Highly dispersive optical soliton perturbation with Kudryashov's sextic-power law nonlinear refractive index by semi-inverse variation.

Results in Physics 27 (2021) 104539

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Highly dispersive optical soliton perturbation with Kudryashov's sextic-power law nonlinear refractive index by semi-inverse variation

Anjan Biswas [a,b,c,d], Mehmet Ekici [e,*], Anelia Dakova [f,g], Salam Khan [d], Seithuti P. Moshokoa [h], Hashim M. Alshehri [b], Milivoj R. Belic [i]

[a] Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Hwy, Moscow 115409, Russian Federation
[b] Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa 0204, Pretoria, South Africa
[d] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762–4900, USA
[e] Department of Mathematics, Faculty of Science and Arts, Yozgat Bozok University, 66100 Yozgat, Turkey
[f] Physics and Technology Faculty, University of Plovdiv "Paisii Hilendarski", 24 Tsar Asen Street, 4000 Plovdiv, Bulgaria
[g] Institute of Electronics, Bulgarian Academy of Sciences, 72 Tzarigradcko Shossee, 1784 Sofia, Bulgaria
[h] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[i] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Kudryashov
Solitons
Semi-inverse
Cardano
Perturbation

ABSTRACT

A single bright highly dispersive optical soliton solution is recovered from semi-inverse variational principle. The model contains six power law nonlinear terms that constitute self-phase modulation effect which was proposed by Kudryashov. The amplitude–width relation of the soliton was finally recovered by Cardano's approach with appropriate and necessary constraints that are presented.

Introduction

A wide variety of features are studied in the context of optical solitons that travel down an optical fiber for inter-continental distances [1–20]. One of the most innovative concepts that has been conceived in telecommunications engineering is "highly dispersive" (HD) and "cubic–quartic" optical solitons. This was first reported in 2020 and 2021 respectively for preposterous consequences with low count of chromatic dispersion [3,12]. It was first studied for polarization-preserving fibers and later extended to birefringent fibers. For the scalar models, semi-inverse variational principle (SVP) was implemented to recover soliton solutions to a couple of nonlinear forms of refractive index. These are polynomial law and quadratic–cubic law. Today's paper moves a step further where HD solitons are studied with the newly proposed form of self-phase modulation (SPM), attributed to Nikolay Kudryashov. This form of SPM contains six nonlinear terms and is being referred to as sextic-power law nonlinearity [19]. The model is studied with a few perturbation terms of Hamiltonian type, which show up maximum intensity.

Stepping back in time, it is worthwhile to mention that SVP was first proposed a couple of decades ago by J. H. He and all along this principle has gained its popularity and it has sustained ever since. This principle of SVP has been successfully applied to various other problems in optics, apart from Kudryashov's law of refractive index. Typically, it was studied in dispersive optical solitons, optical metamaterials, non-Kerr law solitons [11] including polynomial law and triple-power law [13]. It was also successfully applied to other scientific problems that had stemmed from Fluid Dynamics [7], Mathematical Biology [5] and Nuclear Physics [4]. It is not out of place to mention that this principle retrieved results in multi-dimensional optical solitons as well [9]. One of the shortcomings of this approach is that the retrieved soliton solution is not exact although it is analytical since this solutions is based on a certain principle. Although not an exact solution,

* Corresponding author.
E-mail address: mehmet.ekici@bozok.edu.tr (M. Ekici).

https://doi.org/10.1016/j.rinp.2021.104539
Received 13 June 2021; Received in revised form 29 June 2021; Accepted 9 July 2021
Available online 15 July 2021

it works when standard algorithms is rendered to be non-integrable for retrieving an exact solution. Subsequent sections illustrate the details.

Governing model

The governing equation having Kudryashov's sextic form of nonlinear refractive index with the inclusion of perturbation terms is structured as [19]:

$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + \left(b_1 |q|^n + b_2 |q|^{2n} + b_3 |q|^{3n} + b_4 |q|^{4n} + b_5 |q|^{5n} + b_6 |q|^{6n}\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta\left(|q|^{2m}\right)_x q + \mu |q|^{2m} q_x\right]. \quad (1)$$

Here, q is the dependent variable standing for the complex-valued wave function which falls in the scope of nonlinear quantum optics. The coefficients a_j $(1 \le j \le 6)$ that are real-valued represent dispersive effects and stem from inter-modal dispersion, chromatic dispersion, third-, fourth–, fifth- and sixth-order dispersion effects respectively. Next, b_j for $1 \le j \le 6$ are the six nonlinear forms of refractive index that was proposed by Kudryashov. The parameter n attributes to power-law effect. The first term in (1) refers to linear temporal evolution whose coefficient is $i = \sqrt{-1}$. From right side of the model, λ gives the effect of self-steepening while θ and μ represent nonlinear dispersion. Once again, the power-law parameter m accounts for maximum intensity.

This model has been very recently studied with the right side set to zero namely its unperturbed version [14]. In this case, bright, dark and singular solitons have been identified. However, (1) with right side set to zero will yield a bright 1-soliton solution for $n = 2$. Thus, the unperturbed equation (1) that yields bright 1-soliton solution is:

$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + \left(b_1 |q|^2 + b_2 |q|^4 + b_3 |q|^6 + b_4 |q|^8 + b_5 |q|^{10} + b_6 |q|^{12}\right) q = 0, \quad (2)$$

whose bright 1-soliton is as below [19]:

$$q(x,t) = A\,\mathrm{sech}^{\frac{1}{2}}\left[B(x - vt)\right] e^{i(-\kappa x + \omega t + \theta_0)}, \quad (3)$$

where A is the soliton amplitude, while B gives its inverse width and v stands for the soliton speed. Then, κ, ω and θ_0 respectively represent the frequency, wave number and phase constant.

The current work will study the perturbed version of Eq. (2) that is framed as:

$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + \left(b_1 |q|^2 + b_2 |q|^4 + b_3 |q|^6 + b_4 |q|^8 + b_5 |q|^{10} + b_6 |q|^{12}\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta\left(|q|^{2m}\right)_x q + \mu |q|^{2m} q_x\right]. \quad (4)$$

Eq. (4) will be integrated by the aid of SVP to locate its bright 1-soliton solution. This will yield an analytical solution for the governing model studied in (4) that is not exact.

Mathematical foundations

To commence, the starting presumptive hypothesis is [1–7,11–13]

$$q(x,t) = g(s) e^{i\phi(x,t)}, \quad (5)$$

where $g(s)$ stands for the amplitude component of the wave while $\phi(x,t)$ is its phase part. Here,

$$s = x - vt, \quad \text{and} \quad \phi(x,t) = -\kappa x + \omega t + \theta_0.$$

Substitute (5) into (4). Then, imaginary part yields:

$$\left[v - a_1 + 2a_2\kappa + 3a_3\kappa^2 - 4a_4\kappa^3 + 5a_5\kappa^4 - 6a_6\kappa^5 + \left\{(2m+1)\lambda + 2m\theta + \mu\right\} g^2\right] g' - \left(a_3 - 4a_4\kappa - 10a_5\kappa^2 + 20a_6\kappa^3\right) g''' - \left(a_5 - 6a_6\kappa\right) g^{(v)} = 0. \quad (6)$$

This implies the speed of the soliton is:

$$v = a_1 - 2a_2\kappa - 3a_3\kappa^2 + 4a_4\kappa^3 - 5a_5\kappa^4 + 6a_6\kappa^5. \quad (7)$$

The following parameter restrictions also come out of (6):

$$(2m+1)\lambda + 2m\theta + \mu = 0, \quad (8)$$

$$a_3 - 4a_4\kappa - 10a_5\kappa^2 + 20a_6\kappa^3, \quad (9)$$

and

$$a_5 - 6a_6\kappa = 0. \quad (10)$$

The real part now implies

$$P_1 g - P_2 g'' + P_3 g^{(iv)} - a_6 g^{(vi)} + \kappa(\lambda+\mu) g^{2m+1} - b_1 g^3 - b_2 g^5 - b_3 g^7 - b_4 g^9 - b_5 g^{11} - b_6 g^{13} = 0, \quad (11)$$

with the notations

$$P_1 = \omega - a_1\kappa + a_2\kappa^2 + a_3\kappa^3 - a_4\kappa^4 - a_5\kappa^5 + a_6\kappa^6, \quad (12)$$

$$P_2 = a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4, \quad (13)$$

and

$$P_3 = a_4 + 5a_5\kappa - 15a_6\kappa^2, \quad (14)$$

are implied. Multiplying (11) by g' and integrating leaves us with:

$$420P_1 g^2 - 420P_2 \left(g'\right)^2 + 420P_3 \left(g''\right)^2 - 420a_6 \left(g'''\right)^2 + \frac{420\kappa(\lambda+\mu)}{m+1} g^{2m+2} - 210b_1 g^4 - 140b_2 g^6 - 105b_3 g^8 - 84b_4 g^{10} - 70b_5 g^{12} - 60b_6 g^{14} = K, \quad (15)$$

where K is the integration constant.

Implementation of SVP

The stationary integral is now defined as [1–7,11–13]:

$$J = \int_{-\infty}^{\infty} \left\{420P_1 g^2 - 420P_2 \left(g'\right)^2 + 420P_3 \left(g''\right)^2 - 420a_6 \left(g'''\right)^2 + \frac{420\kappa(\lambda+\mu)}{m+1} g^{2m+2} - 210b_1 g^4 - 140b_2 g^6 - 105b_3 g^8 - 84b_4 g^{10} - 70b_5 g^{12} - 60b_6 g^{14}\right\} dx \quad (16)$$

From (3), we observe that

$$g(s) = A\,\mathrm{sech}^{\frac{1}{2}}(Bs). \quad (17)$$

Substituting (17) into (16) and performing the integration gives

$$J = \frac{5040\pi P_1 A^2}{B} - 630\pi P_2 A^2 B - 630\pi P_3 A^2 B^3 - 315\pi a_6 A^2 B^5 + \frac{10,080\kappa(\lambda+\mu) G A^{2m+2}}{m(m+1)B} - \frac{5040 b_1 A^4}{B} - \frac{840\pi b_2 A^6}{B} - \frac{1680 b_3 A^8}{B} - \frac{378\pi b_4 A^{10}}{B} - \frac{896 b_5 A^{12}}{B} - \frac{225\pi b_6 A^{14}}{B}, \quad (18)$$

where the notation

$$G = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{m}{2}\right)} \quad (19)$$

was adopted.

SVP points out that the bright 1-soliton solution to (4) is the same as the solution to (2) as given. However, its amplitude (A) and inverse width (B) will differ. In fact, they will be given by the coupled system of equations [1–7,11–13]:

$$\frac{\partial J}{\partial A} = 0, \quad (20)$$

and

$$\frac{\partial J}{\partial B} = 0, \tag{21}$$

For the reduced stationary integral as given by (18), the coupled system of Eqs. (20) and (21) formulates as:

$$1680\pi P_1 - 210\pi P_2 B^2 + 210\pi P_3 B^4 - 105\pi a_6 B^6 + \frac{3360\kappa(\lambda+\mu)GA^{2m}}{m} - 3360 b_1 A^2 - 560\pi b_2 A^4 - 2240 b_3 A^6 - 630\pi b_4 A^8 - 1792 b_5 A^{10} - 525\pi b_6 A^{12} = 0, \tag{22}$$

and

$$5040\pi P_1 + 630\pi P_2 B^2 + 1890\pi P_3 B^4 + 1575\pi a_6 B^6 + \frac{10,080\kappa(\lambda+\mu)GA^{2m}}{m(m+1)} - 540 b_1 A^2 - 840\pi b_2 A^4 - 1680 b_3 A^6 - 378\pi b_4 A^8 - 896 b_5 A^{10} - 225\pi b_6 A^{12} = 0, \tag{23}$$

respectively. Upon uncoupling, one acquires the hexic polynomial equation for the inverse width B as:

$$630\pi P_2 B^2 + 630\pi P_3 B^4 + 945\pi a_6 B^6 + \frac{540\kappa(\lambda+\mu)GA^{2m}}{m} + 2520 b_1 A^2 + 420\pi b_2 A^4 + 2520 b_3 A^6 + 756\pi b_4 A^8 + 2240 b_5 A^{10} + 675\pi b_6 A^{12} = 0. \tag{24}$$

Eq. (24) can be rewritten as the cubic polynomial equation:

$$au^3 + bu^2 + cu + d = 0, \tag{25}$$

where the following notations are utilized:

$$B^2 = u, \tag{26}$$

$$a = 945\pi a_6, \tag{27}$$

$$b = 630\pi P_3, \tag{28}$$

$$c = 630\pi P_2, \tag{29}$$

$$d = \frac{540\kappa(\lambda+\mu)GA^{2m}}{m} + 2520 b_1 A^2 + 420\pi b_2 A^4 + 2520 b_3 A^6 + 756\pi b_4 A^8 + 2240 b_5 A^{10} + 675\pi b_6 A^{12}. \tag{30}$$

While (19) implicates that $m > 0$, one concludes from (30),

$$m \neq 0. \tag{31}$$

By virtue of Cardano's method, applied to (25) and subsequently from (26), one recovers the width of the soliton in terms of its amplitude as

$$B = \left[\left\{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} + \left\{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} - \frac{b}{3a}\right]^{\frac{1}{2}}. \tag{32}$$

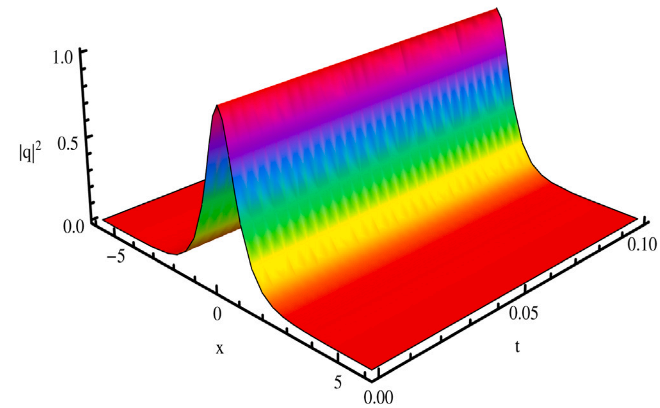

Fig. 1. $|q|^2$ soliton.

The constraints for this solution to exist are:

$$a_6 \neq 0, \tag{33}$$

along with the discriminant

$$\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3 > 0, \tag{34}$$

as well as

$$\left\{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} + \left\{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} > \frac{b}{3a}. \tag{35}$$

The solution to (4) is given by (3), where the soliton width B is related to the amplitude A as in (32) with all of the notations and constraints as discussed in the work.

Fig. 1 demonstrates the numerical simulation of a bright 1-soliton as given by (3) for perturbed Kudryashov's model. The system parameter values are: $m = 1$, $a_1 = 1$, $a_2 = 0.1$, $a_4 = 2$, $a_6 = -0.3$, $b_j = 0.1$ $(j = 1, 2, \ldots, 6)$, $\omega = 1$, $\kappa = \lambda = \mu = 0.5$, $A = 1$.

Conclusions

This paper is about SVP applied successfully to perturbed Kudryashov's equation that contains six nonlinear forms of SPM. In this context, bright HD 1-soliton solution is revealed. This analytical solution is in its closed form although not an exact solution. A version of the variational principle, commonly referred to as SVP, has made this retrieval possible. A numerical simulation is also presented. The perturbation terms are with maximum intensity. No known integration scheme can possibly identify a closed form solution. Thus, SVP is to the rescue. One other shortcoming, however, of the scheme is that it fails to retrieve dark or singular soliton solution. It is the stationary integral that is rendered divergent and thus the current approach meets its limitation. Later, this approach will be implemented with other models as introduced by Kudryashov that will lead to its bright 1-soliton solution. They are surely going to be visible somewhere. The readers are suggested to stay tuned!

CRediT authorship contribution statement

Anjan Biswas: Supervision, Investigation; Writing – original draft, Methodology, Software. **Mehmet Ekici:** Supervision, Investigation, Writing – original draft. **Anelia Dakova:** Validation, Investigation. **Salam Khan:** Supervision, Validation. **Seithuti P. Moshokoa:** Supervision, Validation. **Hashim M. Alshehri:** Writing – review & editing. **Milivoj R. Belic:** Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Alqahtani RT, Babatin MM, Biswas A. Bright optical solitons for Lakshmanan–Porsezian–Daniel model by semi–inverse variational principle. Optik 2018;154:109–14.
[2] Biswas A, Asma M, Guggilla P, Mullick L, Moraru L, Ekici M, Alzahrani AK, Belic MR. Optical soliton perturbation with Kudryashov's equation by semi–inverse variational principle. Phys Lett A 2020;384:126830.
[3] Biswas A, Edoki J, Guggilla P, Khan S, Alzahrani AK, Belic MR. Cubic–quartic optical soliton perturbation with Lakshmanan–Porsezian–Daniel model by semi–inverse variational principle. Ukr J Phys Opt 2021;22(3):122–7.
[4] Biswas A, Milovic D. Chiral solitons with Bohm potential by He's variational principle. Phys Atomic Nuclei 2011;74(5):755–7.
[5] Biswas A, Milovic D, Milic D. Solitons in α–helix proteins by He's variational principle. Int J Biomath 2011;4(4):423–9.
[6] Biswas A, Johnson S, Fessak M, Siercke B, Zerrad E, Konar S. Dispersive optical solitons by semi–inverse variational principle. J Modern Opt 2012;59(3):213–7.
[7] Biswas A, Milovic D, Kumar S, Yildirim A. Perturbation of shallow water waves by semi–inverse variational principle. Indian J Phys 2013;87(6):567–9.
[8] He JH, Sun C. A variational principle for a thin film equation. J Math Chem 2019;57:2075–81.
[9] He JH. Variational principle and periodic solution of the Kundu–Mukherjee–Naskar equation. Results Phys 2020;17:103031.
[10] Kheir H, Jabbari A, Yildirim A, Alomari AK. He's semi–inverse method for soliton solutions of Boussinesq system'. World J Model Simul 2013;9(1):3–13.
[11] Kohl R, Milovic D, Zerrad E, Biswas A. Optical solitons by He's variational principle in a non–Kerr law mwdia. J Infrared Millim Terahertz Waves 2009;30(5):526–37.
[12] Kohl RW, Biswas A, Ekici M, Zhou Q, Khan S, Alshomrani AS, Belic MR. Sequel to highly dispersive optical soliton perturbation with cubic–quintic–septic refractive index by semi-inverse variational principle. Optik 2020;203:163451.
[13] Kohl RW, Biswas A, Zhou Q, Ekici M, Alzahrani AK, Belic MR. Optical soliton perturbation with polynomial and triple–power laws of refractive index by semi–inverse variational principle. Chaos Solitons Fractals 2020;135:109765.
[14] Li XW, Li Y, He JH. On the semi–inverse method and variational principle. Therm Sci 2013;17(5):1565–8.
[15] Liu HM. The variational principle for Yang–Mills equation by semi–inverse method. Facta Univ Ser: Mech Autom Control Robot 2004;4(16):169–71.
[16] Liu WY, Yu YJ, Chen LD. Variational principles for Ginzburg–Landau equation by He's semi–inverse method. Chaos Solitons Fractals 2007;33(5):1801–3.
[17] Ozis T, Yildirim A. Application of He's semi–inverse method to the nonlinear Schrödinger equation. Comput Math Appl 2007;54:1039–42.
[18] Singh SS. Bright and dark 1–soliton solutions to perturbed Schrödinger–Hirota equation with power law nonlinearity via semi–inverse variation method and ansatz method'. Int J Phys Res 2017;5(2):39–42.
[19] Zayed EME, Shohib RMA, Alngar MEM, Biswas A, Ekici M, Khan S, Alzahrani AK, Belic MR. Optical solitons and conservation laws associated with Kudryashov's sextic power–law nonlinearity of refractive index. Ukr J Phys Opt 2021;22(1):38–49.
[20] Zhang J, Yu JY, Pan N. Variational principles for nonlinear fiber optics. Chaos Solitons Fractals 2005;24:309–11.