t) = \left\{ \frac{\bar{P}_1}{R_1 + \sin^2\left[Q_j \left(x - \left\{ \frac{a_2(\xi_2+\sigma_2) - 3\gamma_2(c_2+d_2)}{b_2(\xi_2+\sigma_2)} \right\} t \right) \right]} \right\} \times \exp\left[i \left\{ -\kappa_2 x + \left(\frac{\kappa_2\Theta_2 + b_2\Xi_2(3c_2+3d_2+2\kappa_2(\xi_2+\sigma_2))}{b_2\chi_0(\xi_2+\sigma_2)} \right) t + \theta_2 \right\} \right], \tag{60}$$

for $\delta_2 = \delta_3$.

4. Conclusions

Bright and singular soliton solutions are retrieved in this paper for SHE that models dispersive soliton dynamics in birefringent fibers. The adopted integration algorithm is the extended trial equation scheme. This powerful integration scheme is very effective when other algorithms fail to secure soliton solutions. Therefore, this methodology will be later applied to various other scenarios and situations in mathematical photonics. A few such examples are Bragg gratings, DWDM systems, dispersion-managed solitons and several others. The results of those research activities are currently awaited and will be reported in future.

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