Optik 156 (2018) 999–1006

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Chirped optical solitons of Chen–Lee–Liu equation by extended trial equation scheme

Anjan Biswas[a,b,c], Mehmet Ekici[d], Abdullah Sonmezoglu[d], Ali Saleh Alshomrani[b], Qin Zhou[e,*], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

Article history:
Received 12 November 2017
Accepted 19 December 2017

Keywords:
Solitons
Chirp
Chen–Lee–Liu equation

ABSTRACT

This paper employs the extended trial equation method to retrieve chirped optical soliton solutions to Chen–Lee–Liu model that describes pulse propagation through optical fibers. Bright and singular optical soliton solutions are recovered by the application of this algorithm. As a byproduct, singular periodic solutions fall out. These soliton solutions are obtained with constraint conditions that are also presented.

1. Introduction

Optical solitons is the basic fabric for telecommunication purposes across the globe. There are several models that describe the dynamics of soliton propagation through optical fibers, metamaterials and PCF [1–15]. Some of the most visible models are the nonlinear Schrödinger's equation, Manakov equation, Sasa–Satsuma equation, Thirring model, complex Ginzburg–Landau equation, Gerdjikov–Ivanov equation, Lakshmanan–Posezian–Daniel equation and several others. There are a number of papers visible with such models and their corresponding soliton dynamics. This paper will explore one such model that first appeared during 1979 [1]. This is the Chen–Lee–Liu equation. The extended trial equation approach is implemented in this paper to secure soliton solutions that will be of great asset in telecommunications industry. Bright and singular soliton solutions are retrieved using this approach. The details are enumerated in the subsequent sections.

2. Mathematical analysis

We consider the propagation of an optical pulse inside in a monomode fiber modeled by a family of the following Chen–Lee–Liu equation [1,10–13]:

$$iq_t + aq_{xx} + ib|q|^2 q_x = 0, \tag{1}$$

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2017.12.094

where $q = q(x, t)$ is the complex wave function, while a and b are constants. In the context of optical fiber physics, the term involving the parameter b is usually associated with the self-steepening phenomena [4], while terms involving a is group velocity dispersion. In such contexts, the coordinates t and x denote propagation distance and retarded time, but represent slow time and spatial coordinate traveling with the group velocity in hydrodynamics, respectively [7].

We are interested to find the exact nonlinearly chirped soliton solution for Eq. (1). To do so we put the unknown function of the complex field $q(x, t)$ in the traveling-wave form

$$q(x,t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

with the phase $\phi(x, t)$ as

$$\phi(x,t) = -\kappa x + \omega t + \theta(s), \tag{3}$$

where the traveling coordinate s is given by

$$s = x - vt. \tag{4}$$

Here in (2)–(4), $g(s)$ is real amplitude of the wave, while $\theta(s)$ is an extra phase function depending upon the traveling variable s. Also, κ is the frequency of the soliton, and ω is the wave number of the soliton.

The corresponding chirp is given by

$$\delta\omega = -\frac{\partial\phi}{\partial x} = \kappa - \theta'(s). \tag{5}$$

Inserting the wave solution (2) into Eq. (1) and separating the real and imaginary parts, we obtain the following pair of coupled equations in g and θ:

$$-\omega g + v\theta' g + a g'' - a g\theta'^2 - a\kappa^2 g + 2a\kappa\theta' g - b\theta' g^3 + b\kappa g^3 = 0, \tag{6}$$

and

$$a\left(g\theta'' + 2g'\theta'\right) - vg' - 2a\kappa g' + bg^2 g' = 0, \tag{7}$$

where $g' = dg/ds$, $g'' = d^2g/ds^2$, $\theta' = d\theta/ds$, and $\theta'' = d^2\theta/ds^2$.

Multiplying both sides of (7) by g and integrating gives

$$\theta' = \kappa + \frac{v}{2a} - \frac{b}{4a}g^2 + \frac{A}{2ag^2}, \tag{8}$$

where A is an integration constant, to be fixed by initial conditions.

Relations (5) and (8) show a nontrivial dependence of the chirp on the wave intensity [with $I = |q|^2 = g^2$]. It is worth remarking that this chirp has two intensity dependent terms. It also follows from Eq. (8) that the self-steepening and GVD parameters can be used to control the amplitude of chirp.

Now substituting (8) into (6) gives

$$g'' + \frac{3b^2}{16a^2}g^5 - \frac{bv}{2a^2}g^3 + \left(\frac{v^2}{4a^2} + \frac{v\kappa}{a} - \frac{\omega}{a} - \frac{bA}{4a^2}\right)g - \frac{A^2}{4a^2g^3} = 0, \tag{9}$$

Multiplying of Eq. (9) by g' and integrate with respect to s, we obtain the following auxiliary ordinary differential equation

$$g'^2 + \frac{b^2}{16a^2}g^6 - \frac{bv}{4a^2}g^4 + \left(\frac{v^2}{4a^2} + \frac{v\kappa}{a} - \frac{\omega}{a} - \frac{bA}{4a^2}\right)g^2 + \frac{A^2}{4a^2g^2} = 2B. \tag{10}$$

where B is the second integration constant.

Eq. (10) is a nonlinear differential equation describing the evolution of the wave amplitude in a nonlinear medium that is governed by the Chen–Lee–Liu equation (1). Before discussing exact solutions to Eq. (10), let us rewrite it in a more simplified form. It is of interest to introduce the following change of variable $F = g^2$. Applying this transformation to Eq. (10) yields

$$F'^2 = -c_0 + c_1F + c_2F^2 + c_3F^3 - c_4F^4, \tag{11}$$

where

$$c_0 = \frac{A^2}{a^2}, \quad c_1 = 8B, \quad c_2 = \frac{4\omega}{a} + \frac{bA}{a^2} - \frac{v^2}{a^2} - \frac{4v\kappa}{a}, \quad c_3 = \frac{bv}{a^2}, \quad c_4 = \frac{b^2}{4a^2}. \tag{12}$$

The elliptic equation (11) is known to admit a variety of solutions such as periodic, kink, and solitary-wave-type solutions [5,6]. However, for all the previously reported soliton or solitary wave solutions, they are found for auxiliary ordinary differential equations solved under the vanishing boundary conditions. Here we concentrate on discussing it with nonvanishing boundary conditions by employing the extended trial equation scheme [7–13]. It is worth observing that the coefficients c_0 and c_1 contain the integration constants A and B, thus implying that they have effects on the shape of propagating envelopes.

3. Extended trial equation scheme

To start off with the extended trial equation scheme, the following assumption for the solution structure of (11) is made:

$$F=\sum_{i=0}^{\varsigma}\tau_i\Omega^i, \tag{13}$$

where

$$(\Omega')^2=\Delta(\Omega)=\frac{\Phi(\Omega)}{\Upsilon(\Omega)}=\frac{\mu_\varrho\Omega^\varrho+\cdots+\mu_1\Omega+\mu_0}{\chi_\rho\Omega^\rho+\cdots+\chi_1\Omega+\chi_0}. \tag{14}$$

Here $\tau_0,\ldots,\tau_\varsigma$; $\mu_0,\ldots,\mu_\varrho$ and $\chi_0,\ldots,\chi_\rho$ are constants to be determined later. From (13) and (14), the term F'^2 is derived as

$$F'^2=\frac{\Phi(\Omega)}{\Upsilon(\Omega)}\left(\sum_{i=0}^{\varsigma}i\tau_i\Omega^{i-1}\right)^2, \tag{15}$$

where $\Phi(\Omega)$ and $\Upsilon(\Omega)$ are polynomials of Ω. Eq. (14) can be reduced to

$$\pm(s-s_0)=\int\frac{d\Omega}{\sqrt{\Delta(\Omega)}}=\int\sqrt{\frac{\Upsilon(\Omega)}{\Phi(\Omega)}}d\Omega. \tag{16}$$

Balancing F'^2 with F^4 in Eq. (11) gives

$$\varrho=\rho+2\varsigma+2. \tag{17}$$

When $\varrho=4$, $\rho=0$ and $\varsigma=1$ in Eq. (17), then extended trial equation scheme admits the use of the finite expansion

$$F=\tau_0+\tau_1\Omega, \tag{18}$$

where τ_0 and τ_1 are constants to be determined later such that $\tau_1\neq 0$, and Ω satisfies Eq. (14). Substituting (18) into (11), collecting the coefficients of Ω, and solving the resulting system we have the following sets of solutions

$$\begin{aligned}
&\mu_1=\mu_1,\quad \chi_0=\chi_0,\quad \tau_0=\tau_0,\quad \tau_1=\tau_1,\\
&\mu_0=\frac{2a^2\mu_1\tau_0\tau_1+\chi_0\left(16a^2\tau_0B-4A^2+b^2\tau_0^4-2bv\tau_0^3\right)}{4a^2\tau_1^2},\\
&\mu_2=\frac{a^2\mu_1\tau_1+\chi_0\left[b\tau_0^2(3v-2b\tau_0)-8a^2B\right]}{2a^2\tau_0},\\
&\mu_3=\frac{b\tau_1\chi_0(v-b\tau_0)}{a^2},\quad \mu_4=-\frac{b^2\tau_1^2\chi_0}{4a^2},\\
&\omega=\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}.
\end{aligned} \tag{19}$$

Substituting these set of solutions into Eqs. (14) and (16) gives

$$\pm(s-s_0)=\Pi\int\frac{d\Psi}{\sqrt{\Delta(\Omega)}}, \tag{20}$$

where

$$\Delta(\Omega)=\Omega^4+\frac{\mu_3}{\mu_4}\Omega^3+\frac{\mu_2}{\mu_4}\Omega^2+\frac{\mu_1}{\mu_4}\Omega+\frac{\mu_0}{\mu_4},\quad \Pi=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{21}$$

As a consequence, the following traveling wave solutions are obtained:

For $\Delta(\Omega)=(\Omega-\eta_1)^4$,

$$\begin{aligned}
q(x,t)=&\left\{\tau_0+\tau_1\eta_1\pm\frac{\tau_1\Pi}{x-vt-s_0}\right\}^{\frac{1}{2}}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right].
\end{aligned} \tag{22}$$

If $\Delta(\Omega)=(\Omega-\eta_1)^3(\Omega-\eta_2)$ and $\eta_2>\eta_1$,

$$q(x,t)=\left\{\tau_0+\tau_1\eta_1+\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt-s_0)]^2}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right]. \tag{23}$$

However, when $\Delta(\Omega)=(\Omega-\eta_1)^2(\Omega-\eta_2)^2$,

$$q(x,t)=\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_2-\eta_1)}{\exp\left[\frac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right], \tag{24}$$

and

$$q(x,t)=\left\{\tau_0+\tau_1\eta_1+\frac{\tau_1(\eta_1-\eta_2)}{\exp\left[\frac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right]. \tag{25}$$

Whenever $\Delta(\Omega)=(\Omega-\eta_1)^2(\Omega-\eta_2)(\Omega-\eta_3)$ and $\eta_1>\eta_2>\eta_3$,

$$q(x,t)=\left\{\tau_0+\tau_1\eta_1-\frac{2\tau_1(\eta_1-\eta_2)(\eta_1-\eta_3)}{2\eta_1-\eta_2-\eta_3+(\eta_3-\eta_2)\cosh\left[\frac{\sqrt{(\eta_1-\eta_2)(\eta_1-\eta_3)}}{\Pi}(x-vt)\right]}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right]. \tag{26}$$

Finally, if $\Delta(\Omega)=(\Omega-\eta_1)(\Omega-\eta_2)(\Omega-\eta_3)(\Omega-\eta_4)$ and $\eta_1>\eta_2>\eta_3>\eta_4$,

$$q(x,t)=\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_1-\eta_2)(\eta_4-\eta_2)}{\eta_4-\eta_2+(\eta_1-\eta_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\eta_1-\eta_3)(\eta_2-\eta_4)}}{2\Pi}(x-vt-s_0),m\right]}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right], \tag{27}$$

where

$$m^2=\frac{(\eta_2-\eta_3)(\eta_1-\eta_4)}{(\eta_1-\eta_3)(\eta_2-\eta_4)}. \tag{28}$$

The chirp corresponding to Eqs. (22)–(27) can be obtained readily as

$$\delta\omega(x,t)=-\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_1\pm\frac{\tau_1\Pi}{x-vt-s_0}\right\}-\frac{A}{2a}\left\{\tau_0+\tau_1\eta_1\pm\frac{\tau_1\Pi}{x-vt-s_0}\right\}^{-1}, \tag{29}$$

$$\begin{aligned}\delta\omega(x,t)= & -\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_1+\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt-s_0)]^2}\right\}\\ & -\frac{A}{2a}\left\{\tau_0+\tau_1\eta_1+\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt-s_0)]^2}\right\}^{-1},\end{aligned} \tag{30}$$

$$\begin{aligned}\delta\omega(x,t)= & -\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_2-\eta_1)}{\exp\left[\dfrac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}\\ & -\frac{A}{2a}\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_2-\eta_1)}{\exp\left[\dfrac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}^{-1},\end{aligned} \tag{31}$$

$$\begin{aligned}\delta\omega(x,t)= & -\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_1+\frac{\tau_1(\eta_1-\eta_2)}{\exp\left[\dfrac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}\\ & -\frac{A}{2a}\left\{\tau_0+\tau_1\eta_1+\frac{\tau_1(\eta_1-\eta_2)}{\exp\left[\dfrac{\eta_1-\eta_2}{\Pi}(x-vt-s_0)\right]-1}\right\}^{-1},\end{aligned} \tag{32}$$

$$\begin{aligned}\delta\omega(x,t)= & -\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_1-\frac{2\tau_1(\eta_1-\eta_2)(\eta_1-\eta_3)}{2\eta_1-\eta_2-\eta_3+(\eta_3-\eta_2)\cosh\left[\dfrac{\sqrt{(\eta_1-\eta_2)(\eta_1-\eta_3)}}{\Pi}(x-vt)\right]}\right\}\\ & -\frac{A}{2a}\left\{\tau_0+\tau_1\eta_1-\frac{2\tau_1(\eta_1-\eta_2)(\eta_1-\eta_3)}{2\eta_1-\eta_2-\eta_3+(\eta_3-\eta_2)\cosh\left[\dfrac{\sqrt{(\eta_1-\eta_2)(\eta_1-\eta_3)}}{\Pi}(x-vt)\right]}\right\}^{-1},\end{aligned} \tag{33}$$

$$\begin{aligned}\delta\omega(x,t)= & -\frac{v}{2a}+\frac{b}{4a}\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_1-\eta_2)(\eta_4-\eta_2)}{\eta_4-\eta_2+(\eta_1-\eta_4)\,\mathrm{sn}^2\left[\pm\dfrac{\sqrt{(\eta_1-\eta_3)(\eta_2-\eta_4)}}{2\Pi}(x-vt-s_0),m\right]}\right\}\\ & -\frac{A}{2a}\left\{\tau_0+\tau_1\eta_2+\frac{\tau_1(\eta_1-\eta_2)(\eta_4-\eta_2)}{\eta_4-\eta_2+(\eta_1-\eta_4)\,\mathrm{sn}^2\left[\pm\dfrac{\sqrt{(\eta_1-\eta_3)(\eta_2-\eta_4)}}{2\Pi}(x-vt-s_0),m\right]}\right\}^{-1}.\end{aligned} \tag{34}$$

It is important to note that η_j for $j=1,\ldots,4$ are the roots of

$$\Delta(\Omega)=0. \tag{35}$$

For $\tau_0=-\tau_1\eta_1$ and $s_0=0$, the solutions (22)–(26) can be reduced to plane wave solutions

$$\begin{aligned}q(x,t)= & \left\{\pm\frac{\tau_1\Pi}{x-vt}\right\}^{\frac{1}{2}}\\ & \times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],\end{aligned} \tag{36}$$

$$
\begin{aligned}
q(x,t)= &\left\{\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt)]^2}\right\}^{\frac{1}{2}}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0\left(b\tau_0-3v\right)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{37}
$$

singular soliton solutions

$$
\begin{aligned}
q(x,t)= &\left\{\frac{\tau_1(\eta_2-\eta_1)}{2}\left(1\mp\coth\left[\frac{\eta_1-\eta_2}{2\Pi}(x-vt)\right]\right)\right\}^{\frac{1}{2}}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0\left(b\tau_0-3v\right)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{38}
$$

and bright soliton solution

$$
\begin{aligned}
q(x,t)= &\left\{\frac{M}{(R+\cosh\left[N(x-vt)\right])^{\frac{1}{2}}}\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0\left(b\tau_0-3v\right)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
M&=\left(\frac{2\tau_1(\eta_1-\eta_2)(\eta_1-\eta_3)}{\eta_3-\eta_2}\right)^{\frac{1}{2}},\\
N&=\frac{\sqrt{(\eta_1-\eta_2)(\eta_1-\eta_3)}}{\Pi},\\
R&=\frac{2\eta_1-\eta_2-\eta_3}{\eta_3-\eta_2}.
\end{aligned}
\tag{40}
$$

Here M is the amplitude of the soliton, while N is the inverse width of the soliton. These solitons exist for $\tau_1<0$. The chirp corresponding to Eqs. (36)–(39) can be derived as

$$
\delta\omega(x,t)=-\frac{v}{2a}+\frac{b}{4a}\left\{\pm\frac{\tau_1\Pi}{x-vt}\right\}-\frac{A}{2a}\left\{\pm\frac{\tau_1\Pi}{x-vt}\right\}^{-1},
\tag{41}
$$

$$
\delta\omega(x,t)=-\frac{v}{2a}+\frac{b}{4a}\left\{\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt)]^2}\right\}-\frac{A}{2a}\left\{\frac{4\tau_1\Pi^2(\eta_2-\eta_1)}{4\Pi^2-[(\eta_1-\eta_2)(x-vt)]^2}\right\}^{-1},
\tag{42}
$$

$$
\begin{aligned}
\delta\omega(x,t)= &-\frac{v}{2a}+\frac{b}{4a}\left\{\frac{\tau_1(\eta_2-\eta_1)}{2}\left(1\mp\coth\left[\frac{\eta_1-\eta_2}{2\Pi}(x-vt)\right]\right)\right\}\\
&-\frac{A}{2a}\left\{\frac{\tau_1(\eta_2-\eta_1)}{2}\left(1\mp\coth\left[\frac{\eta_1-\eta_2}{2\Pi}(x-vt)\right]\right)\right\}^{-1},
\end{aligned}
\tag{43}
$$

$$
\delta\omega(x,t)=-\frac{v}{2a}+\frac{b}{4a}\left\{\frac{M^2}{R+\cosh\left[N(x-vt)\right]}\right\}-\frac{A}{2a}\left\{\frac{M^2}{R+\cosh\left[N(x-vt)\right]}\right\}^{-1}.
\tag{44}
$$

On the other hand, if $\tau_0=-\tau_1\eta_2$ and $s_0=0$, Jacobi elliptic function solutions (27) can be transformed to

$$
\begin{aligned}
q(x,t)= &\left\{\frac{M_1}{\left(R_1+\operatorname{sn}^2\left[N_j(x-vt),\frac{(\eta_2-\eta_3)(\eta_1-\eta_4)}{(\eta_1-\eta_3)(\eta_2-\eta_4)}\right]\right)^{\frac{1}{2}}}\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0\left(b\tau_0-3v\right)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{45}
$$

where

$$
\begin{aligned}
M_1 &= \left(\frac{\tau_1(\eta_1-\eta_2)(\eta_4-\eta_2)}{\eta_1-\eta_4}\right)^{\frac{1}{2}},\\
R_1 &= \frac{\eta_4-\eta_2}{\eta_1-\eta_4},\\
N_j &= \frac{(-1)^j\sqrt{(\eta_1-\eta_3)(\eta_2-\eta_4)}}{2\Pi} \quad \text{for} \quad j=1,2.
\end{aligned}
\tag{46}
$$

The corresponding chirp is given by

$$
\begin{aligned}
\delta\omega(x,t) = &-\frac{v}{2a}+\frac{b}{4a}\left\{\frac{M_1^2}{R_1+\operatorname{sn}^2\left[N_j(x-vt),\frac{(\eta_2-\eta_3)(\eta_1-\eta_4)}{(\eta_1-\eta_3)(\eta_2-\eta_4)}\right]}\right\}\\
&-\frac{A}{2a}\left\{\frac{M_1^2}{R_1+\operatorname{sn}^2\left[N_j(x-vt),\frac{(\eta_2-\eta_3)(\eta_1-\eta_4)}{(\eta_1-\eta_3)(\eta_2-\eta_4)}\right]}\right\}^{-1}.
\end{aligned}
\tag{47}
$$

Remark 1. When the modulus $m\to 1$, singular optical soliton solutions are

$$
\begin{aligned}
q(x,t) = &\left\{\frac{M_1}{\left(R_1+\tanh^2\left[N_j(x-vt)\right]\right)^{\frac{1}{2}}}\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{48}
$$

where $\eta_3=\eta_4$.

$$
\delta\omega(x,t) = -\frac{v}{2a}+\frac{b}{4a}\left\{\frac{M_1^2}{R_1+\tanh^2\left[N_j(x-vt)\right]}\right\}-\frac{A}{2a}\left\{\frac{M_1^2}{R_1+\tanh^2\left[N_j(x-vt)\right]}\right\}^{-1}. \tag{49}
$$

Remark 2. However, if $m\to 0$, periodic singular solutions are

$$
\begin{aligned}
q(x,t) = &\left\{\frac{M_1}{\left(R_1+\sin^2\left[N_j(x-vt)\right]\right)^{\frac{1}{2}}}\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{a^2\mu_1\tau_1+\chi_0\left(\tau_0\left[2(v(v+4a\kappa)-bA)+b\tau_0(b\tau_0-3v)\right]-8a^2B\right)}{8a\tau_0\chi_0}\right)t+\theta(s)\right\}\right],
\end{aligned}
\tag{50}
$$

where $\eta_2=\eta_3$.

The chirp is given as

$$
\delta\omega(x,t) = -\frac{v}{2a}+\frac{b}{4a}\left\{\frac{M_1^2}{R_1+\sin^2\left[N_j(x-vt)\right]}\right\}-\frac{A}{2a}\left\{\frac{M_1^2}{R_1+\sin^2\left[N_j(x-vt)\right]}\right\}^{-1}. \tag{51}
$$

4. Conclusions

This paper obtained bright and singular optical soliton solutions of Chen–Lee–Liu equation by the application of extended trial equation method. This algorithm also gave way to singular periodic solutions to the model as a byproduct. The results appear with their respective existence criteria that are presented as constraint conditions. This paper paves way for additional results in this field. The model will be further studied with the inclusion of several Hamiltonian and non-local perturbation terms that will give a broader picture to the soliton dynamics in telecommunication systems. The results of those research will be soon reported. Later, the vector form of the model will be studied as an application to fiber optical sciences. This is just a tip of the iceberg.

Conflict of interest

The authors also declare that there is no conflict of interest.

Acknowledgements

The work of the fifth author (QZ) was supported by the National Science Foundation for Young Scientists of Wuhan Donghu University under grant number 2017dhzk001. The sixth author (SPM) would like to thank the research support provided by the Department of Mathematics and Statistics at Tshwane University of Technology and the support from the South African National Foundation under Grant Number 92052 IRF1202210126. The research work of seventh author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP 8-028-1-001.

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