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Optical solitons with DWDM technology and four-wave mixing

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ABSTRACT

This paper obtains bright and singular optical soliton solutions to DWDM system in presence of four-wave mixing. The extended trial function scheme is adopted. The two types of nonlinear media studied are Kerr law and parabolic law. There are other types of waves that appears as a byproduct to this scheme.

1. Introduction

DWDM technology is an essential feature that needs to be incorporated in fiber-optic communication system [1–10]. This hugely multiplies the information carrying capacity through these fibers. Thus, parallel transmission of data is possible across trans-continental and trans-oceanic distances in just a matter of a few femto-seconds. Such an engineering marvel can only be achieved by perfecting this DWDM technology. However, an unwanted, as well as unavoidable, feature of four-wave mixing (FWM) naturally creeps in. This therefore poses a challenge to retrieve soliton solutions to such systems. This paper adopts extended trial equation scheme that overcomes this challenge and retrieves exact soliton solutions to DWDM system with FWM. One restriction that is imposed while extracting soliton solutions is the phase-matching condition. Thus, bright and singular soliton solutions are recovered for the system. Additional solutions naturally emerge with the implementation of this integration scheme. These are plane waves and doubly periodic functions. In the limiting case, when the modulus of ellipticity approaches zero or unity, soliton solutions or periodic singular waves fall out. There are two types of nonlinear optical fibers studied in this paper. They are with Kerr law and parabolic law. The details are discussed in the subsequent section.

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2. The model

The model for DWDM system stems out of nonlinear Schrödinger's equation that is considered in this paper with spatio-temporal dispersion (STD) in addition to the usual group velocity dispersion (GVD). This only makes the problem well-posed. The two laws of nonlinearity that will be studied are Kerr law and parabolic law. The study will now be split into the next couple of subsections where each of these laws are worked out in detail.

2.1. Kerr law

For Kerr law nonlinearity, DWDM model reads [6–9]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left\{ c_l \left| q^{(l)} \right|^2 + \sum_{n \neq l}^{N} \alpha_{ln} \left| q^{(n)} \right|^2 \right\} q^{(l)} + \sum_{n \neq l}^{N} \beta_{ln} q^{(l)*} \left| q^{(n)} \right|^2 = 0, \tag{1}$$

where $1 \leq l \leq N$. The first term in (1) on left hand side is the linear evolution term, while a_l represents the coefficient of GVD; b_l represents the STD. Then, c_l is the coefficient of self-phase modulation (SPM) while α_{ln} are the coefficients of cross-phase modulation (XPM), while β_{ln} accounts for FWM. The independent variables are x and t that represents the spatial and temporal variables respectively. The dependent variable is $q^{(l)}(x,t)$ that represents soliton profile in every single channel for $1 \leq l \leq N$.

This subsection will obtain exact bright, dark and singular 1-soliton solutions to (1) by the extended trial scheme approach [2–5,10]. In order to solve Eq. (1), we use the following wave transformation

$$q^{(l)}(x,t) = P^{(l)}(\xi) e^{i\Phi(x,t)}, \tag{2}$$

where

$$\xi = k(x - vt), \tag{3}$$

$$\Phi(x,t) = -\kappa x + \omega t + \theta. \tag{4}$$

In Eq. (2), the function $\Phi(x,t)$ is the phase component of the soliton. Then, in Eq. (4), κ represents the frequency that stays the same in all channels, while ω is the same wave number, while θ is the phase constant that also stays the same in all channels, while v is the same soliton velocity. The amplitude part is $P^{(l)}(\xi)$. Substituting the wave transformation (2) into (1) and splitting into real and imaginary parts leads to [1]

$$k^2(a_l - b_l v)\left(P^{(l)}\right)'' + \left(b_l \omega \kappa - \omega - a_l \kappa^2\right) P^{(l)} + c_l \left(P^{(l)}\right)^3 + \left\{ \sum_{n \neq l}^{N} (\alpha_{ln} + \beta_{ln}) \left(P^{(n)}\right)^2 \right\} P^{(l)} = 0, \tag{5}$$

and

$$k\{b_l \omega - 2a_l \kappa - v(1 - b_l \kappa)\}\left(P^{(l)}\right)' = 0. \tag{6}$$

The imaginary part equation leads to the speed of the soliton that is given by

$$v = \frac{b_l \omega - 2a_l \kappa}{1 - b_l \kappa}. \tag{7}$$

The speed of the soliton given by (7) remains valid for

$$b_l \kappa \neq 1. \tag{8}$$

Using the balancing principle leads to

$$P^{(l)} = P^{(n)}. \tag{9}$$

Consequently, Eq. (5) reduces to

$$k^2(a_l - b_l v)\left(P^{(l)}\right)'' + \left(b_l \omega\kappa - \omega - a_l\kappa^2\right)P^{(l)} + \left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \beta_{ln})\right\}\left(P^{(l)}\right)^3 = 0. \tag{10}$$

The extended trial function scheme will be applied to Eq. (10) to retrieve bright, dark and singular solitons. To start with the extraction of solutions to (10), the following assumption for the soliton structure is made

$$P^{(l)} = \sum_{i=0}^{\varsigma} \tau_i^{(l)} \Psi^i, \tag{11}$$

where

$$(\Psi')^2 = \Lambda(\Psi) = \frac{\Phi(\Psi)}{\Upsilon(\Psi)} = \frac{\mu_\sigma \Psi^\sigma + \cdots + \mu_1\Psi + \mu_0}{\chi_\rho \Psi^\rho + \cdots + \chi_1\Psi + \chi_0}. \tag{12}$$

Here $\tau_0^{(l)}, \ldots, \tau_\varsigma^{(l)}$; $\mu_0, \ldots, \mu_\sigma$ and $\chi_0, \ldots, \chi_\rho$ are constants to be determined later. Using the relations (11) and (12), we derive the terms $((P^{(l)})')^2$ and $(P^{(l)})''$ as below:

$$\left(\left(P^{(l)}\right)'\right)^2 = \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i^{(l)}\Psi^{i-1}\right)^2, \tag{13}$$

and

$$\left(P^{(l)}\right)'' = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i^{(l)}\Psi^{i-1}\right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i(i-1)\tau_i^{(l)}\Psi^{i-2}\right), \tag{14}$$

where $\Phi(\Psi)$ and $\Upsilon(\Psi)$ are polynomials of Ψ. We reduce Eq. (12) to the elementary integral form as follows:

$$\pm(\xi - \xi_0) = \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}} = \int \sqrt{\frac{\Upsilon(\Psi)}{\Phi(\Psi)}}d\Psi. \tag{15}$$

According to the balance principle, we determine a relation of σ, ρ and ς is given by

$$\sigma = \rho + 2\varsigma + 2. \tag{16}$$

When $\sigma = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (16), we then assume that Eq. (10) have the following formal solutions:

$$P^{(l)} = \tau_0^{(l)} + \tau_1^{(l)}\Psi, \tag{17}$$

where $\tau_0^{(l)}$ and $\tau_1^{(l)}$ are constants to be determined later such that $\tau_1^{(l)} \neq 0$, and Ψ satisfies Eq. (12). Substituting these formal solutions and their necessary derivatives into Eq. (10), and solving the resulting system of algebraic equations one recovers the following solution set:

$$\begin{aligned}
&\mu_0 = \mu_0, \quad \mu_2 = \mu_2, \quad \mu_3 = \mu_3, \quad \tau_0^{(l)} = \tau_0^{(l)}, \quad \tau_1^{(l)} = \tau_1^{(l)},\\
&\mu_1 = \frac{2\tau_0^{(l)}\left(\mu_2\tau_1^{(l)} - \mu_3\tau_0^{(l)}\right)}{\left(\tau_1^{(l)}\right)^2}, \quad \mu_4 = \frac{\mu_3\tau_1^{(l)}}{4\tau_0^{(l)}},\\
&\chi_0 = -\frac{k^2\mu_3(a_l - b_l v)}{2\tau_0^{(l)}\tau_1^{(l)}\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{\ln} + \beta_{\ln})\right\}},\\
&\omega = \frac{a_l\kappa^2\mu_3 - \tau_0^{(l)}\left(3\mu_3\tau_0^{(l)} - 2\mu_2\tau_1^{(l)}\right)\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{\ln} + \beta_{\ln})\right\}}{\mu_3(b_l\kappa - 1)}.
\end{aligned} \tag{18}$$

Substituting the solution set (18) into Eqs. (12) and (15), we find that

$$\pm(\xi-\xi_0)=W\int\frac{d\Psi}{\sqrt{\Lambda(\Psi)}}, \tag{19}$$

where

$$\Lambda(\Psi)=\Psi^4+\frac{\mu_3}{\mu_4}\Psi^3+\frac{\mu_2}{\mu_4}\Psi^2+\frac{\mu_1}{\mu_4}\Psi+\frac{\mu_0}{\mu_4},\quad W=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{20}$$

Consequently, we have the traveling wave solutions to Eq. (1) as follows:
When $\Lambda(\Psi)=(\Psi-\lambda_1)^4$, we obtain solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1\pm\frac{\tau_1^{(l)}W}{k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{21}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^3(\Psi-\lambda_2)$ and $\lambda_2>\lambda_1$, we get solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1+\frac{4\tau_1^{(l)}W^2(\lambda_2-\lambda_1)}{4W^2-\left[(\lambda_1-\lambda_2)\left(k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{22}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)^2$, we have solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_2+\frac{\tau_1^{(l)}(\lambda_2-\lambda_1)}{\exp\left[\frac{\lambda_1-\lambda_2}{W}\left(k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{23}$$

and

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)}{\exp\left[\frac{\lambda_1-\lambda_2}{W}\left(k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{24}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)(\Psi-\lambda_3)$ and $\lambda_1>\lambda_2>\lambda_3$, we recover solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1-\frac{2\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{2\lambda_1-\lambda_2-\lambda_3+(\lambda_3-\lambda_2)\cosh\left[\frac{k\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{W}\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{25}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)(\Psi-\lambda_3)(\Psi-\lambda_4)$ and $\lambda_1>\lambda_2>\lambda_3>\lambda_4$, we find solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_2+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)\mathrm{sn}^2\left[\pm\frac{\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W}\left[k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right],m\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{26}$$

where

$$m^2=\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}. \tag{27}$$

We note that λ_i ($i=1,\ldots,4$) are the roots of the polynomial equation

$$\Lambda(\Psi)=0. \tag{28}$$

When $\tau_0^{(l)}=-\tau_1^{(l)}\lambda_1$ and $\xi_0=0$, the solutions (21)–(25) are reduced to plane wave solutions

$$q^{(l)}(x,t)=\left\{\pm\frac{\tau_1^{(l)}W}{k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{29}$$

$$q^{(l)}(x,t)=\left\{\frac{4\tau_1^{(l)}W^2(\lambda_2-\lambda_1)}{4W^2-\left[k(\lambda_1-\lambda_2)\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{30}$$

singular soliton solutions

$$q^{(l)}(x,t)=\frac{\tau_1^{(l)}(\lambda_2-\lambda_1)}{2}\left\{1\mp\coth\left[\frac{k(\lambda_1-\lambda_2)}{2W}\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{31}$$

and bright soliton solutions

$$q^{(l)}(x,t)=\left\{\frac{A}{C+\cosh\left[B\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{32}$$

where

$$A=\frac{2\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{\lambda_3-\lambda_2},\quad B=\frac{k\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{W},\quad C=\frac{2\lambda_1-\lambda_2-\lambda_3}{\lambda_3-\lambda_2}. \tag{33}$$

Here, A is the amplitude of the solitons, while B is the inverse width of the solitons. So, the solitons exist for $\tau_1^{(l)}<0$. Furthermore, when $\tau_0^{(l)}=-\tau_1^{(l)}\lambda_2$ and $\xi_0=0$, Jacobi elliptic function solutions (26) can be reduced to

$$q^{(l)}(x,t)=\left\{\frac{A_1}{C_1+\mathrm{sn}^2\left[B_j\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right),\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{34}$$

where

$$A_1=\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_1-\lambda_4},\quad C_1=\frac{\lambda_4-\lambda_2}{\lambda_1-\lambda_4},\quad B_j=\frac{(-1)^j k\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W}\ \text{ for }\ j=1,2. \tag{35}$$

Remark-1. When the modulus $m\to 1$, a second form of singular optical soliton solutions fall out

$$q^{(l)}(x,t)=\left\{\frac{A_1}{C_1+\tanh^2\left[B_j\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{36}$$

where $\lambda_3=\lambda_4$.

Remark-2. However, if $m\to 0$, periodic singular waves emerge

$$q^{(l)}(x,t)=\left\{\frac{A_1}{C_1+\sin^2\left[B_j\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{a_l\kappa^2\mu_3-\tau_0^{(l)}\left(3\mu_3\tau_0^{(l)}-2\mu_2\tau_1^{(l)}\right)\left\{c_l+\sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln})\right\}}{\mu_3(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{37}$$

where $\lambda_2=\lambda_3$.

2.2. Parabolic law

This law is alternatively known as the cubic-quintic nonlinearity and arises in the nonlinear interaction between Langmuir waves and electrons. It describes the nonlinear interaction between the high frequency Langmuir waves and the ion acoustic waves by ponderomotive forces. For parabolic law nonlinearity, DWDM, with FWM, is modeled as [7,8]:

$$\begin{aligned}&iq_t^{(l)}+a_lq_{xx}^{(l)}+b_lq_{xt}^{(l)}+\left[c_l\left|q^{(l)}\right|^2+\sum_{n\neq l}^{N}\alpha_{ln}\left|q^{(n)}\right|^2\right]q^{(l)}+\left[d_l\left|q^{(l)}\right|^4+\sum_{n\neq l}^{N}\left\{\left|q^{(n)}\right|^2\left(\beta_{ln}\left|q^{(n)}\right|^2+\gamma_{ln}\left|q^{(l)}\right|^2\right)\right\}\right]q^{(l)}\\&+\delta_{ln}\sum_{n\neq l}^{N}q^{(l)*}\left(\left|q^{(n)}\right|^2\right)+\lambda_{ln}\sum_{n\neq l}^{N}\left|q^{(l)}\right|^2q^{(l)*}\left(q^{(n)}\right)^2+\nu_{ln}\sum_{n\neq l}^{N}q^{(l)*}\left|q^{(n)}\right|^2\left(q^{(n)}\right)^2+\sigma_{ln}\sum_{n\neq l}^{N}\left(q^{(l)*}\right)^2\left(q^{(n)}\right)^3=0,\end{aligned} \tag{38}$$

for $1\leq l\leq N$. In (38), SPM terms are the coefficients of c_l and d_l, while XPM coefficients are α_{ln}, β_{ln} and γ_{ln}. Also, the terms with δ_{ln}, λ_{ln}, ν_{ln} and σ_{ln} are accounted for FWM in parabolic law medium.

In this case, substituting (2) into (38), leads to the same imaginary part as given by (6). Again, the speed will be the same as (7). The real part equation however from (38) is

$$\begin{aligned}&k^2(a_l-b_lv)\left(P^{(l)}\right)''+\left(b_l\omega\kappa-\omega-a_l\kappa^2\right)P^{(l)}+c_l\left(P^{(l)}\right)^3+d_l\left(P^{(l)}\right)^5+\left\{\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\left(P^{(n)}\right)^2\right\}P^{(l)}+(\beta_{ln}\\&+\nu_{ln})P^{(l)}\left(P^{(n)}\right)^4+(\gamma_{ln}+\lambda_{ln})\left(P^{(l)}\right)^3\left(P^{(n)}\right)^2+\sigma_{ln}\left(P^{(l)}\right)^2\left(P^{(n)}\right)^3=0.\end{aligned} \tag{39}$$

Using the balancing principle leads to

$$P^{(l)}=P^{(n)}. \tag{40}$$

Consequently, Eq. (39) reduces to

$$k^2(a_l - b_l v)\left(P^{(l)}\right)'' + \left(b_l\omega\kappa - \omega - a_l\kappa^2\right)P^{(l)} + \left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right\}\left(P^{(l)}\right)^3 + (d_l + \beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\left(P^{(l)}\right)^5 = 0. \tag{41}$$

Balancing $(P^{(l)})''$ with $(P^{(l)})^5$ in Eq. (41), we obtain

$$M^{(l)} = \frac{1}{2}. \tag{42}$$

To obtain an analytic solution, we employ the transformation

$$P^{(l)} = \left(U^{(l)}\right)^{\frac{1}{2}}, \tag{43}$$

in Eq. (41) to find

$$k^2(a_l - b_l v)\left\{2U^{(l)}\left(U^{(l)}\right)'' - \left(\left(U^{(l)}\right)'\right)^2\right\} + 4\left(b_l\omega\kappa - \omega - a_l\kappa^2\right)\left(U^{(l)}\right)^2 + 4\left\{c_l + \sum_{n\neq l}^{N}(\alpha_{ln} + \delta_{ln})\right\}\left(U^{(l)}\right)^3 + 4(d_l + \beta_{ln} + \nu_{ln} + \gamma_{ln} + \lambda_{ln} + \sigma_{ln})\left(U^{(l)}\right)^4 = 0, \tag{44}$$

where $1 \leq l \leq N$.

The extended trial function scheme will be applied to Eq. (44) to retrieve bright, dark and singular solitons. Balancing the orders of $U^{(l)}(U^{(l)})''$ and $(U^{(l)})^4$ in Eq. (44), we determine a relation of σ, ρ and ς as

$$\sigma = \rho + 2\varsigma + 2. \tag{45}$$

Let us choose $\sigma = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (45). This means that the extended trial function scheme suggests the use of the finite expansion

$$U^{(l)} = \tau_0^{(l)} + \tau_1^{(l)}\Psi, \tag{46}$$

where $\tau_0^{(l)}$ and $\tau_1^{(l)}$ are constants to be determined later such that $\tau_1^{(l)} \neq 0$, and Ψ satisfies Eq. (12). Substituting Eq. (46) into Eq. (44), and solving the resulting system of algebraic equations, we obtain

$$\begin{aligned}
&\mu_0 = \mu_0, \quad \mu_2 = \mu_2, \quad \chi_0 = \chi_0, \quad \tau_0^{(l)} = \tau_0^{(l)}, \quad \tau_1^{(l)} = \tau_1^{(l)},\\
&\mu_1 = \frac{k^2\mathcal{H}_1(a_l - b_l v) + 2\chi_0\left(\tau_0^{(l)}\right)^3\left\{c_l + 2\tau_0^{(l)}\mathcal{H}_3 + \sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{k^2\tau_0^{(l)}\tau_1^{(l)}(a_l - b_l v)},\\
&\mu_3 = -\frac{2\chi_0\tau_1^{(l)}\left\{3c_l + 8\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{3k^2(a_l - b_l v)},\\
&\mu_4 = -\frac{4\chi_0\left(\tau_1^{(l)}\right)^2\mathcal{H}_3}{3k^2(a_l - b_l v)},\\
&\omega = \frac{\mathcal{H}_2 + b_l k^2 v\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)},
\end{aligned} \tag{47}$$

where

$$\mathcal{H}_1 = \mu_2\left(\tau_0^{(l)}\right)^2 + \mu_0\left(\tau_1^{(l)}\right)^2, \quad \mathcal{H}_2 = a_l\left(4\kappa^2\chi_0 - k^2\mu_2\right), \quad \mathcal{H}_3 = d_l + \beta_{\ln} + \gamma_{\ln} + \lambda_{\ln} + \nu_{\ln} + \sigma_{\ln}. \tag{48}$$

Substituting the solution set (47) into Eqs. (12) and (15), we recover that

$$\pm(\xi - \xi_0) = Q\int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}}, \tag{49}$$

where

$$\Lambda(\Psi) = \Psi^4 + \frac{\mu_3}{\mu_4}\Psi^3 + \frac{\mu_2}{\mu_4}\Psi^2 + \frac{\mu_1}{\mu_4}\Psi + \frac{\mu_0}{\mu_4}, \quad Q = \sqrt{\frac{\chi_0}{\mu_4}}. \tag{50}$$

Consequently, we have the traveling wave solutions to Eq. (38) in the forms:
When $\Lambda(\Psi) = (\Psi - \lambda_1)^4$, we obtain solutions as

$$q^{(l)}(x,t) = \left\{\tau_0^{(l)} + \tau_1^{(l)}\lambda_1 \pm \frac{\tau_1^{(l)}Q}{k\left(x - \left\{\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right\}t\right) - \xi_0}\right\}^{\frac{1}{2}} \times \exp\left[i\left\{-\kappa x + \left(\frac{\mathcal{H}_2 + b_l k^2\nu\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)}\right)t + \theta\right\}\right]. \tag{51}$$

When $\Lambda(\Psi) = (\Psi - \lambda_1)^3(\Psi - \lambda_2)$ and $\lambda_2 > \lambda_1$, we get solutions as

$$q^{(l)}(x,t) = \left\{\tau_0^{(l)} + \tau_1^{(l)}\lambda_1 + \frac{4\tau_1^{(l)}Q^2(\lambda_2 - \lambda_1)}{4Q^2 - \left[(\lambda_1 - \lambda_2)\left(k\left(x - \left\{\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right\}t\right) - \xi_0\right)\right]^2}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left\{-\kappa x + \left(\frac{\mathcal{H}_2 + b_l k^2\nu\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)}\right)t + \theta\right\}\right]. \tag{52}$$

When $\Lambda(\Psi) = (\Psi - \lambda_1)^2(\Psi - \lambda_2)^2$, we have solutions as

$$q^{(l)}(x,t) = \left\{\tau_0^{(l)} + \tau_1^{(l)}\lambda_2 + \frac{\tau_1^{(l)}(\lambda_2 - \lambda_1)}{\exp\left[\frac{\lambda_1 - \lambda_2}{Q}\left(k\left(x - \left\{\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right\}t\right) - \xi_0\right)\right] - 1}\right\}^{\frac{1}{2}} \times \exp\left[i\left\{-\kappa x + \left(\frac{\mathcal{H}_2 + b_l k^2\nu\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)}\right)t + \theta\right\}\right], \tag{53}$$

and

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)}{\exp\left[\frac{\lambda_1-\lambda_2}{Q}\left(k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right)\right]-1}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{54}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)(\Psi-\lambda_3)$ and $\lambda_1>\lambda_2>\lambda_3$, we recover solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1-\frac{2\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{2\lambda_1-\lambda_2-\lambda_3+(\lambda_3-\lambda_2)\cosh\left[\frac{k\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{Q}\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right]. \tag{55}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)(\Psi-\lambda_3)(\Psi-\lambda_4)$ and $\lambda_1>\lambda_2>\lambda_3>\lambda_4$, we find solutions as

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_2+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)\mathrm{sn}^2\left[\pm\frac{\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2Q}\left[k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)-\xi_0\right],m\right]}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{56}$$

where

$$m^2=\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}. \tag{57}$$

We note that λ_i $(i=1,\ldots,4)$ are the roots of the polynomial equation

$$\Lambda(\Psi)=0. \tag{58}$$

When $\tau_0^{(l)}=-\tau_1^{(l)}\lambda_1$ and $\xi_0=0$, the solutions (51)–(55) are reduced to plane wave solutions

$$q^{(l)}(x,t)=\left\{\pm\frac{\tau_1^{(l)}Q}{k\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{ln}+\delta_{ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{59}$$

$$q^{(l)}(x,t)=\left\{\frac{4\tau_1^{(l)}Q^2(\lambda_2-\lambda_1)}{4Q^2-\left[k(\lambda_1-\lambda_2)\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]^2}\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{\ln}+\delta_{\ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{60}$$

singular soliton solutions

$$q^{(l)}(x,t)=\frac{\tau_1^{(l)}(\lambda_2-\lambda_1)}{2}\left\{1\mp\coth\left[\frac{k(\lambda_1-\lambda_2)}{2Q}\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]\right\}^{\frac{1}{2}}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{\ln}+\delta_{\ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{61}$$

and bright soliton solutions

$$q^{(l)}(x,t)=\left\{\frac{A_2}{\left(C_2+\cosh\left[B_3\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]\right)^{\frac{1}{2}}}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{\ln}+\delta_{\ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{62}$$

where

$$A_2=\left(\frac{2\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{\lambda_3-\lambda_2}\right)^{\frac{1}{2}},\quad B_3=\frac{k\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{Q},\quad C_2=\frac{2\lambda_1-\lambda_2-\lambda_3}{\lambda_3-\lambda_2}. \tag{63}$$

Here, A_2 is the amplitude of the solitons, while B_3 is the inverse width of the solitons. So, the solitons exist for $\tau_1^{(l)}<0$. Furthermore, when $\tau_0^{(l)}=-\tau_1^{(l)}\lambda_2$ and $\xi_0=0$, Jacobi elliptic function solutions (56) can be reduced to

$$q^{(l)}(x,t)=\left\{\frac{A_3}{\left(C_3+\mathrm{sn}^2\left[B_j\left(x-\left\{\frac{b_l\omega-2a_l\kappa}{1-b_l\kappa}\right\}t\right),\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\right]\right)^{\frac{1}{2}}}\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_2+b_lk^2\nu\mu_2-2\chi_0\tau_0^{(l)}\left\{3c_l+4\tau_0^{(l)}\mathcal{H}_3+3\sum_{n\neq l}^{N}(\alpha_{\ln}+\delta_{\ln})\right\}}{4\chi_0(b_l\kappa-1)}\right)t+\theta\right\}\right], \tag{64}$$

where

$$A_3=\left(\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_1-\lambda_4}\right)^{\frac{1}{2}},\quad C_3=\frac{\lambda_4-\lambda_2}{\lambda_1-\lambda_4},\quad B_j=\frac{(-1)^jk\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2Q}\ \text{ for } j=4,5. \tag{65}$$

Remark-3. When the modulus $m \to 1$, a second form of singular optical soliton solutions fall out

$$q^{(l)}(x,t) = \left\{ \frac{A_3}{\left(C_3 + \tanh^2\left[B_j\left(x - \left\{\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]\right)^{\frac{1}{2}}} \right\} \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_2 + b_l k^2 \nu\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)} \right) t + \theta \right\} \right], \tag{66}$$

where $\lambda_3 = \lambda_4$.

Remark-4. However, if $m \to 0$, periodic singular waves emerge

$$q^{(l)}(x,t) = \left\{ \frac{A_3}{\left(C_3 + \sin^2\left[B_j\left(x - \left\{\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right\}t\right)\right]\right)^{\frac{1}{2}}} \right\} \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_2 + b_l k^2 \nu\mu_2 - 2\chi_0\tau_0^{(l)}\left\{3c_l + 4\tau_0^{(l)}\mathcal{H}_3 + 3\sum_{n\neq l}^{N}(\alpha_{\ln} + \delta_{\ln})\right\}}{4\chi_0(b_l\kappa - 1)} \right) t + \theta \right\} \right], \tag{67}$$

where $\lambda_2 = \lambda_3$.

3. Conclusions

This paper recovered optical soliton solutions to DWDM systems that is studied with FWM. Two types of nonlinear media are considered and they are with Kerr law and parabolic law. Bright and singular soliton solutions are recovered by the extended trial equation method that is implemented in this paper. A few additional forms of wave solutions also emerged as a byproduct of the integration scheme. These are plane waves and Jacobi's elliptic functions. These doubly periodic solutions, in the limiting case of modulus of ellipticity, led to periodic singular solutions and singular soliton solutions.

The results of this paper are thus very encouraging. Other integration schemes will be applied to secure soliton solutions to DWDM systems with FWM. Moreover, in future, it is necessary to consider additional features such as conservation laws, time-dependent coefficients, stochastic coefficients, fractional temporal and spatial evolution and others. All of these results are on the horizon, only to be fully visible in no time.

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References

[1] J. Du, Y. Xu, F. Han, W. Yang. Impact of Quasi-Linear Pulse Evolution on FWM Noise in Dispersion-Managed DPSK DWDM Systems. To appear in Optik.
[2] M. Ekici, M. Mirzazadeh, M. Eslami, Solitons and other solutions to Boussinesq equation with power law nonlinearity and dual dispersion, Nonlinear Dyn. 84 (2) (2016) 669–676.
[3] Y. Gurefe, E. Misirli, A. Sonmezoglu, M. Ekici, Extended trial equation method to generalized nonlinear partial differential equations, Appl. Math. Comput. 219 (2013) 5253–5260.
[4] C.-S. Liu, Representations and classification of traveling wave solutions to sinh-Gordon equation, Commun. Theor. Phys. 49 (1) (2008) 153–158.
[5] C.-S. Liu, Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations, Comput. Phys. Commun. 181 (2) (2010) 317–324.
[6] M. Mirzazadeh, M. Eslami, M. Savescu, A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, A. Biswas, Optical solitons in DWDM system with spatio-temporal dispersion, J. Nonlinear Opt. Phys. Mater. 24 (1) (2015), 1550006 (28 pages).

[7] M. Mirzazadeh, M. Eslami, Q. Zhou, A. Biswas, M. Belic, Optical solitons in DWDM system by G'/G-expansion scheme, J. Optoelectron. Adv. Mater. 17 (3–4) (2015) 499–504.
[8] M. Savescu, A.A. Alshaery, E.M. Hilal, A.H. Bhrawy, Q. Zhou, A. Biswas, Optical solitons in DWDM system with four-wave mixing, Optoelectron. Adv. Mater.-Rapid Commun. 9 (1–2) (2015) 14–19.
[9] M.Z. Ullah, A. Biswas, S.P. Moshokoa, Q. Zhou, M. Mirzazadeh, M. Belic, Dispersive optical solitons in DWDM systems, Optik 132 (2017) 210–215.
[10] Q. Zhou, M. Ekici, A. Sonmezoglu, M. Mirzazadeh, M. Eslami, Optical solitons with Biswas-Milovic equation by extended trial equation method, Nonlinear Dyn. 84 (4) (2016) 1883–1900.