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Optical soliton perturbation with full nonlinearity for Kundu–Eckhaus equation by modified simple equation method

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ABSTRACT

This paper secures dark and singular optical soliton solution to the perturbed Kundu–Eckhaus equation where the perturbation terms appear with full nonlinearity. The integration algorithm adopted in this paper is the modified simple equation approach. The solitons appear with certain parameter restrictions that are listed as constraints.

1. Introduction

The dynamics of optical soliton propagation in a dispersive media is modeled by Kundu–Eckhaus (KE) equation that belongs to the class of nonlinear Schrödinger's equation which is more commonly visible in mathematical photonics field. There has been a lot of results reported with KE equation during the past few years [1–10]. The extraction of soliton solution by the aid of Lie symmetry, method of undetermined coefficients, extended trial equation method, extended Kudryashov's method, $\tan(\phi(\xi))$-expansion method and others are reported [2,3,5,6,8]. The conservation law for this model is also given

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recently [9]. The rogue wave solutions and chaotic dynamics to this model have also been studied [1,10]. The Adomian decomposition method have been successfully implemented to address this model numerically [4]. This paper will study the perturbed KE equation where the perturbation terms are Hamiltonian and appear with full nonlinearity. The modified simple equation algorithm will be successfully implemented to obtain dark and singular soliton solutions. The constraint conditions for the existence of these solitons naturally emerge from the structure of the soliton solutions.

1.1. Governing model

The KE equation is studied in the following form:

$$iq_t + aq_{xx} + b|q|^4 q + c\left(|q|^2\right)_x q = 0. \tag{1}$$

In (1), the two independent variables are x and t that represent spatial and temporal variables respectively. The dependent variable $q(x, t)$ is the nonlinear wave profile. The first term in Eq. (1) takes care of the evolution of the wave, while the real-valued constants a, b and c represents group velocity dispersion, quintic nonlinearity, and nonlinear dispersion respectively.

In presence of perturbation terms, the KE equation that is analyzed in this paper is given by

$$iq_t + aq_{xx} + b|q|^4 q + c\left(|q|^2\right)_x q = i\left[\alpha q_x + \lambda\left(|q|^{2m} q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where α is the inter-modal dispersion, λ represents the coefficient of self-steepening for short pulses and μ is the higher-order dispersion coefficient. The parameter m conforms to full nonlinearity.

2. Quick review of modified simple equation method

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step 1: We use the transformation

$$u(x, t) = U(\xi), \quad \xi = x - ct, \tag{4}$$

where c is a constant to be determined, to reduce Eq. (3) to the following ODE:

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{5}$$

where Q is a polynomial in $U(\xi)$ and its total derivatives, while $' = d/d\xi$.

Step 2: We suppose that Eq. (5) has the formal solution

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right)^l, \tag{6}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi\left(\xi\right)$ is an unknown function to be determined later.

Step 3: We determine the positive integer N in Eq. (6) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step 4: We substitute (6) into (5), then we calculate all the necessary derivatives U', U'', ... of the unknown function $U(\xi)$ and we account the function $U(\xi)$. As a result of this substitution, we get a polynomial of $\psi'\left(\xi\right)/\psi\left(\xi\right)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}\left(\xi\right)$, $j = 0, 1, 2, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi\left(\xi\right)$. Consequently, we can get the exact solutions of Eq. (3).

3. Soliton solutions

In order to solve Eq. (2) by the modified simple equation method, we use the following wave transformation

$$q(x, t) = U(\xi)e^{i\phi(x,t)} \tag{7}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.

Substituting Eq. (7) into Eq. (2) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v = -2a\kappa - \alpha - ((2m + 1)\lambda + 2m\mu)U^{2m} \tag{8}$$

while the real part gives

$$aU'' - \left(\omega + a\kappa^2 + \alpha\kappa\right) U + 2cU^2U' - \kappa\lambda U^{2m+1} + bU^5 = 0. \tag{9}$$

Eq. (8) gives the velocity of the soliton while Eq. (9) can be integrated to determine the soliton profile. From equation (8), one can write:

$$v = -\alpha - 2a\kappa \tag{10}$$

together with the constraint condition:

$$(2m+1)\lambda + 2m\mu = 0. \tag{11}$$

These follow from the coefficients of linearly independent functions.

By using transformation $U = V^{1/2}$, Eq. (9) becomes

$$a\left(-\left(V'\right)^2 + 2VV''\right) - 4V^2\left(\omega + \alpha\kappa + a\kappa^2\right) + 4cV^2V' - 4\lambda\kappa V^{m+2} + 4bV^4 = 0 \tag{12}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (12), then we get $N = 1$. Consequently, we reach

$$V\left(\xi\right) = a_0 + a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right) \tag{13}$$

Substituting Eq. (13) into Eq. (12) along with $m = 1$ and then setting the coefficients of ψ^{-j}, $j = 0, 1, 2, 3, 4$ to zero, then we obtain a set of algebraic equations as follows:

ψ^{-4} coeff.:

$$a_1^2\left(\psi'\right)^4\left(4ba_1^2 - 4ca_1 + 3a\right) = 0, \tag{14}$$

ψ^{-3} coeff.:

$$-4a_1\left(\psi'\right)^2\left(\left(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0\right)\psi' + \left(-ca_1^2 + aa_1\right)\psi''\right) = 0, \tag{15}$$

ψ^{-2} coeff.:

$$-a_1\left(\left(4\kappa^2 aa_1 + 12\kappa\lambda a_0a_1 - 24ba_0^2a_1 + 4\kappa\alpha a_1 + 4ca_0^2 + 4\omega a_1\right)\left(\psi'\right)^2 + \left(-8ca_0a_1 + 6aa_0\right)\psi'\psi'' - 2aa_1\psi'\psi''' + aa_1\left(\psi''\right)^2\right) = 0, \tag{16}$$

ψ^{-1} coeff.:

$$-2a_0a_1\left(\left(4\kappa^2 a + 6\kappa\lambda a_0 - 8ba_0^2 + 4\kappa\alpha + 4\omega\right)\psi' - 2ca_0\psi'' - a\psi'''\right) = 0, \tag{17}$$

ψ^0 coeff.:

$$-4a_0^2\left(\kappa^2 a + \kappa\lambda a_0 - ba_0^2 + \kappa\alpha + \omega\right) = 0. \tag{18}$$

Solving this system, we obtain

$$a_0 = \frac{4f\lambda\kappa}{4ab - c^2}, \tag{19}$$

$$4ba_1^2 - 4ca_1 + 3a = 0, \tag{20}$$

$$\begin{aligned}\omega \;=& -\frac{\kappa}{4\left(4ab - c^2\right)^2}\left(-24\kappa abc\lambda^2a_1 + 32\kappa bcf\lambda^2a_1 + 4\kappa c^3\lambda^2a_1 + 64\kappa a^3b^2 - 32\kappa a^2bc^2\right.\\ &\left.+36\kappa a^2b\lambda^2 - 32\kappa abf\lambda^2 + 4\kappa ac^4 - 3\kappa ac^2\lambda^2 - 16\kappa c^2f\lambda^2 + 64a^2\alpha b^2 - 32a\alpha bc^2 + 4\alpha c^4\right),\end{aligned} \tag{21}$$

$$64bf^2 + (32bca_1 - 96ab)f - 24a_1abc + 4a_1c^3 + 36a^2b - 3ac^2 = 0, \tag{22}$$

and

$$\psi'' = -\frac{\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0}{a_1\left(-ca_1 + a\right)}\psi', \tag{23}$$

$$\psi''' = \frac{\left(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0\right)^2}{a_1^2(-ca_1 + a)^2}\psi'. \tag{24}$$

From Eqs. (23) and (24), we can deduce that

$$\psi' = -\frac{a_1(-ca_1+a)}{\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0} c_1 e^{-(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0)/a_1(-ca_1+a)\xi}, \tag{25}$$

and

$$\psi = \frac{a_1^2(-ca_1+a)^2}{\left(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0\right)^2} c_1 e^{-(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0)/a_1(-ca_1+a)\xi} + c_2, \tag{26}$$

where c_1 and c_2 are constants of integration. Substituting Eqs. (25) and (26) into Eq. (13), we obtain following exact solution to Eq. (2)

$$q(x,t) = \left\{ \frac{4f\lambda\kappa}{4ab-c^2} + a_1 \left(\frac{-(a_1(-ca_1+a)/(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0))c_1 e^{-(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0)/a_1(-ca_1+a)(x-vt)}}{a_1^2(-ca_1+a)^2/\left(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0\right)^2 c_1 e^{-(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0)/a_1(-ca_1+a)(x-vt)} + c_2} \right) \right\}^{1/2} e^{i(-\kappa x+\omega t+\theta)}$$

where a_0, a_1, ω and f are given by Eqs. (19)–(22). If we set

$$c_1 = \frac{\left(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0\right)^2}{a_1^2(-ca_1+a)^2} e^{-(\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0)/a_1(-ca_1+a)\xi_0}, \quad c_2 = \pm 1$$

we obtain:

$$q(x,t) = \left\{ \frac{4f\lambda\kappa}{2\left(4ab-c^2\right)} \left(1 \pm \tanh\left[-\frac{\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0}{2a_1(-ca_1+a)} \left(x - vt + \xi_0\right) \right] \right) \right\}^{1/2} e^{i(-\kappa x+\omega t+\theta)}, \tag{27}$$

$$q(x,t) = \left\{ \frac{4f\lambda\kappa}{2\left(4ab-c^2\right)} \left(1 \pm \coth\left[-\frac{\kappa\lambda a_1^2 - 4ba_0a_1^2 + 2ca_0a_1 - aa_0}{2a_1(-ca_1+a)} \left(x - vt + \xi_0\right) \right] \right) \right\}^{1/2} e^{i(-\kappa x+\omega t+\theta)}, \tag{28}$$

where Eqs. (27) and (28) represent dark soliton and singular soliton solutions respectively.

4. Conclusions

This paper successfully applied the modified simple equation approach to secure dark and singular optical soliton solution to the perturbed Kundu–Eckhaus equation where the perturbation terms were considered with full nonlinearity. The results appear with constraints on the soliton parameters that are needed for these solitons to exist. It is interesting to note that bright solitons are not recoverable by the aid of this integration algorithm. This proves the limitation of this integration scheme. The bright solitons for KE equation were recovered by the extended trial equation method that was reported earlier [9]. The results of this paper nevertheless carry prospect for further extension of these results. Later, stochastic perturbation terms will be considered and the mean free velocity of the soliton will be recovered after formulating the corresponding Langevin equation. This is just a tip of the iceberg.

Conflict of interest

The authors also declare that there is no conflict of interest.

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