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Optical soliton perturbation with anti-cubic nonlinearity by semi-inverse variational principle

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ABSTRACT

The perturbed nonlinear Schrödinger's equation with anti-cubic nonlinearity is studied with the aid of semi-inverse variational principle. The perturbation terms that are included are inter-modal dispersion, third and fourth order dispersion, nonlinear dispersion and self-steepening term, the last two of which appear with full nonlinearity.

1. Introduction

Optical solitons is one of the fastest growing areas of research in the field of telecommunications. There are various aspects of these solitons that are constantly investigated upon. A few such topics are polarizatin preserving fibers, birefringent fibers, DWDM systems and dispersion-flattened fibers, Bragg gratings, optical switching, magneto-optic waveguides, integrability, perturbation theory, super-continuum generation, rogue waves and several others. This paper is going to address one such aspect of optical solitons that are studied with anti-cubic nonlinearity. It is the integrability issue. There are several perturbation terms that the governing nonlinear Schrödinger's equation (NLSE) is studied with. These are inter-modal dispersion (IMD), third order dispersion (3OD), fourth order dispersion 4OD), self-steepening term and nonlinear dispersion, the last two of which are studied with full nonlinearity. There are several analytical tools available to study these type of nonlinear models [1–15]. Some of the most visible ones are G'/G-expansion method, extended trial equation method, Lie symmetry analysis, traveling wave hypothesis. This paper is going to employ the semi-inverse variational principle (SVP) to secure analytical solutions to the model. Although this will not be an exact solution, the variational principle will lead to a closed form analytical bright soliton solution. There are a few restrictions that are going to be implemented for the solitons to exist and these conditions will be listed.

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2. Governing model

The governing NLSE with perturbation terms having anti-cubic nonlinearity is [3,5,6,11]:

$$iq_t + aq_{xx} + \left(b_1|q|^{-4} + b_2|q|^2 + b_3|q|^4\right) q = i\left[\alpha q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda\left(|q|^{2m}q\right)_x + \theta\left(|q|^{2m}\right)_x q\right] \tag{1}$$

In (1), $q(x, t)$ is the dependent variable that represents the complex-valued wave profile while x and t are independent spatial and temporal variables respectively. The coefficient of a is the group velocity dispersion (GVD). The three nonlinear terms are the coefficients of b_j for $j = 1, 2, 3$. The term with b_1 acccounts for the anti-cubic nonlinearity. However, b_2 and b_3 are with cubic and quintic nonlinear terms. On the right hand side α is the coefficient of inter-modal dispersion. This happens when group velocity of light propagating in multimode fibers or other waveguides depend on optical frequency as well as the propagation mode involved. The coefficients of 3OD and 4OD are γ and σ respectively. The self-steepening term is with λ while nonlinear dispersion is given by θ. Finally, the full nonlinearity parameter is governed by the parameter m.

3. Semi-inverse variational principle

To solve (1) by SVP, an initial hypothesis is [1–3]:

$$q(x, t) = g(s)e^{i\phi(x,t)} \tag{2}$$

where

$$s = x - vt \tag{3}$$

and the phase component ϕ is:

$$\phi(x, t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

In (2) and (3), $g(x, t)$ represents the amplitude component of the wave and v is the speed of the wave. From (4), κ is the soliton frequency, ω is its wave number and θ_0 is the phase constant. Substituting (2) into (1) and splitting into real and imaginary parts yield [3,13]

$$\sigma g^{(iv)} - P_2 g'' + P_1 g + \left(b_1 g^{-4} + b_2 g^2 + b_3 g^4\right) g + \lambda\kappa g^{2m+1} = 0 \tag{5}$$

and

$$\left(v + 2a\kappa + \alpha + 3\gamma\kappa^2 + 4\sigma\kappa^3\right) g' - (\gamma + 4\sigma\kappa) g'' + \left\{(2m + 1)\lambda + 2m\theta\right\} g^{2m} g' = 0, \tag{6}$$

respectively, where the notations $g' = dg/ds$ and $g'' = d^2g/ds^2$ etc are adopted. Here, in (5)

$$P_1 = \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \sigma\kappa^4, \tag{7}$$

$$P_2 = a + 3\gamma\kappa + 6\sigma\kappa^3. \tag{8}$$

From (6), setting the coefficients of linearly independent functions to zero implies:

$$v = -2a\kappa - \alpha - 3\gamma\kappa^2 - 4\sigma\kappa^3, \tag{9}$$

$$\gamma + 4\sigma\kappa = 0, \tag{10}$$

and

$$(2m + 1)\lambda + 2m\theta = 0. \tag{11}$$

Thus, Eq. (9) gives the speed of the soliton in presence of the perturbation terms while relations (10) and (11) are the constraints on the perturbation parameters.

Next, multiplying both sides of (5) by g' and integrating once yields

$$\sigma\left(g''\right)^2 - 2\sigma g' g''' + P_1 g^2 + P_2\left(g'\right)^2 - \frac{b_1}{g^2} + \frac{b_2}{2}g^4 + \frac{b_3}{3}g^6 + \frac{\lambda g^{2m+2}}{m+1} = K, \tag{12}$$

where K is the integration constant. The stationary integral is then defined as [3,4,13]:

$$J = \int_{-\infty}^{\infty} K ds = \int_{-\infty}^{\infty} \left[3\sigma\left(g''\right)^2 + P_1 g^2 + P_2\left(g'\right)^2 - \frac{b_1}{g^2} + \frac{b_2}{2}g^4 + \frac{b_3}{3}g^6 + \frac{\lambda\kappa g^{2m+2}}{m+1}\right] ds \tag{13}$$

Now choose [3,13]

$$g(s) = A\sqrt{\text{sech}(Bs)} \tag{14}$$

where A is the soliton amplitude, B is its inverse width. SVP states that the solution of the perturbed Eq. (1) will be same as the unperturbed model. However, its amplitude and width will vary according to the coupled system of equations [3,4,13]:

$$\frac{\partial J}{\partial A} = 0, \tag{15}$$

and

$$\frac{\partial J}{\partial B} = 0. \tag{16}$$

Substituting (14) into (13) and carrying out the integrations gives

$$J = \frac{57\sigma\pi A^2 B^3}{64} + \frac{P_2\pi A^2 B}{8} + \frac{P_1 A^2\pi}{B} - \frac{b_1 I}{A^2 B} + \frac{b_2 A^4}{B} + \frac{b_3 A^6\pi}{6B} + \frac{\lambda\kappa A^{2m+2}}{(m+1)B}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}, \tag{17}$$

where $\Gamma(x)$ is Euler's gamma function and

$$I = \int_{-\infty}^{\infty} \cosh u \quad du. \tag{18}$$

This is a divergent integral and can be approximated numerically between finite limits of integration.

Next, for J given by (17), Eqs. (15) and (16) reduce to

$$\frac{57\sigma\pi B^4}{32} + \frac{P_2\pi B^2}{4} + 2P_1\pi + \frac{2b_1 I}{A^4} + 4b_2A^2 + b_3\pi A^4 + 2\lambda\kappa A^{2m}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)} = 0 \tag{19}$$

and

$$\frac{171\sigma\pi B^4}{64} + \frac{P_2\pi B^2}{8} - P_1\pi + \frac{b_1 I}{A^4} - b_2A^2 + \frac{b_3\pi A^4}{6} - \frac{\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)} = 0 \tag{20}$$

Upon uncoupling (19) and (20) leads to a biquadratic equation for the width B in terms of the soliton amplitude A that is given by

$$\frac{57\sigma\pi B^4}{16} + \frac{P_2\pi B^2}{4} + 2\frac{b_1 I}{A^4} + b_2A^2 + \frac{b_3\pi A^4}{3} + \frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)} = 0 \tag{21}$$

which solves to

$$B = \sqrt{\frac{8}{57\sigma\pi}\left[-\frac{P_2\pi}{4} + \left\{\frac{P_2^2\pi^2}{16} - \frac{57\sigma\pi}{4}\left(\frac{2b_1 I}{A^4} + b_2A^2 + \frac{b_3\pi A^4}{3} + \frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right)\right\}^{1/2}\right]} \tag{22}$$

This relation between the soliton amplitude and width will remain valid provided

$$\sigma\left[-\frac{P_2\pi}{4} + \left\{\frac{P_2^2\pi^2}{16} - \frac{57\sigma\pi}{4}\left(\frac{2b_1 I}{A^4} + b_2A^2 + \frac{b_3\pi A^4}{3} + \frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right)\right\}^{1/2}\right] > 0 \tag{23}$$

and

$$\left|57\sigma\left\{\frac{2b_1 I}{A^4} + b_2A^2 + \frac{b_3\pi A^4}{3} + \frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right\}\right| < \frac{P_2\pi}{4} \tag{24}$$

Thus, finally, the 1-soliton solution to (1) is given by

$$q(x,t) = A\sqrt{\operatorname{sech}[B(x-vt)]}\,e^{i\left(-\kappa x+\omega t+\theta_0\right)} \tag{25}$$

where the parameter relations and restrictions are all described above.

4. Conclusions

This paper obtained an analytical soliton solution to the pertured NLSE with anti-cubic nonlinearity. The perturbation terms are of Hamiltonian type and two of these perturbations appear with full nonlinearity. SVP yielded the analytical solution although this is not an exact solution. It is not possible to secure an exact soliton solution to the model studied in this paper since the perturbation terms appear with full nonlinearity. The restrictions, or constraint conditions, to the parameters are also listed for existence of these bright solitons.

The results of this paper carry a lot of future prospects. This integration scheme can be applied to other situations as well, whenever an exact soliton solution is not available. Some of these situations are in birefringent fibers, DWDM systems, optical switching, as well as other models such as complex Ginzburg-Landau equation, Gerdjikov–Ivanov equation and others. Those results will be visible fairly soon.

Conflict of interest

The authors also declare that there is no conflict of interest.

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