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Extending the application of fuzzy COPRAS to optimal product design

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Abstract

The availability of customized and multifunctional designs of a particular product in the market due to varieties in users' requirement and demand necessitates that product design must be properly done before progressing to developmental stage. This implies that the importance of decision making in the selection of optimal conceptual design cannot be overstressed. The main contribution of this article is the implementation of Fuzzy COPRAS (COmplex PRoportional ASsessment) model as a decision tool for identification optimal design concept. The weights of the design features and sub-features were determined in the form linguistic terms generated from a triangular fuzzy number in order to avoid apportioning of crisp values in the decision process. An analytical approach was applied by creating a sub-aggregate matrix for each design feature, considering all the design alternatives in order to develop a sub-decision matrix. The relative significance or priorities of the design alternatives obtained from the maximization and minimization indices are defuzzified by applying the minimum degree of possibility in order to rank the design alternatives. The findings from the application of the Fuzzy COPRAS methodology to four design alternatives of pipe bending machines using eight design features and several sub-features indicates that the methodology can identify an optimal design concept from a set of alternative designs. The application of the methodology also shows that there are slight differences in the priority values of design alternatives next to the optimal design concept. The implication and recommendation from the application is that choosing any of the design alternatives other than the optimal design concept will provide the same risk

Keywords: Multicriteria Decision Making; Fuzzy COPRAS; Conceptual design and decision making; Pipe bending machine; Design concept selection

1. Introduction

The dynamic nature of the market as a result of varieties in users' customization still remains a challenge to manufacturers. Manufacturers are usually faced with the challenge of making decision on optimal design of a product particularly at the conceptual phase due to diverse design features required by the customers. In order to meet up with the emerging technologies and have a larger share of the competitive market, intensified efforts are needed in the conceptual design of products, particularly at the concept generation stage and decision making on the optimal conceptual design. The importance of concept selection in the design phase of a product cannot be overemphasized because it plays a key role in the achievement of optimal conceptual design [1]. In order to effectively select an optimal design from a set of alternative designs it is necessary to consider information that are pertaining to design requirements, design features and sub-features [2]. The essence of considering all the information in the concept selection phase is to improve decision-making process because it is the desire of the manufacturer to have a product with extended design life cycle that can be introduced to the dynamic market flooded with various designs. Hence, the process of selecting optimal design concept is expected to produce effective results if a holistic approach is applied in the decision process. This approach involves the consideration of design features and sub-features. [3]-[4]. However, the approach also require decision making models that goes beyond the conventional concept selection process because of the multifarious units of

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the design features and their sub-features [5]-[6]. One of the major challenges in the decision process is apportioning of weights to the design features and sub features without bias. Another challenge is how to obtain a decision matrix that will contain the relative performance of the design concepts considering their design features [7]-[8]. Since the conventional design concept selection process cannot provide an all-inclusive computational process considering different units of the design features and sub features, there is a need to analyze the selection of design concept as a Multi-Criteria Decision-Making model (MCDM) in order to solve the challenges of apportioning weights to design features and developing a decision matrix [9]-[10].

The MCDM models have been applied in different field of engineering and can be classified as Multi-Objective Decision Making models (MODM) and Multi-Attribute Decision Making models (MADM) [11]. The MADM models are suitable for decision making when the relative importance of the design features is considered in the optimal design. In this case, the design alternatives are evaluated by considering their performance in terms of the design features. The weights of the design features are decided by the design engineer based on the expected performance of the optimal design [5]. Some examples of the MADM models are Weighted Decision Matrix (WDM) [4], Analytic Hierarch Process (AHP) [12], COmplex PRoportional ASsessment (COPRAS) [13]-[14], Additive Ratio ASsessment (ARAS) [15]-[16] among others. The MADM models are applied in situations involving both the design features and their sub-features. The dimensions and units of the design features and sub features are considered in the decision process and their weights are sometimes determined by design experts serving as the decision team. In order to cater for the multifarious dimensions and units of the design features and their sub features, the MADM employs the fuzzy sets theory because it is difficult to ascribe crisp values to them [17]. In view of this, MADM models can be fuzzified to form robust decision models. Examples of these models are Fuzzy TOPSIS (Technique for Order Preference by similarity to Ideal Solution) Fuzzy VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) and lots more. Also, depending on the nature of the decision process two or more MADM models can also be hybridized to obtain a robust MADM model [9].

In order to highlight the contribution and motivation for this article, it is evident to know that the COPRAS method as a MCDM tool have gained application in different field of decision making as identified [13]-[14]. In view of this, it will be interesting to investigate if this MCDM method is also suitable for identification of optimal design considering the feature of its methodology that separates decision criteria into cost and beneficial categories. It is worthwhile to know that other MCDM tools have been applied to identify optimal design concept [1]-[6], [8], [18]-[21]. However, the choice of which tool to apply depends on the importance of the outcome of the decision process and the nature of the design features and requirements. Considering situations where the optimal design is to be identified in order to establish cost requirements and beneficial features of the product, it is necessary to investigate the suitability of a MCDM tool such as COPRAS. This is necessary in order to predict the performance of the product at the design stage and throughout the product life cycle. Concerning the multidimensional nature of design features and the complications of apportioning crisp values to evaluate the performance of design features, it is necessary to introduce the fuzzy membership function into the COPRAS methodology. Aside from fuzzy set, the rough number theory can also be applied but the fuzzy membership function is suitable considering its ability to categorize a decision output into several members within a set rather than allotting a specific value. Another reason for consideration of fuzzy membership function is that, it will neutralize the units of measurements that comes with the cost and beneficial features of the design requirements. In essence, all the design features will be normalized into the membership function thereby removing the effect of their units which the decision experts may find difficult relating together.

Two major tasks involved in the decision-making process for identification of optimal design concept are assigning weights to design features and sub-features and development of a decision matrix to represent the performance of the design alternatives relative to the design features [7], [22]. Methods applied by researchers to address the challenge of apportioning weights to design features and sub-features includes development of pairwise comparison and preference matrices by several decision makers or design experts [3]. Also, a method to obtain the decision matrix is to evaluate the availability of the sub-features in each of the design concepts considering their weights. Selecting an effective decision-making model is an important task that will be carried out after obtaining the decision matrix. Several attempts have been made to improve the integrity of the computational process and optimize the decision-making process. One of the attempts is hybridizing two or more MADM models. This provides good results [23]-[24] but with the challenge of computational complexity. Hence, there is a need to continually explore the suitability of several decision-making models in order to identify the MADM model with less computational complexity. However, the choice of decision model is a function of the nature of the design features and sub-features and importance attached to the outcome of the decision process. It is necessary to make a choice on the decision model because application of various decision models will provide different or relatively similar solutions [6].

Further, the decision process for identification of optimal design concept requires that the design engineer weighs and integrates the importance of several design features and sub-features in order to satisfy requirements of the customers. These features are different entities having different units and dimensions. The Fuzzified multicriteria decision making models have been applied in all facet of life that requires decision making on optimal choice. Fuzzy COPRAS is an example of these fuzzified multicriteria decision-making models. Some of the areas of application of fuzzy COPRAS includes; risk analysis of critical infrastructures [25], social media platform selection [26], machine tool evaluation [27], supplier evaluation [28], site selection [13], and performance measurement of production maintenance [29]-[30]. However,

to the best of knowledge of the authors, an extension of the fuzzy COPRAS for decision making on optimal design has not been considered. Hence, there is a need to explore the application of fuzzy COPRAS considering its computational capability in terms classifying decision criteria into cost and beneficial features. In essence, considering the literature survey, the application and demonstration of fuzzy COPRAS in the identification of optimal design concept has not been considered and this will be the area of concentration of this article.

To this end, it can be stated that, there is a need to research on the suitability of fuzzy COPRAS for identification of optimal design concept in order to address the challenges of design feature characterization and apportioning of crisp values in the decision process. This is necessary because, there is a need to identify the optimal design concept from a set of alternative designs before an initial fabrication can be done. This will assist the design engineers to save cost in fabrication because, identification of design features in the selection process would have provided a holistic view of the design before commencement of fabrication. Also, complications that can occur in the fabrication process of the product would have been identified in the concept selection process. In essence the application of fuzzy COPRAS will provide an avenue for the separation of the design features into cost and beneficial features which will solve the problem of determining which design feature is to be minimized or maximized. In order to avoid prejudice apportioning of weights and values, three design experts view is proposed in the computation in order to provide preference to the relative contribution of the sub-features to the main design features and availability of sub-features in the design alternatives. The decision process is fuzzified in order to avoid assigning crisp values to diverse dimensions of the design features and sub-features and the performance indices of the design alternatives are defuzzified using the minimum degree of possibility.

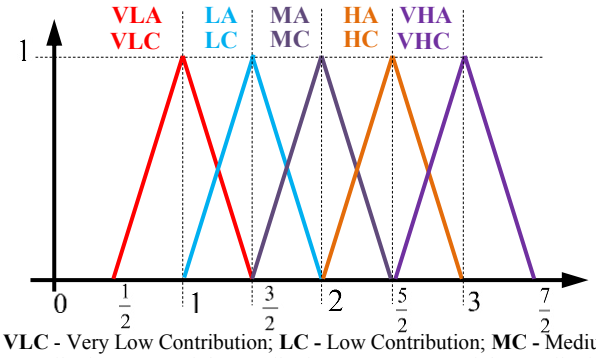
2. Methodology

2.1. Fuzzy COPRAS applied to selection of optimal conceptual design

For clarity of analysis, let there be m number of design concepts (DCm) to be evaluated using n number of design features (DFn). These design features can be further characterized by i sets of sub-features (Sf) which varies according to the number of sub-features under a design feature. It is desired to obtain the weights of the design features (Wdf) from the relative contributions, importance, and performance of their sub-features. In this article, the weights of the sub-features are derived from design experts view and opinion considering their relevance in the optimality of the design features. Also, the weights of the design features are cumulative weights of the weights of the sub-features. This implies that, the weight of the design feature depends on the weights of its sub-features. In order to determine the weights of the sub-features, a linguistic scale is allotted to membership functions of a Triangular fuzzy Number (TFN) to describe the availability and contributions of the sub-features to the design features as presented in Fig. 1. An expert's opinion on the relative contribution of the sub-feature to its linked design feature can be represented by a fuzzified matrix containing TFNs which membership function $\mu_m(y)$ is contained in [0 1] as defined in equation 1 [31].

$$\mu_m(y)=\begin{cases}0 & y<1,\\ \dfrac{1}{b-a}y-\dfrac{a}{b-a} & y\in[a\quad b],\\ \dfrac{1}{b-c}y-\dfrac{c}{b-c} & y\in[b\quad c],\\ 0 & y>c\end{cases} \tag{1}$$

In equation 1, $a\le b\le c$ and *a, b* and *c* represent the lower, modal and upper values of the fuzzy number *M* respectively. A matrix containing the ratings of design expert's decisions from which the weights of the sub features (W_{sf}) and design features (W_{df}) can be obtained is presented in equation 2.



VLC - Very Low Contribution; **LC** - Low Contribution; **MC** - Medium Contribution; **HC** - High Contribution; **VHC** - Very High Contribution
VLA - Very Low Availability; **LA** - Low Availability; **MA** - Medium Availability; **HA** - High Availability; **VHA** - Very High Availability

Fig.1. Membership Function for contributions of sub features to design features and availability of features

$$\begin{array}{c|ccccccc} & S^n_{F1} & S^n_{F2} & S^n_{F3} & \cdots & S^n_{Fi} & C^{dfn}_j & W_{dfn}\\ DE_1 & d\tilde{E}^{n,1}_1 & d\tilde{E}^{n,2}_1 & d\tilde{E}^{n,3}_1 & \cdots & d\tilde{E}^{n,i}_1 & \cdots & \\ DE_2 & d\tilde{E}^{n,1}_2 & d\tilde{E}^{n,2}_2 & d\tilde{E}^{n,3}_2 & \cdots & d\tilde{E}^{n,i}_2 & \cdots & \\ DF_n \quad \vdots & \vdots & \vdots & \vdots & & \vdots & & \cdots\\ DE_j & d\tilde{E}^{n,1}_j & d\tilde{E}^{n,2}_j & d\tilde{E}^{n,3}_j & \cdots & d\tilde{E}^{n,i}_j & \cdots & \\ W^{dfn}_{sf} & \cdots & \cdots & \cdots & & \cdots & & \end{array} \tag{2}$$

In equation 2, $d\tilde{E}^{n,i}_j$ is a TFN representing rating of the j[th] design expert for sub feature *i* of design feature *n*. C^{dfn}_j, W_{dfn} and W^{dfn}_{sf} represents the cumulative of ratings of j[th] design expert, weight of design feature *n* and weight of sub feature *i* allotted to design feature *n* respectively. From equation 2 C^{dfn}_j, W_{dfn} and W^{dfn}_{sf} can be obtained from equations 3 to 5 respectively.

$$C^{dfn}_j=\sum_{i=1}^{i=i} d\tilde{E}^{n,i}_j \Big|^{\forall\ n\ =\ 1,\ 2,\ 3....\ n}_{\forall\ j\ =\ 1,\ 2....\ j} \tag{3}$$

$$W_{sf}^{dfn} = \left. \frac{\sum_{j=1}^{j=j} d\tilde{E}_j^{n,i}}{\sum j} \right|_{\forall\, i = 1,\, 2 \ldots i}^{\forall\, n = 1,\, 2,\, 3 \ldots n} \tag{4}$$

$$W_{dfn} = \sum_{i=1}^{i=i} W_{sf}^{dfn}\Big|_i = \left. \frac{\sum_{j=1}^{j=j} C_j^{dfn}}{\sum j} \right| \forall\, n = 1,\ 2,\ 3 \ldots\ldots n \tag{5}$$

Following the membership function described in Figure 1, it can be deduced that, a fuzzified sub-decision matrix can also be developed from the design experts view to represent the availability of sub-features in the design concepts using the weights of the sub-features. A form of this sub-decision matrix is presented in equation 6.

$$\begin{array}{cccccccc} & & DC_1 & & DC_2 & & DC_m & \\ D_F & S_F & DE_1 & DE_j & DE_1 & DE_j & DE_1 & DE_j \\ & \tilde{W}_{sf}^{1,n} & d\tilde{E}_{1,n}^{1,1} & d\tilde{E}_{1,n}^{j,1} & d\tilde{E}_{2,n}^{1,1} & d\tilde{E}_{2,n}^{j,1} & d\tilde{E}_{m,n}^{1,1} & d\tilde{E}_{m,n}^{j,1} \\ & \tilde{W}_{sf}^{2,n} & d\tilde{E}_{1,n}^{1,2} & d\tilde{E}_{1,n}^{j,2} & d\tilde{E}_{2,n}^{1,2} & d\tilde{E}_{2,n}^{j,2} & d\tilde{E}_{m,n}^{1,2} & d\tilde{E}_{m,n}^{j,2} \\ \tilde{W}_{dfn} & \tilde{W}_{sf}^{3,n} & d\tilde{E}_{1,n}^{1,3} & d\tilde{E}_{1,n}^{j,3} & d\tilde{E}_{2,n}^{1,3} & d\tilde{E}_{2,n}^{j,3} & d\tilde{E}_{m,n}^{1,3} & d\tilde{E}_{m,n}^{j,3} \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & \tilde{W}_{sf}^{i,n} & d\tilde{E}_{1,n}^{1,i} & d\tilde{E}_{1,n}^{j,i} & d\tilde{E}_{2,n}^{1,i} & d\tilde{E}_{2,n}^{j,i} & d\tilde{E}_{m,n}^{1,i} & d\tilde{E}_{m,n}^{j,i} \\ & \left[\tilde{A}_g\right]_m^j & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \left[\tilde{A}\right]_m^n & & & & & & \end{array} \tag{6}$$

In equation 6, $d\tilde{E}_{m,n}^{j,i}$ is a TFN representing the decision of j[th] design expert for m[th] design concept, considering i[th] sub feature relative to n[th] design feature. Where $W_{sf}^{i,n}$ is the weight corresponding to the i[th] sub feature that can be obtained from equation 5. Also, $\left[\tilde{A}_g\right]_m^j$ and $\left[\tilde{A}\right]_m^n$ are the aggregate TFN for m[th] design concept considering the decision of j[th] design expert and overall TFN for m[th] design concept relative to n[th] design feature respectively $\left[\tilde{A}_g\right]_m^j$ and $\left[\tilde{A}\right]_m^n$ can be obtained from equations 7 and 8 respectively.

$$\left[\tilde{A}_g\right]_m^j\Big|_{\forall\, n = 1,\, 2,\, 3 \ldots n} = \sum_{i=1}^{i=i}\left[\tilde{W}_{sf}^{i,n} * d\tilde{E}_{m,n}^{j,i}\right]\Big|_{\forall\, m = 1,\, 2,\, 3 \ldots m}^{\forall\, j = 1,\, 2,\, 3 \ldots j} \tag{7}$$

$$\left[\tilde{A}\right]_m^n = \left. \frac{\left[\tilde{A}_g\right]_m^j}{\sum j} \right|_{\forall\, m = 1,\, 2,\, 3 \ldots m}^{\forall\, n = 1,\, 2,\, 3 \ldots n} \tag{8}$$

The overall aggregates obtained from equation 8 can be harnessed to develop the decision matrix alongside the weights of the design features as presented in equation 9. In order to implement the fuzzy COPRAS decision model, the design features are classified into cost (c) and beneficial (b) features. The fuzzified decision matrix is normalized using equations 10 and 11 [32]-[34], and the normalized decision matrix is weighted using the normalized weight of the design features. A form of the weighted normalized decision matrix is presented in equation 12 [30].

$$\begin{array}{ccccc} & \tilde{W}_{df1} & \tilde{W}_{df2} & \cdots & \tilde{W}_{dfn} \\ DC_1 & \left[\tilde{A}\right]_1^1 & \left[\tilde{A}\right]_1^2 & \cdots & \left[\tilde{A}\right]_1^n \\ DC_2 & \left[\tilde{A}\right]_2^n & \left[\tilde{A}\right]_2^2 & \cdots & \left[\tilde{A}\right]_2^n \\ \vdots & \vdots & \vdots & & \vdots \\ DC_m & \left[\tilde{A}\right]_m^1 & \left[\tilde{A}\right]_m^2 & \cdots & \left[\tilde{A}\right]_m^n \end{array} \tag{9}$$

$$\left(z_{ij}\right)_N = \left[\frac{a_{ij} - a_j^{\text{Min}}}{\Delta_{\text{Min}}^{\text{Max}}},\ \frac{b_{ij} - a_j^{\text{Min}}}{\Delta_{\text{Min}}^{\text{Max}}},\ \frac{c_{ij} - a_j^{\text{Min}}}{\Delta_{\text{Min}}^{\text{Max}}}\right], \tag{10}$$

$$i = 1,\ldots\ldots n;\ j \in \Omega_b$$

$$\left(z_{ij}\right)_N = \left[\frac{c_{ij} - a_j^{\text{Max}}}{\Delta_{\text{Min}}^{\text{Max}}},\ \frac{b_{ij} - a_j^{\text{Max}}}{\Delta_{\text{Min}}^{\text{Max}}},\ \frac{a_{ij} - a_j^{\text{Max}}}{\Delta_{\text{Min}}^{\text{Max}}}\right], \tag{11}$$

$$i = 1,\ldots\ldots n;\ j \in \Omega_c$$

In equations 10 and 11, $a_j^{\text{Min}} = \text{Min}\, a_{ij}$ and $c_j^{\text{Max}} = \text{Max}\, c_{ij}$ for $i = 1,\ldots\ldots n$ $\Delta_{\text{Min}}^{\text{Max}} = c_j^{\text{Max}} - a_j^{\text{Min}}$. Also, Ω_b and Ω_c are sets of benefit and cost attributes respectively.

$$\begin{array}{ccccc} DC_1 & \left[\tilde{W}_{df1} * \tilde{A}\right]_1^1\Big|_N & \left[\tilde{W}_{df2} * \tilde{A}\right]_1^2\Big|_N & \cdots & \left[\tilde{W}_{dfn} * \tilde{A}\right]_1^n\Big|_N \\ DC_2 & \left[\tilde{W}_{df1} * \tilde{A}\right]_2^n\Big|_N & \left[\tilde{W}_{df2} * \tilde{A}\right]_2^2\Big|_N & \cdots & \left[\tilde{W}_{dfn} * \tilde{A}\right]_2^n\Big|_N \\ \vdots & \vdots & \vdots & & \vdots \\ DC_m & \left[\tilde{W}_{df1} * \tilde{A}\right]_m^1\Big|_N & \left[\tilde{W}_{df2} * \tilde{A}\right]_m^2\Big|_N & \cdots & \left[\tilde{W}_{dfn} * \tilde{A}\right]_m^n\Big|_N \end{array} \tag{12}$$

It is worthwhile to know that the maximization (S_{max}) and minimization (S_{min}) indices are attributed to the beneficial and cost features respectively [28]. Hence, if the design features associated with beneficial and cost features are denoted as $\tilde{W}_{dfn}^b$ and $\tilde{W}_{dfn}^c$ respectively, then from equation 12 it can be implied that the TFN representing the maximization and minimization indices can be obtained as described in equations 13 and 14 respectively [27].

$$\tilde{S}_{max} = \sum_{n=1}^{n=n_b}\left[\tilde{W}_{dfn}^b * \tilde{A}\right]_m^n\Big|_N \tag{13}$$

$$\tilde{S}_{min} = \sum_{n=1}^{n=n_c}\left[\tilde{W}_{dfn}^c * \tilde{A}\right]_m^n\Big|_N \tag{14}$$

In equations 13 and 14, n_b and n_c are the number of design features that can be classified as beneficial and cost attributes respectively. Further, the relative significance or priorities of the design concepts can be obtained from the maximization and minimization indices in the form of TFN as presented in equation 15. The TFNs of the relative

performance for the design concepts can be ranked using the minimum degree of possibility as presented in equations 16 and 17 [25].

$$Q_m = \tilde{S}_{max} + \frac{\sum_{m=1}^{m=m} S_{min}}{S_{min} * \sum_{m=1}^{m=m}\left[\frac{1}{S_{min}}\right]} \tag{15}$$

$$V\left(Q_m \geq Q_y\right) = \ heights\ \left(Q_y \cap Q_m\right) =$$

$$\begin{cases} 1 & if\ b_m \geq b_y \\ 0 & if\ a_y \geq c_m \\ \dfrac{a_y - c_m}{\left(b_m - c_m\right) - \left(b_y - a_y\right)} & \text{otherwise} \end{cases} \tag{16}$$

$$Min\ V(Q \geq Q_1,\ Q_2 \ldots\ldots\ldots\ Q_m) \tag{17}$$

The normalized weight vector $\left[Q_m\right]_N$ or the priority value for the design alternatives will be ranked in order to obtain the best design concept. This normalized weight vector can be obtained from equation 18.

$$\left[Q_m\right]_N = \frac{P'_m}{\sum_{m=1}^{m=m} P'_m} \tag{18}$$

In order to simplify the analysis, a framework for applying the fuzzy COPRAS model to decision making on optimal conceptual design using the developed equations is presented in Fig. 2.

2.2. Application to the design of pipe bending machine

In order to ascertain the suitability of the fuzzy COPRAS, it was considered for design of pipe bending machine with eight design features and several sub-features as presented in Fig. 3. The weights of the sub-features and design features are determined from the expert's view using equations 4 and 5 respectively. The fuzzy COPRAS procedure was applied using equations 6 to 15 and the minimum degree of possibility was implemented using equations 16 to 18. The results obtained from the implementation of the fuzzy COPRAS are presented in Table 1 to 11 in the Appendix.

3. Results

The relative contributions of the sub features to importance of the design features were developed from three design experts in order to arrive at the weights of the sub features and design features by applying equations 4 and 5 respectively as presented in Table 1. As presented in Tables 2 to 9, the availability of the sub features in the design concepts are also evaluated by three design experts in order to create sub decision matrices that can be harnessed and normalized alongside the normalized weights as presented in Table 10.

The aggregation of the sub decision matrices and their normalization are obtained from equations 8 and 10 respectively. Design features related to cost characteristics or having sub features related to cost are grouped to determine the minimization indices in terms of TFNs applying equation 13 while features related to beneficial characteristics are grouped to determine the maximization indices by applying equation 14 as presented in Table 11. The performance indices of the design alternatives are also obtained from equation 15 in the form of TFNs as presented in Table11. In order to rank the performance indices, the degree of possibilities and minimum degree of possibilities are obtained from equations 16 and 17. The design concepts are ranked according to the highest priority value as presented in equation 27.

Considering Table 11 and applying equation 16, the degree of possibility of $P_y = \left(a_y,\ b_y,\ c_y\right) \geq P_m = \left(a_m,\ b_m,\ c_m\right)$ can be expressed as follows;

$$V\left(\text{Concept } 1 \geq \text{Concept } 2\right) = \frac{\left(0.315 - 1.010\right)}{\left(1.010 - 2.470\right) - \left(1.096 - 0.315\right)} = 0.310 \tag{19}$$

$$V\left(\text{Concept } 1 \geq \text{Concept } 3\right) = 1 \text{ since } b_1 > b_3 \tag{20}$$

$$V\left(\text{Concept } 1 \geq \text{Concept } 4\right) = \frac{\left(0.296 - 1.010\right)}{\left(1.010 - 2.470\right) - \left(1.028 - 0.296\right)} = 0.326 \tag{21}$$

In the same manner the degree of possibilities for other design concepts can be derived and from equation 17, the minimum degree of possibility can also be obtained as follows;

$$MinV\left(\text{Concept } 1 \geq \text{Concept } 2, \text{Concept } 3, \text{Concept } 4\right) = MinV\left(0.310,\ 1,\ 0.326\right) = 0.310 \tag{22}$$

$$MinV\left(\text{Concept } 2 \geq \text{Concept } 1, \text{Concept } 3, \text{Concept } 4\right) = MinV\left(1,\ 1,\ 1\right) = 1 \tag{23}$$

$$MinV\left(\text{Concept } 3 \geq \text{Concept } 1, \text{Concept } 2, \text{Concept } 4\right) = MinV\left(0.303,\ 0.273, 0.297\right) = 0.273 \tag{24}$$

$$MinV\left(\text{Concept } 4 \geq \text{Concept } 1, \text{Concept } 2, \text{Concept } 3\right) = MinV\left(1,\ 0.316,\ 1\right) = 0.316 \tag{25}$$

In essence the weight vector for the design concepts can be written as;

$$\begin{bmatrix} Q_{\text{concept1}} \\ Q_{\text{concept2}} \\ Q_{\text{concept3}} \\ Q_{\text{concept4}} \end{bmatrix} = \begin{bmatrix} 0.310 \\ 1.000 \\ 0.273 \\ 0.316 \end{bmatrix} \tag{26}$$

From equation 18, the priority values for the design concept and ranking can be expressed as;

$$\begin{bmatrix} \left[Q_{\text{concept1}}\right]_N \\ \left[Q_{\text{concept2}}\right]_N \\ \left[Q_{\text{concept3}}\right]_N \\ \left[Q_{\text{concept4}}\right]_N \end{bmatrix} = \begin{bmatrix} 0.163 \\ 0.527 \\ 0.144 \\ 0.166 \end{bmatrix} \Rightarrow \begin{bmatrix} 3^{rd} \\ 1^{st} \\ 4^{th} \\ 2^{nd} \end{bmatrix} \tag{27}$$

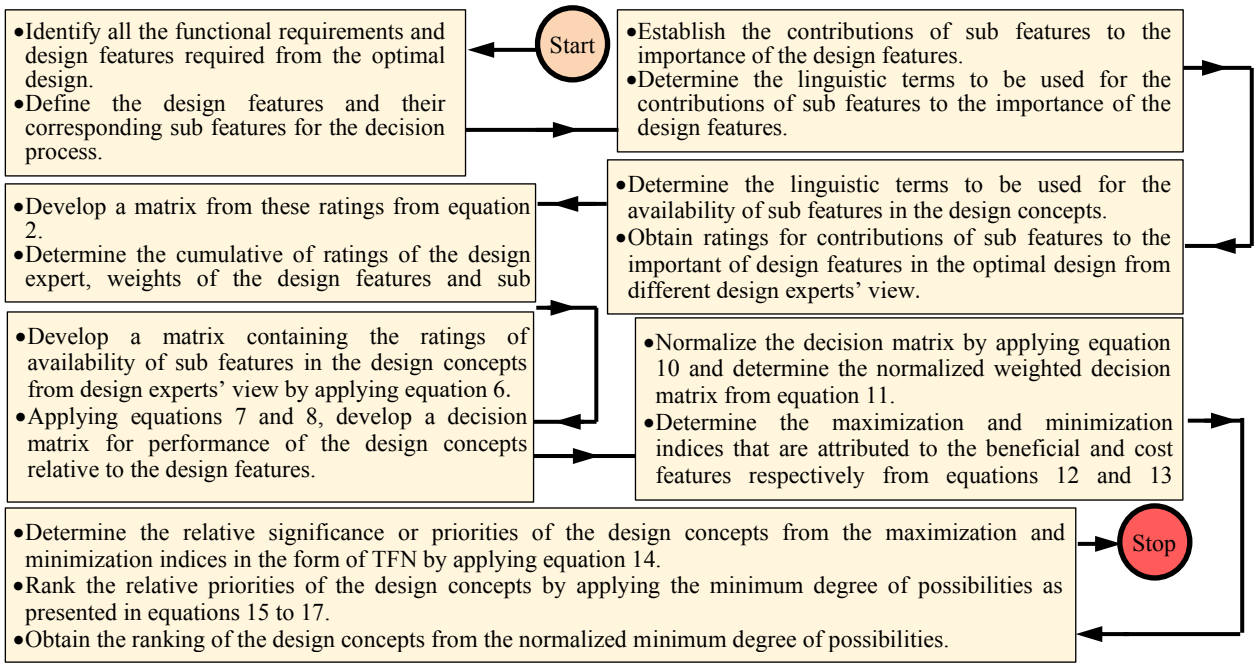


Fig. 2 Framework for Application of Fuzzy COPRAS to Decision Making on Conceptual Design

4. Discussion

The priority values of the design concepts show that design concept 2 is the optimal design in terms of cost and beneficial features. Apart from the observable distinction in the priority value for design concept 2, other design concepts have close priority values as presented in equation 27. This is an indication that fuzzy COPRAS can identify and make a decision on optimal design concept from a group of alternative designs. Also, the closeness of priority values for other design concepts is an indication that the risk of choosing them as an alternative to design concept two is apparently the same. Although the slight difference in the priority values of design concepts one, three and four can be used a preference to select any of the designs as an alternative to design concept two but the fact still remains that the closeness in the final priority values does not select any of these designs as an outstanding alternative to design concept two. Further, the close priority values of design concepts one, three and four can been attributed to the large number of design features and their sub features used in the comparison process because the fact that a design concept is good considering a sub feature does not mean that other designs are have nothing to offer but a major determinant factor is the contribution of the sub feature to the importance of the design feature in the optimal design. An outstanding advantage of the computational process is the separation of the design features into cost and beneficial characteristics because it plays a role in comparing the design concepts in terms of strengths and weaknesses in order determine their priority values. Although, the result of the case study considered in this study shows that concept two is the optimal design that has satisfactory performance in some set of cost and beneficial design features, but this result may change if the decision experts decide to introduce some other design features or constraints in the process of weighting the sub features or design features. This effect will be observed from the normalized and weighted normalized decision matrices in Tables 10 and 11 respectively. The introduction of constraints into the weighting of the design features and sub features may be as a result of market or user's response from the management team which may be related to acceptability of the product in the market. Also, the decision expert may decide to prioritize some the sub features of the cost or beneficial design features which will affect the performance of the design concepts as this can be seen from the results in Tables 2 to 9. Considering Tables 2 to 9, it is evident that the opinion or response of the decision experts are affected by the weights of the design features which have been aggregated by some other experts. This will ensure that there is no bias or prejudice in the decision process because whatever values obtained from this process will determine the values of the decision matrix and also the performance indices of the design concepts. Comparing this result with other MCDM models and hybridized models for identifying optimal design concepts, [8], [18]-[21], [35] it can be observed that the performance values of the design concepts is also a function of the weights of the design features and sub features in some cases. Although the design features and concepts may not be the same but the processes have all pointed to the fact that the decisions are based on the weights of the design features and sub features. Hence, it can be stated that the fuzzy COPRAS method can be applied to identify optimal design concept from a group of alternative designs because it provides priority values to the designs based on their performance in terms of availability of sub features in them and contributions of these sub features to the cost and beneficial requirements of the optimal design. Further, the fuzzy COPRAS method may have shown some satisfactory performance in this study, but there is a need for improvement

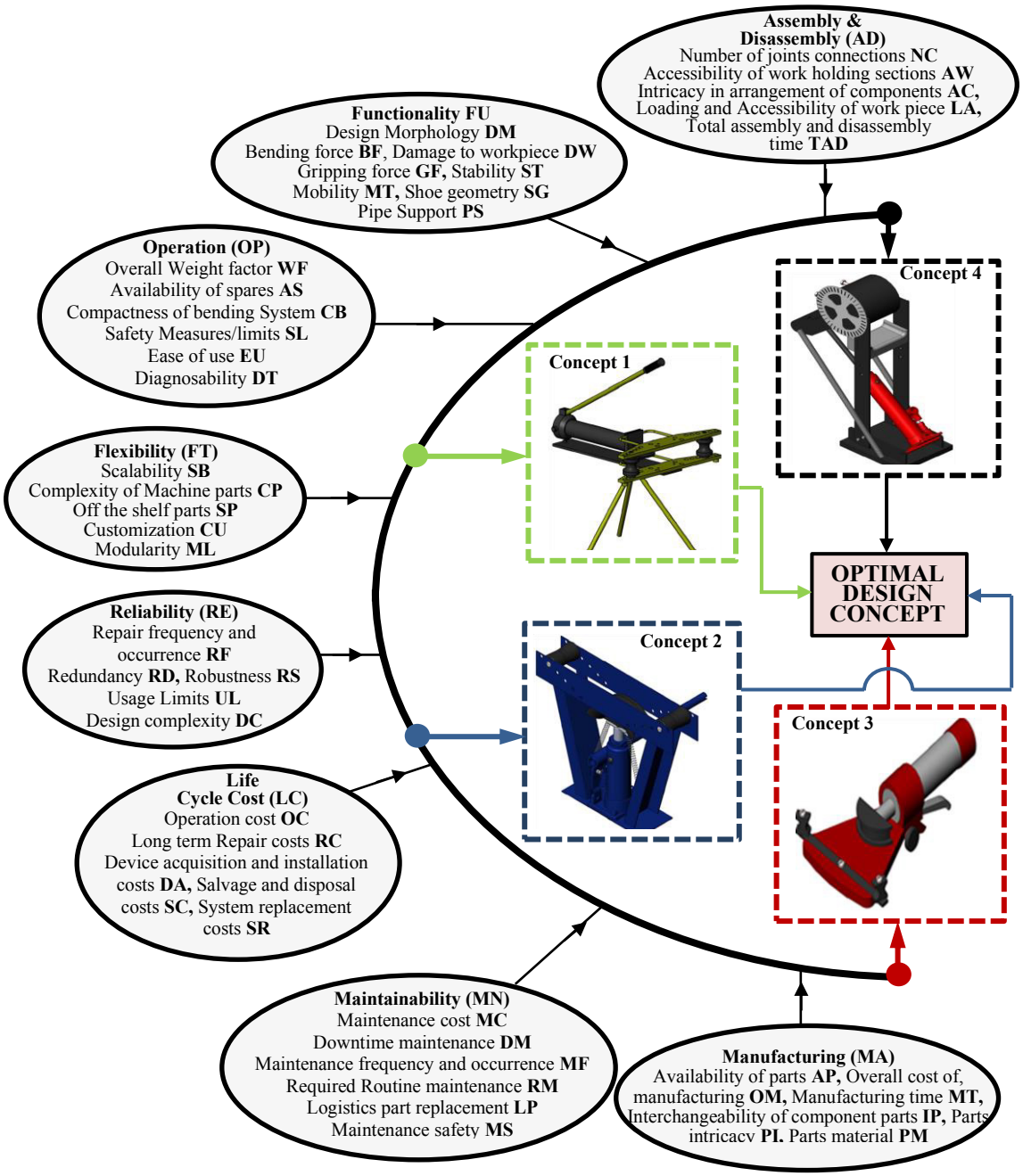


Fig. 3 Evaluating design concepts of pipe bending Machine

in the process in order to increase the credibility of the result and robustness of the decision process. These improvements can be introduced in the weighting process of the sub features and design features. rather than making the weights of the design features to be dependent of the weights of the sub features other weighting process can be introduced because it appears that a design feature with more sub feature has more weight than the one with less sub feature. This may not be valid at all times because some design features with high impact and importance for consideration in the decision process may exist with less sub features.

5. Conclusion

Selection of optimal design concept is an important task in the design phase of a product because it is an integral part of developing the best product with a splendid design life cycle before phasing out. The unique research contribution of this article is the demonstration of how fuzzy COPRAS can be applied and implemented in the selection of optimal design concept from a group of alternative designs. Results obtained from the decision process shows that the decision model is suitable for concept selection considering the results obtained from its application to design of pipe bending machine. The results obtained from the implementation of the fuzzy COPRAS has been able to identify the design concept with satisfactory performance in terms of cost and beneficial design features. This will provide an insight to the management in order to know the right product to embark upon in order to gain a substantial amount of the turbulent market. Also, the impact of this result is that the design engineer can also look at the performance of the design

concepts in terms of the cost and beneficial design features and think of a new design concept by combining two or more designs or by adding some features to existing design concepts in order to improve their performance. In essence, the result obtained from this decision process may not necessarily conclude that one design concept should be selected while other design concepts are discarded but rather, it will point to areas of improvement in the design concepts or even in the identified optimal design. However, in addition to the future work identified in the discussion section, it is still possible to improve the process by developing a model that will require the captioning of the fuzzified membership functions of design features and sub feature in order to increase the computational integrity of the decision process. Also, the Fuzzy COPRAS process can be hybridized with other MADM models and developed into application models to simplify the computational drudgery involved in the implementation of the mathematical models.

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Appendix

Table1. Determination of Weights for Design Features and Sub features

Design Features	Design Experts	Sub Features								Wdf
		DM	**BF**	**DW**	**GF**	**ST**	**MT**	**SG**	**PS**	
FU	DE1	MC	HC	VHC	HC	VHC	MC	HC	VHC	16.5 20.5 24.5
	DE2	LC	HC	HC	VHC	HC	HC	VHC	VHC	
	DE3	HC	VHC	VHC	HC	HC	HC	VHC	HC	
	Wsf	1.5 2.0 2.5	2.2 2.7 3.2	2.3 2.8 3.3	2.2 2.7 3.2	2.2 2.7 3.2	1.8 2.3 2.8	2.3 2.8 3.3	2.3 2.8 3.3	
		NC	**AW**	**AC**	**LA**	**TAD**	----	----	----	Wdf
AD	DE1	HC	HC	VHC	HC	VHC	----	----	----	10.5 13.0 15.5
	DE2	HC	MC	HC	VHC	VHC	----	----	----	
	DE3	VHC	HC	MC	HC	HC	----	----	----	
	Wsf	2.2 2.7 3.2	1.8 2.3 2.8	2.0 2.5 3.0	2.2 2.7 3.2	2.3 2.8 3.3	----	----	----	
		WF	**AS**	**CB**	**SL**	**EU**	**DT**	----	----	Wdf
OP	DE1	HC	MC	HC	MC	VHC	LC	----	----	11.3 14.3 17.3
	DE2	VHC	HC	MC	HC	HC	MC	----	----	
	DE3	VHC	MC	VHC	HC	VHC	LC	----	----	
	Wsf	2.3 2.8 3.3	1.7 2.2 2.7	2.0 2.5 3.0	1.8 2.3 2.8	2.3 2.8 3.3	1.2 1.7 2.2	----	----	
		SB	**CP**	**SP**	**CU**	**ML**	----	----	----	Wdf
FT	DE1	VHC	HC	VHC	MC	HC	----	----	----	9.7 12.2 14.7
	DE2	HC	MC	VHC	HC	MC	----	----	----	
	DE3	HC	MC	HC	HC	MC	----	----	----	
	Wsf	2.2 2.7 3.2	1.7 2.2 2.7	2.3 2.8 3.3	1.8 2.3 2.8	1.7 2.2 2.7	----	----	----	
		RF	**RD**	**RS**	**UL**	**DC**	----	----	----	Wdf
RE	DE1	HC	MC	LC	MC	MC	----	----	----	7.7 10.2 12.7
	DE2	MC	LC	LC	LC	HC	----	----	----	
	DE3	MC	HC	MC	HC	HC	----	----	----	
	Wsf	1.7 2.2 2.7	1.5 2.0 2.5	1.2 1.7 2.2	1.5 2.0 2.5	1.8 2.3 2.8	----	----	----	
		OC	**RC**	**DA**	**SC**	**SR**	----	----	----	Wdf
LC	DE1	HC	VHC	MC	HC	LC	----	----	----	8.8 11.3 13.8
	DE2	MC	HC	MC	VHC	LC	----	----	----	
	DE3	MC	VHC	LC	VHC	MC	----	----	----	
	Wsf	1.7 2.2 2.7	2.3 2.8 3.3	1.3 1.8 2.3	2.3 2.8 3.3	1.2 1.7 2.2	----	----	----	

Continued: Table1. Determination of Weights for Design Features and Sub features

Design Features / Design Experts		Sub Features								Wdf
		AP	**OM**	**MT**	**IP**	**PI**	**PM**	----	----	Wdf
MA	DE1	VHC	HC	HC	HC	VHC	MC	----	----	11.8 14.8 17.8
	DE2	VHC	VHC	HC	MC	HC	MC	----	----	
	DE3	HC	VHC	HC	MC	HC	LC	----	----	
	Wsf	2.3 2.8 3.3	2.3 2.8 3.3	2.0 2.5 3.0	1.7 2.2 2.7	2.2 2.7 3.2	1.3 1.8 2.3	----	----	
		MC	**DM**	**MF**	**RM**	**LP**	**MS**	----	----	Wdf
MN	DE1	MC	HC	MC	HC	MC	LC	----	----	10 13 16
	DE2	HC	HC	MC	HC	MC	LC	----	----	
	DE3	HC	MC	LC	VHC	HC	MC	----	----	
	Wsf	1.8 2.3 2.8	1.8 2.3 2.8	1.3 1.8 2.3	2.2 2.7 3.2	1.7 2.2 2.7	1.2 1.7 2.2	----	----	

Table 2. Availability of Sub Features of Assembly and Disassembly in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
NC (2.2 2.7 3.2)	MA	HA	HA	HA	HA	VHA	LA	LA	MA	HA	MA	MA
AW (1.8 2.3 2.8)	HA	MA	HA	HA	HA	VHA	MA	MA	HA	LA	MA	HA
AC (2.0 2.5 3.0)	MA	MA	HA	HA	HA	HA	LA	MA	MA	HA	MA	MA
LA (2.2 2.7 3.2)	HA	HA	MA	VHA	HA	HA	HA	MA	HA	MA	HA	MA
TAD (2.3 2.8 3.3)	HA	MA	MA	MA	MA	LA	HA	HA	VHA	MA	LA	MA
Sub-DM	18.5 29.4 42.9			20.5 31.9 45.8			17.3 27.9 40.9			16.4 26.8 39.8		

Table 3. Availability of Sub Features of Operation in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
WF (2.3 2.8 3.3)	MA	HA	LA	HA	HA	MA	LA	MA	MA	HA	MA	MA
AS (1.7 2.2 2.7)	HA	HA	MA	HA	VHA	HA	HA	MA	MA	HA	HA	HA
CB (2.0 2.5 3.0)	HA	MA	MA	VHA	HA	VHA	MA	LA	LA	HA	HA	MA
SL (1.8 2.3 2.8)	HA	MA	HA	HA	MA	MA	MA	LA	MA	MA	HA	HA
EU (2.3 2.8 3.3)	MA	MA	HA	VHA	HA	MA	HA	HA	VHA	HA	MA	HA
DT (1.2 1.7 2.2)	HA	MA	HA	MA	HA	MA	VHA	HA	VHA	MA	HA	MA
Sub-DM	19.3 31.6 46.9			22.2 35.2 51.2			18.4 30.6 45.8			20.5 33.1 48.6		

Table 4. Availability of Sub Features of Flexibility in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
SB (2.2 2.7 3.2)	HA	VHA	HA	MA	HA	HA	VHA	HA	VHA	HA	MA	MA
CP (1.7 2.2 2.7)	MA	MA	VHA	HA	HA	VHA	MA	MA	HA	HA	VHA	VHA
SP (2.3 2.8 3.3)	MA	LA	MA	HA	MA	MA	LA	LA	MA	MA	MA	LA
CU (1.8 2.3 2.8)	HA	MA	MA	MA	MA	LA	LA	MA	MA	HA	HA	MA
ML (1.7 2.2 2.7)	HA	HA	VHA	HA	MA	HA	VHA	HA	VHA	MA	HA	MA
Sub-DM	17.5 28.2 41.4			17.5 27.5 40.5			16.9 27.4 40.4			16.8 27.3 40.3		

Table 5. Availability of Sub Features of Reliability in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
RF (1.7 2.2 2.7)	MA	MA	HA	HA	VHA	HA	HA	HA	MA	MA	MA	LA
RD (1.5 2.0 2.5)	HA	MA	HA	MA	HA	HA	MA	MA	HA	HA	MA	MA
RS (1.2 1.7 2.2)	MA	LA	MA	VHA	HA	VHA	LA	LA	MA	MA	HA	HA
UL (1.5 2.0 2.5)	MA	MA	LA	HA	VHA	VHA	LA	MA	LA	MA	MA	LA
DC (1.8 2.3 2.8)	LA	MA	HA	VHA	VHA	MA	MA	LA	LA	VHA	HA	HA
Sub-DM	11.8 20.8 32.2			16.6 27.1 40.1			10.8 19.4 30.5			12.8 22.1 33.8		

Table 6. Availability of Sub Features of Functionality in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
DM (1.5 2.0 2.5)	HA	MA	HA	HA	VHA	VHA	LA	LA	MA	HA	HA	MA
BF (2.2 2.7 3.2)	MA	MA	LA	HA	HA	VHA	LA	MA	LA	MA	MA	LA
DW (2.3 2.8 3.3)	MA	LA	MA	LA	LA	MA	LA	LA	LA	MA	MA	LA
GF (2.2 2.7 3.2)	MA	MA	HA	HA	VHA	HA	MA	LA	MA	HA	HA	MA
ST (2.2 2.7 3.2)	MA	MA	HA	VHA	HA	HA	MA	LA	LA	MA	MA	HA
MT (1.8 2.3 2.8)	HA	VHA	HA	HA	MA	HA	VHA	HA	VHA	HA	HA	HA
SG (2.3 2.8 3.3)	MA	HA	HA	HA	VHA	VHA	MA	HA	MA	VHA	HA	MA
PS (2.3 2.8 3.3)	MA	HA	HA	VHA	HA	HA	LA	LA	MA	MA	MA	LA
Sub-DM	28.3 45.5 66.7			34.0 52.5 75.0			22.8 38.7 58.5			27.6 44.7 65.7		

Table 7. Availability of Sub Features of Life Cycle Cost in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
OC (1.7 2.2 2.7)	HA	VHA	HA	MA	MA	HA	VHA	VHA	HA	LA	MA	MA
RC (2.3 2.8 3.3)	MA	MA	HA	VHA	VHA	HA	HA	VHA	HA	MA	HA	MA
DA (1.3 1.8 2.3)	HA	HA	MA	LA	LA	MA	VHA	VHA	HA	MA	LA	MA
SC (2.3 2.8 3.3)	MA	MA	LA	VHA	HA	VHA	LA	LA	MA	HA	HA	MA
SR (1.2 1.7 2.2)	HA	MA	HA	MA	MA	LA	HA	HA	VHA	LA	MA	MA
Sub-DM	15.2	25.2	37.8	16.7	27.1	39.3	17.3	28.0	41.2	13.7	23.1	35.0

Table 8. Availability of Sub Features of Manufacturing in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
AP (2.3 2.8 3.3)	HA	HA	MA	VHA	HA	VHA	MA	MA	HA	HA	MA	MA
OM (2.3 2.8 3.3)	MA	HA	VHA	MA	MA	LA	HA	VHA	VHA	LA	MA	MA
MT (2.0 2.5 3.0)	HA	VHA	HA	MA	LA	MA	VHA	HA	VHA	MA	LA	LA
IP (1.7 2.2 2.7)	MA	HA	MA	VHA	HA	HA	LA	LA	MA	HA	MA	HA
PI (2.2 2.7 3.2)	HA	HA	MA	HA	HA	VHA	MA	MA	LA	MA	MA	HA
PM (1.3 1.8 2.3)	HA	MA	MA	HA	VHA	HA	MA	HA	MA	MA	HA	MA
Sub-DM	22.2	34.4	50.2	22.3	35.5	51.6	21.0	33.7	49.3	18.2	30.2	45.3

Table 9. Availability of Sub Features of Maintainability in the Design Concepts

Sub Features	CONCEPT 1			CONCEPT 2			CONCEPT 3			CONCEPT 4		
	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3	DE1	DE2	DE3
MC (1.8 2.3 2.8)	MA	MA	LA	HA	HA	VHA	MA	HA	HA	HA	MA	HA
DM (1.8 2.3 2.8)	MA	HA	HA	LA	MA	MA	LA	LA	MA	MA	LA	MA
MF (1.3 1.8 2.3)	HA	HA	MA	HA	VHA	HA	HA	HA	VHA	LA	MA	LA
RM (2.2 2.7 3.2)	HA	MA	HA	MA	HA	MA	MA	MA	HA	MA	HA	HA
LP (1.7 2.2 2.7)	VHA	VHA	HA	HA	VHA	HA	HA	HA	VHA	HA	VHA	HA
MS (1.2 1.7 2.2)	MA	HA	MA	HA	MA	MA	HA	VHA	HA	VHA	HA	VHA
Sub-DM	18.0	29.9	44.8	18.4	30.5	45.5	18.1	30.1	45.2	17.6	29.4	44.3

Table 10. Normalized Decision Matrix Considering the Design Features

Design Features	Design Concepts			
	Concept 1	Concept 2	Concept 3	Concept 4
FU (0.53 0.76 1.00)	0.27 0.54 0.87	0.36 0.65 1.00	0.19 0.43 0.74	0.26 0.53 0.85
AD (0.16 0.31 0.45)	0.12 0.29 0.50	0.15 0.33 0.55	0.10 0.27 0.47	0.09 0.25 0.45
OP (0.21 0.39 0.56)	0.13 0.32 0.56	0.18 0.38 0.63	0.12 0.31 0.54	0.15 0.35 0.59
FT (0.11 0.26 0.41)	0.10 0.27 0.48	0.10 0.26 0.46	0.09 0.26 0.46	0.09 0.26 0.46
RE (0.00 0.14 0.29)	0.02 0.15 0.33	0.09 0.25 0.46	0.00 0.13 0.31	0.03 0.18 0.36
LC (0.07 0.21 0.36)	0.07 0.22 0.42	0.09 0.25 0.44	0.10 0.27 0.47	0.04 0.19 0.38
MA (0.24 0.41 0.59)	0.18 0.37 0.61	0.18 0.38 0.64	0.16 0.36 0.60	0.11 0.30 0.54
MN (0.13 0.31 0.48)	0.11 0.30 0.53	0.12 0.31 0.54	0.11 0.30 0.54	0.11 0.29 0.52

Table 11. Weighted Normalized decision matrix, Maximization and Minimization TFNs and performance Indices

Nature of Features	Design Features	Design Concepts											
		Concept 1			Concept 2			Concept 3			Concept 4		
Beneficial Features	**FU**	0.145	0.413	0.868	0.192	0.496	0.997	0.099	0.331	0.740	0.139	0.403	0.852
	AD	0.020	0.089	0.226	0.025	0.101	0.247	0.016	0.082	0.213	0.014	0.077	0.204
	OP	0.028	0.125	0.315	0.038	0.147	0.353	0.025	0.119	0.305	0.032	0.134	0.330
	FT	0.012	0.070	0.193	0.011	0.067	0.187	0.011	0.067	0.187	0.011	0.067	0.186
S_{max}		0.204	0.698	1.602	0.265	0.811	1.784	0.152	0.599	1.445	0.196	0.680	1.572
Cost Features	**RE**	0.000	0.022	0.096	0.000	0.036	0.132	0.000	0.019	0.088	0.000	0.025	0.104
	LC	0.004	0.047	0.150	0.006	0.054	0.159	0.007	0.057	0.169	0.003	0.040	0.134
	MA	0.043	0.152	0.361	0.043	0.160	0.375	0.038	0.148	0.353	0.028	0.126	0.317
	MN	0.015	0.092	0.256	0.016	0.094	0.261	0.015	0.093	0.259	0.014	0.089	0.252
S_{min}		0.062	0.314	0.863	0.065	0.344	0.926	0.060	0.316	0.869	0.045	0.281	0.806
Q_m		0.257	1.010	2.470	0.315	1.096	2.592	0.206	0.909	2.306	0.269	1.028	2.500

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