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Chirped *w*-shaped optical solitons of Chen–Lee–Liu equation

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Propagating chirped soliton solutions for the Chen–Lee–Liu equation also called the derivative nonlinear Schrödinger II equation are investigated by application of the ansatz method. The model incorporating self-steepening term has many applications in nonlinear optical fibers and plasma physics. A nonlinear differential equation describing the evolution of the wave amplitude in the nonlinear media is derived by means of the coupled amplitude-phase formulation. Special exact chirped soliton solution that takes the shape of "*W*" is determined for the first time in presence of all physical effects. It is shown that the nonlinear chirp associated with this type of solitons is crucially dependent on the wave intensity and related to self-steepening and group velocity dispersion parameters. Parametric conditions on system parameters for the existence of the chirped soliton structure are also presented. This soliton solution exists due to a balance among group velocity dispersion and self-steepening effect solely.

1. Introduction

It is well known that the nonlinear Schrödinger (NLS) equation [1]

$$iq_t + \frac{1}{2}q_{xx} + |q|^2q = 0, \tag{1}$$

describes the optical pulse propagation in single-mode nonlinear optical fibers when the pulse width is greater than 100 femtosecond [2,3]. This equation includes only basic effects on waves such as lowest-order dispersion and lowest-order nonlinearity. It is relevant to mention that the self-phase modulation (SPM) is the nonlinear effect due to the lowest dominant nonlinear susceptibility $\chi^{(3)}$ in silica fibers [4]. The possibility of bright (dark) solitons in optical fibers is due to the exact counterbalancing between the effects of anomalous (normal) group velocity dispersion and self-phase modulation [5].

To study the effect of higher-order perturbations, various modifications and generalizations of the nonlinear Schrödinger's (NLS) equations have been proposed and studied [6–10]. Among all theoretical models, derivative nonlinear Schrödinger

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(DNLS) equations play a pivotal role in modeling the wave propagation in different physical branches. Well known DNLS equations include the Kaup–Newell equation [6]

$$iq_t + q_{xx} + i(|q|^2 q)_x = 0, \tag{2}$$

the Chen–Lee–Liu equation [7]

$$iq_t + q_{xx} + i|q|^2 q_x = 0, \tag{3}$$

and the Gerjikov–Ivanov equation [8]

$$iq_t + q_{xx} - iq^2 q_x^* + \frac{1}{2}|q|^4 q = 0. \tag{4}$$

which are usually called DNLS I, DNLS II, and DNLS III equations, respectively.

In recent years, the chirped femtosecond solitons or solitary waves became of increasing interest [11–16]. Alka et al. have presented chirped bright, dark and double-kink solitons of the cubic-quintic NLS equation with self-steepening and self-frequency shift [11]. In Refs. [12,13], bright, dark, and kink soliton solutions with nonlinear chirp have been obtained for the NLS equation with self-steepening and self-frequency shift effects. Additionally, several other higher-order NLS equations with non-Kerr nonlinear terms have been recently considered to study chirped femtosecond solitons in highly nonlinear optical fibers [14–16]. Such chirped pulses have many application in pulse compression or amplification and thus they are particularly useful in the design of fiber-optic amplifiers, optical pulse compressors, and solitary-wave-based communications links [11,17,18].

In this paper we derive, for the first time to our knowledge, exact nonlinearly chirped soliton solutions in a family of the Chen–Lee–Liu equation. In particular, analytical chirped soliton solution that takes the shape of W is obtained for the first time in presence of all physical effects. Interestingly, this W-shaped soliton structure exhibits a non-trivial phase chirping which varies as a function of the intensity of the wave. The conditions of existence of this localized structure are also presented.

2. Mathematical analysis

We consider the propagation of an optical pulse inside in a monomode fiber modeled by a family of the following Chen–Lee–Liu equation:

$$iq_t + aq_{xx} + ib|q|^2 q_x = 0, \tag{5}$$

where $q = q(x, t)$ is the complex wave function, while a and b are constants. In the context of optical fiber physics, the term involving the parameter b is usually associated with the self-steepening phenomena [19], while terms involving a is group velocity dispersion. In such contexts, the coordinates t and x denote propagation distance and retarded time, but represent slow time and spatial coordinate traveling with the group velocity in hydrodynamics, respectively [19]. In case when $a = b = 1$, Eq. (5) collapses to the case of regular Chen–Lee–Liu equation given in (3).

We are interested to find the exact nonlinearly chirped soliton solution for Eq. (5). To do so we put the unknown function of the complex field $q(x, t)$ in the traveling-wave form

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{6}$$

with the phase $\phi(x, t)$ as

$$\phi(x, t) = -\kappa x + \omega t + \theta s. \tag{7}$$

where the traveling coordinate s is given by

$$s = x - vt, \tag{8}$$

Here, in (6)–(8), $g(s)$ is real amplitude of the wave, while $\theta(s)$ is an extra phase function depending upon the traveling variable s. Also, κ is the frequency of the soliton, and ω is the wave number of the soliton.

The corresponding chirp is given by

$$\delta\omega = -\frac{\partial \phi}{\partial x} = \kappa - \theta' s. \tag{9}$$

Inserting the wave solution (6) into Eq. (5) and separating the real and imaginary parts, we obtain the following pair of coupled equations in g and θ:

$$-\omega g + v\theta' g + ag'' - ag\theta'^2 - a\kappa^2 g + 2a\kappa\theta' g - b\theta' g^3 + b\kappa g^3 = 0, \tag{10}$$

and

$$a\left(g\theta'' + 2g'\theta'\right) - vg' - 2a\kappa g' + bg^2 g' = 0, \tag{11}$$

where $g' = dg/ds$, $g'' = d^2g/ds^2$, $\theta' = d\theta/ds$, and $\theta'' = d^2\theta/ds^2$.

Multiplying both sides of (11) by g and integrating gives

$$\theta' = \kappa + \frac{v}{2a} - \frac{b}{4a}g^2 + \frac{A}{2ag^2}, \tag{12}$$

where A is an integration constant, to be fixed by initial conditions.

Relations (9)–(12) show a nontrivial dependence of the chirping on the wave intensity [with $I = |q|^2 = g^2$]. It is worth remarking that this chirp has two intensity dependent terms. It also follows from Eq. (12) that the self-steepening and GVD parameters can be used to control the amplitude of chirping.

Now substituting (12) into (10) gives

$$g'' + \frac{3b^2}{16a^2}g^5 - \frac{bv}{2a^2}g^3 + \left(\frac{v^2}{4a^2} + \frac{v\kappa}{a} - \frac{\omega}{a} - \frac{bA}{4a^2}\right)g - \frac{A^2}{4a^2g^3} = 0, \tag{13}$$

Multiplying of Eq. (13) by g' and integrate with respect to s, we obtain the following auxiliary ordinary differential equation

$$g'^2 + \frac{b^2}{16a^2}g^6 - \frac{bv}{4a^2}g^4 + \left(\frac{v^2}{4a^2} + \frac{v\kappa}{a} - \frac{\omega}{a} - \frac{bA}{4a^2}\right)g^2 + \frac{A^2}{4a^2g^2} = 2B. \tag{14}$$

where B is the second integration constant.

Eq. (14) is a nonlinear differential equation describing the evolution of the wave amplitude in a nonlinear medium that is governed by the Chen–Lee–Liu equation (5). Before discussing exact solutions to Eq. (14), let us rewrite it in a more simplified form. It is of interest to introduce the following change of variable $F = g^2$. Applying this transformation to Eq. (14) yields

$$F'^2 = -c_0 + c_1F + c_2F^2 + c_3F^3 - c_4F^4, \tag{15}$$

where

$$c_0 = \frac{A^2}{a^2},\ c_1 = 8B,\ c_2 = \frac{4\omega}{a} + \frac{bA}{a^2} - \frac{v^2}{a^2} - \frac{4v\kappa}{a},\ c_3 = \frac{bv}{a^2},\ c_4 = \frac{b^2}{4a^2}. \tag{16}$$

The elliptic Eq. (15) is known to admit a variety of solutions such as periodic, kink, and solitary-wave-type solutions [20,21]. However, for all the previously reported soliton or solitary wave solutions, they are found for auxiliary ordinary differential equations solved under the vanishing boundary conditions. Here we concentrate on discussing it with nonvanishing boundary conditions by employing the ansarz method. It is worth observing that the coefficients c_0 and c_1 contain the integration constants A and B, thus implying that they have effects on the shape of propagating envelopes.

3. Exact chirped *w*-shaped solitons

Having found the nonlinear dynamical equation for the wave amplitude, we turn our attention next to the question of finding chirped localized structures of the model. Here, we study localized soliton ansatz of the type

$$Fs = \beta + \rho\,\text{sech}(\mu s), \tag{17}$$

which allows for W-shaped solitons if the unknown parameters β and ρ satisfy the conditions $\beta\rho < 0$ and $|\rho| > \beta$ [22].

Substituting Eq. (17) in Eq. (15) and equating different powers of *sech* functions, we obtain the following overdetermined system of equations:

$$-c_0 + c_1\beta + c_2\beta^2 + c_3\beta^3 - c_4\beta^4 = 0, \tag{18}$$

$$\rho[c_1 + 2c_2\beta + 3c_3\beta^2 - 4c_4\beta^3] = 0, \tag{19}$$

$$\rho^2[\mu^2 - c_2 - 3c_3\beta + 6c_4\beta^2] = 0, \tag{20}$$

$$\rho^3[c_3 - 4c_4\beta] = 0, \tag{21}$$

$$\rho^2[\mu^2 - c_4\rho^2] = 0, \tag{22}$$

Solving these relations, one finds

$$\beta = \frac{c_3}{4c_4}, \tag{23}$$

$$\rho = \pm\sqrt{\frac{3c_3^2 + 8c_2c_4}{8c_4^2}}, \tag{24}$$

$$\mu = \sqrt{\frac{3c_3^2 + 8c_2c_4}{8c_4}}, \tag{25}$$

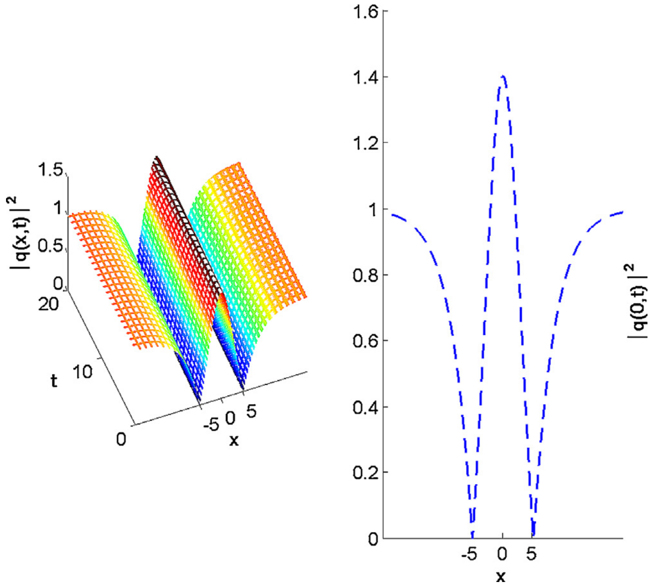

Fig. 1. Evolution of the W-shaped soliton as computed from Eq. (28) up to a distance of 20 dispersion lengths for $\beta = 1$, $\lambda = 0$, $\rho = \sqrt{6}$, $\mu = 0.3$ and $\nu = 0.033$.

and

$$c_0 = -\frac{c_3^2\left(5c_3^2 + 16c_2c_4\right)}{256c_4^3}, \quad c_1 = -\frac{c_3\left(c_3^2 + 4c_2c_4\right)}{8c_4^2}, \tag{26}$$

provided $3c_3^2 + 8c_2c_4 > 0$ and $c_4 > 0$.

Using the last consistency relations in Eq. (26), one arrives at determining the values of the integration constants A and B so that $A^2 = a^2c_0$ and $B = c_1/8$. Moreover, it follows from Eqs. (23) and (24) that we can determine the amplitude ρ in terms of the obtained soliton parameter β as

$$\rho = \pm\sqrt{6\beta^2 + \lambda^2} \tag{27}$$

with $\lambda^2 = c_2/c_4$.

Making use of these findings, we can write the exact chirped soliton solution on a continuous-wave (cw) background of Eq. (1)in the form

$$q(x,t) = \left\{\beta \pm \sqrt{6\beta^2 + \lambda^2}\,\text{sech}[\mu(x - vt)]\right\}^{1/2} e^{i[-\kappa x + \omega t + \theta s]}, \tag{28}$$

The corresponding chirping associated with this propagating envelope can be obtained readily as

$$\begin{aligned}\delta\omega(x,t) = &-\frac{v}{2a} + \frac{b}{4a}\left(\beta \pm \sqrt{6\beta^2 + \lambda^2}\,\text{sech}[\mu(x - vt)]\right)\\ &-\frac{A}{2a\left(\beta \pm \sqrt{6\beta^2 + \lambda^2}\,\text{sech}[\mu(x - vt)]\right)}.\end{aligned} \tag{29}$$

Physically, Eq. (28) describes the evolution of two different type soliton pulses on a cw background for the Chen–Lee–Liu equation (5). In these propagating envelope solutions, parameter β (determined by Eq. (23)) decides the strength of the background, in which these solutions propagate. Here we only consider soliton solution given in Eq. (28) with the sign −. The evolution of the soliton intensity as computed from Eq. (28) is shown in Fig. 1. Thus one can observe that the intensity profile of the soliton takes the shape of W and remains unchanged along a significant propagation distance. This interesting soliton shape possesses a pronounced platform underneath originating from the self-steepening effect. It is relevant to mention that this W-shaped soliton structure was first found for a higher-order NLS equation with third-order dispersion, self steepening, and self-frequency shift effects by Li et al. [22]. Generally speaking, the existence of W-shaped soliton type solution for the NLS family of equations is relatively rare in comparison with the bright and dark soliton solutions. It is worth mentioning that the soliton solution (28) with the sign + corresponds to a bright pulse solution on a cw background for the Chen–Lee–Liu equation (5).

4. Conclusions

In this paper, we have investigated a family of the Chen–Lee–Liu equation with the self-steepening effect (otherwise called the Kerr dispersion), which can be used to govern the soliton propagation in single-mode nonlinear optical fibers. We used the coupled amplitude-phase method to derive a nonlinear differential equation describing the evolution of the wave amplitude in the system. By means of an appropriate ansatz, we obtained the exact W-shaped soliton solution with nonlinear chirp for the model. It is shown that the nonlinear chirp associated with this interesting soliton structure has a nontrivial form involving two intensity dependent terms, in addition to the linear contribution. This type of chirped soliton solution exists due to a balance among group velocity dispersion and self-steepening effects solely. The stability aspects of such privileged exact chirped structures typically require detailed individual analysis based on such a balance aspect. This work is currently in progress.

Conflict of interest

The authors also declare that there is no conflict of interest.

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