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Research Article

Local Fractional Laplace Variational Iteration Method for Solving Diffusion and Wave Equations on Cantor Sets within Local Fractional Operators

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The local fractional Laplace variational iteration method (LFLVIM) is employed to handle the diffusion and wave equations on Cantor set. The operators are taken in the local sense. The nondifferentiable approximate solutions are obtained by using the local fractional Laplace variational iteration method, which is the coupling method of local fractional variational iteration method and Laplace transform. Illustrative examples are included to demonstrate the high accuracy and fast convergence of this new algorithm.

1. Introduction

Local fractional calculus has played an important role in areas ranging from fundamental science to engineering in the past ten years [1–3] and has been applied to a wide class of complex problems encompassing physics, biology, mechanics, and interdisciplinary areas [4–9]. Various methods, for example, the Adomian decomposition method [10], the Rich-Adomian-Meyers modified decomposition method [11], the variational iteration method [12], the homotopy perturbation method [13, 14], the fractal Laplace and Fourier transforms [15], the homotopy analysis method [16], the heat balance integral method [17–19], the fractional variational iteration method [20–22], and the fractional subequation method [23, 24], have been utilized to solve fractional differential equations [3, 15].

The diffusion equations are important in many processes in science and engineering, for example, the diffusion of a dissolved substance in the solvent liquids and neutrons in a nuclear reactor and Brownian motion, while wave equations characterize the motion of a vibrating string (see [25, 26] and the references therein).

The diffusion equation on Cantor sets (called local fractional diffusion equation) was recently described in [27] as

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = a^{2\alpha} \frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}}, \tag{1}$$

where $a^{2\alpha}$ denotes the fractal diffusion constant which is, in essence, a measure for the efficiency of the spreading of the underlying substance, while local fractional wave equation is written in the following form [28, 29]:

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} = a^{2\alpha} \frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}}. \tag{2}$$

The local fractional Laplace operator is given by [28, 29] as follows:

$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}. \quad (3)$$

We notice that the local fractional diffusion equation yields

$$\nabla^{2\alpha} u = \frac{1}{a^{2\alpha}} \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}}, \quad (4)$$

and the local fractional wave equation has the following form:

$$\nabla^{2\alpha} u = \frac{1}{a^{2\alpha}} \frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}}, \quad (5)$$

where $1/a^{2\alpha}$ is a constant. This equation describes vibrations in a fractal medium.

The quantity $u(x,t)$ is interpreted as the local fractional deviation at the time t from the position at rest of the point with rest position given by x, y, and z. The above fractal derivatives were considered as the local fractional operators [30, 31].

The paper is organized as follows. In Section 2, we introduce the notions of local fractional calculus theory used in this paper. In Section 3, we give the local fractional Laplace variational iteration method. Section 4 presents the solutions for diffusion and wave equations in Cantor set conditions. Section 5 is devoted to our conclusions.

2. Mathematical Fundamentals

2.1. Local Fractional Calculus (See [28, 29, 32, 33])

Definition 1. Suppose that there is the relation

$$\left|f(x) - f(x_0)\right| < \varepsilon^{\alpha}, \quad 0 < \alpha \le 1, \quad (6)$$

with $|x - x_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in R$; then the function $f(x)$ is called local fractional continuous at $x = x_0$ and it is denoted by $\lim_{x\to x_0} f(x) = f(x_0)$.

Definition 2. Suppose that the function $f(x)$ satisfies condition (6), for $x \in (a,b)$; it is so called local fractional continuous on the interval (a,b), denoted by $f(x) \in C_{\alpha}(a,b)$.

Definition 3. In fractal space, let $f(x) \in C_{\alpha}(a,b)$; local fractional derivative of $f(x)$ of order α at the point $x = x_0$ is given by

$$\begin{aligned} D_x^{\alpha} f(x_0) &= \left.\frac{d^{\alpha}}{dx^{\alpha}} f(x)\right|_{x=x_0} \\ &= f^{(\alpha)}(x_0) = \lim_{x\to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}}, \end{aligned} \quad (7)$$

where $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\alpha+1)(f(x) - f(x_0))$.

The formulas of local fractional derivatives of special functions used in the paper are as follows:

$$\begin{aligned} D_x^{(\alpha)} a g(x) &= a D_x^{(\alpha)} g(x), \\ \frac{d^{\alpha}}{dx^{\alpha}}\left(\frac{x^{n\alpha}}{\Gamma(1+n\alpha)}\right) &= \frac{x^{(n-1)^{\alpha}}}{\Gamma(1+(n-1)\alpha)}, \quad n \in N. \end{aligned} \quad (8)$$

Definition 4. A partition of the interval $[a,b]$ is denoted by (t_j, t_{j+1}), $j = 0,\ldots,N-1$, $t_0 = a$ and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \ldots\}$. Local fractional integral of $f(x)$ in the interval $[a,b]$ is given by

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^{\alpha} \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t\to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^{\alpha}. \end{aligned} \quad (9)$$

The formulas of local fractional integrals of special functions used in the paper are as follows:

$$\begin{aligned} {}_0I_x^{(\alpha)} a g(x) &= a\, {}_0I_x^{(\alpha)} g(x), \\ {}_0I_t^{(\alpha)}\left(\frac{x^{n\alpha}}{\Gamma(1+n\alpha)}\right) &= \frac{x^{(n+1)^{\alpha}}}{\Gamma(1+(n+1)\alpha)}, \quad n \in N. \end{aligned} \quad (10)$$

2.2. Local Fractional Laplace Transform and Its Inverse Formula (See [28, 29, 32])

Definition 5. Let $(1/\Gamma(1+\alpha))\int_0^{\infty}|f(x)|(dx)^{\alpha} < k < \infty$. The Yang-Laplace transforms of $f(x)$ are given by

$$\begin{aligned} L_{\alpha}\{f(x)\} &= f_s^{L,\alpha}(s) \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^{\infty} E_{\alpha}(-s^{\alpha}x^{\alpha}) f(x)(dx)^{\alpha}, \\ &\qquad 0 < \alpha \le 1, \end{aligned} \quad (11)$$

where the latter integral converges and $s^{\alpha} \in R^{\alpha}$.

Definition 6. The inverse formula of the Yang-Laplace transforms of $f(x)$ is given by

$$L_{\alpha}^{-1}\{f_s^{L,\alpha}(s)\} = f(t) = \frac{1}{(2\pi)^{\alpha}} \int_{\beta-i\omega}^{\beta+i\omega} E_{\alpha}(s^{\alpha}x^{\alpha}) f_s^{L,\alpha}(s)(ds)^{\alpha}, \quad 0 < \alpha \le 1, \quad (12)$$

where $s^{\alpha} = \beta^{\alpha} + i^{\alpha}\omega^{\alpha}$; here i^{α} is fractal imaginary unit os s and $\mathrm{Re}(s) = \beta > 0$.

2.3. Some Basic Properties of Local Fractional Laplace Transform (See [28, 29]). Let $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$ and $L_\alpha\{g(x)\} = g_s^{L,\alpha}(s)$; then we have the following formulas:

$$\begin{gathered} L_\alpha\{af(x)+bg(x)\} = af_s^{L,\alpha}(s) + bg_s^{L,\alpha}(s), \\ L_\alpha\{E_\alpha(c^\alpha x^\alpha) f(x)\} = f_s^{L,\alpha}(s-c), \\ L_\alpha\left\{f^{(k\alpha)}(x)\right\} = s^{k\alpha} f_s^{L,\alpha}(s) - s^{(k-1)\alpha} f(0) - s^{(k-2)\alpha} f^{(\alpha)}(0) - \cdots - f^{((k-1)\alpha)}(0), \\ L_\alpha\{E_\alpha(a^\alpha x^\alpha)\} = \frac{1}{s^\alpha - a^\alpha}, \\ L_\alpha\{\sin_\alpha(a^\alpha x^\alpha)\} = \frac{a^\alpha}{s^{2\alpha}+a^{2\alpha}}, \\ L_\alpha\left\{x^{k\alpha}\right\} = \frac{\Gamma(1+k\alpha)}{s^{(k+1)\alpha}}. \end{gathered} \tag{13}$$

3. Local Fractional Laplace Variational Iteration Method

Let us consider the following local fractional partial differential equations:

$$L_\alpha u(x,t) + R_\alpha u(x,t) = f(x,t), \tag{14}$$

where L_α is the linear local fractional operator, R_α is the linear local fractional operator of order less than L_α, and $f(x,t)$ is a source term of the nondifferential function.

According to the rule of local fractional variational iteration method, the correction functional for (14) is constructed as follows [34–37]:

$$u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)}\left(\frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - f(t)\right]\right), \tag{15}$$

where $\lambda(x-t)^\alpha/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier and L_α in (14) are $k\alpha$ times local fractional partial differential equations.

For initial value problems of (14), we can start with

$$u_0(x) = u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0) + \cdots + \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}u^{((k-1)\alpha)}(0). \tag{16}$$

We now take Yang-Laplace transform of (15); namely,

$$\begin{aligned} &Ł_\alpha\{u_{n+1}(x)\} \\ &= Ł_\alpha\{u_n(x)\} \\ &+ Ł_\alpha\left\{{}_0I_x^{(\alpha)}\left(\frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha \tilde{u}_n(t) + R_\alpha u_n(t) - f(t)\right]\right)\right\}, \end{aligned} \tag{17}$$

or

$$\begin{aligned} &Ł_\alpha\{u_{n+1}(x)\} \\ &= Ł_\alpha\{u_n(x)\} \\ &+ Ł_\alpha\left\{\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)}\right\} Ł_\alpha\{L_\alpha u_n(x) + R_\alpha \tilde{u}_n(x) - f(x)\}. \end{aligned} \tag{18}$$

Take the local fractional variation of (18), which is given by

$$\begin{aligned} &\delta^\alpha\left(Ł_\alpha\{u_{n+1}(x)\}\right) \\ &= \delta^\alpha\left(Ł_\alpha\{u_n(x)\}\right) \\ &+ \delta^\alpha\left(Ł_\alpha\left\{\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)}\right\} Ł_\alpha\{L_\alpha u_n(x) - R_\alpha \tilde{u}_n(x) - f(x)\}\right). \end{aligned} \tag{19}$$

By using computation of (19), we get

$$\begin{aligned} &\delta^\alpha\left(Ł_\alpha\{u_{n+1}(x)\}\right) \\ &= \delta^\alpha\left(Ł_\alpha\{u_n(x)\}\right) + Ł_\alpha\left\{\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)}\right\}\delta^\alpha\left(Ł_\alpha\{L_\alpha u_n(x)\}\right). \end{aligned} \tag{20}$$

Hence, from (20), we get

$$1 + Ł_\alpha\left\{\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)}\right\} s^{k\alpha} = 0, \tag{21}$$

where

$$\begin{aligned} &\delta^\alpha\left(Ł_\alpha\{L_\alpha u_n(x)\}\right) \\ &= \delta^\alpha\left(s^{k\alpha}Ł_\alpha\{u_n(x)\} - s^{(k-1)\alpha}u_n(0) - \cdots - u_n^{((k-1)\alpha)}(0)\right) \\ &= s^{k\alpha}\delta^\alpha\left(Ł_\alpha\{u_n(x)\}\right). \end{aligned} \tag{22}$$

Therefore, we get

$$Ł_\alpha\left\{\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)}\right\} = -\frac{1}{s^{k\alpha}}. \tag{23}$$

Taking the inverse version of the Yang-Laplace transform, we have

$$\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} = Ł_\alpha^{-1}\left\{-\frac{1}{s^{k\alpha}}\right\} = -\frac{(x)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}, \quad k \in N. \tag{24}$$

In view of (24), we obtain

$$\begin{aligned} &Ł_\alpha \{u_{n+1}(x)\} \\ &= Ł_\alpha \{u_n(x)\} \\ &- Ł_\alpha \left\{ {}_0I_x^{(\alpha)} \left(\frac{(x-t)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} \right.\right. \\ &\left.\left. \cdot [L_\alpha \tilde{u}_n(t) + R_\alpha u_n(t) - f(t)] \right) \right\}. \end{aligned} \quad (25)$$

Therefore, we have the following iteration algorithm:

$$\begin{aligned} Ł_\alpha \{u_{n+1}(x)\} &= Ł_\alpha \{u_n(x)\} \\ &- Ł_\alpha \left\{ \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} \right\} Ł_\alpha \{L_\alpha u_n(x) \\ &+ R_\alpha \tilde{u}_n(x) - f(x)\}, \end{aligned} \quad (26)$$

or

$$\begin{aligned} Ł_\alpha \{u_{n+1}(x)\} &= Ł_\alpha \{u_n(x)\} \\ &- \frac{1}{s^{k\alpha}} Ł_\alpha \{L_\alpha u_n(x) + R_\alpha \tilde{u}_n(x) - f(x)\}, \end{aligned} \quad (27)$$

where the initial value reads as follows:

$$\begin{aligned} &u_0(x) \\ &= Ł_\alpha^{-1} \left(\frac{s^{(k-1)\alpha} u(0) + s^{(k-2)\alpha} u^{(\alpha)}(0) + \cdots + u_n^{((k-1)\alpha)}(0)}{s^{k\alpha}} \right) \\ &= u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} u^{(2\alpha)}(0) \\ &+ \cdots + \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} u^{((k-1)\alpha)}(0). \end{aligned} \quad (28)$$

Thus, the local fractional series solution of (14) is

$$u(x,t) = \lim_{n\to\infty} Ł_\alpha^{-1} \left(Ł_\alpha \{u_n(x,t)\} \right). \quad (29)$$

4. Applications to Diffusion and Wave Equations on Cantor Sets

In this section, four examples for diffusion and wave equations on Cantor sets will demonstrate the efficiency of local fractional Laplace variational iteration method.

Example 1. Let us consider the following diffusion equation on Cantor set:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \quad 0 < \alpha \le 1, \quad (30)$$

with the initial value condition

$$u(x,0) = \frac{x^\alpha}{\Gamma(1+\alpha)}. \quad (31)$$

Using relation (26), we structure the iterative relation as

$$\begin{aligned} &Ł_\alpha \{u_{n+1}(x,t)\} \\ &= Ł_\alpha \{u_n(x,t)\} - Ł_\alpha \{1\} Ł_\alpha \left\{ \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} \right\} \\ &= Ł_\alpha \{u_n(x,t)\} \\ &- \frac{1}{s^\alpha} \left(s^\alpha Ł_\alpha \{u_n(x,t)\} - u_n(x,0) - \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}} \right) \\ &= \frac{1}{s^\alpha} u_n(x,0) + \frac{1}{s^\alpha} \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}}. \end{aligned} \quad (32)$$

In view of (28), the initial value reads as follows:

$$u_0(x,t) = u(x,0) = \frac{x^\alpha}{\Gamma(1+\alpha)}. \quad (33)$$

Hence, we get the first approximation; namely,

$$\begin{aligned} Ł_\alpha \{u_1(x,t)\} &= \frac{1}{s^\alpha} u_0(x,0) + \frac{1}{s^\alpha} \frac{\partial^{2\alpha} Ł_\alpha \{u_0(x,t)\}}{\partial x^{2\alpha}} \\ &= \frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{1}{s^\alpha} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} \right\} \\ &= \frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \quad (34)$$

Thus,

$$u_1(x,t) = Ł_\alpha^{-1} \left(\frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)} \right) = \frac{x^\alpha}{\Gamma(1+\alpha)}. \quad (35)$$

The second approximation reads as follows:

$$\begin{aligned} Ł_\alpha \{u_2(x,t)\} &= \frac{1}{s^\alpha} u_1(x,0) + \frac{1}{s^\alpha} \frac{\partial^{2\alpha} Ł_\alpha \{u_1(x,t)\}}{\partial x^{2\alpha}} \\ &= \frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{1}{s^\alpha} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} \right\} \\ &= \frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \quad (36)$$

Therefore, we get

$$u_2(x,t) = Ł_\alpha^{-1} \left(\frac{1}{s^\alpha} \frac{x^\alpha}{\Gamma(1+\alpha)} \right) = \frac{x^\alpha}{\Gamma(1+\alpha)} \cdots. \quad (37)$$

Consequently, the local fractional series solution is

$$\begin{aligned} u(x,t) &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(Ł_\alpha\{u_n(x,t)\}\right) \\ &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}\frac{x^\alpha}{\Gamma(1+\alpha)}\right) \\ &= \lim_{n\to\infty}\frac{x^\alpha}{\Gamma(1+\alpha)} = \frac{x^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \quad (38)$$

The result is the same as the one which is obtained by the local fractional series expansion method [38].

Example 2. Let us consider the following diffusion equation on Cantor set:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}} = 0, \quad 0<\alpha\le 1, \quad (39)$$

with the initial value conditions being as follows:

$$u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (40)$$

Using relation (26), we structure the iterative relation as follows:

$$\begin{aligned} &Ł_\alpha\{u_{n+1}(x,t)\} \\ &= Ł_\alpha\{u_n(x,t)\} \\ &\quad - Ł_\alpha\{1\} Ł_\alpha\left\{\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}\right\} \\ &= Ł_\alpha\{u_n(x,t)\} \\ &\quad - \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_n(x,t)\} - u_n(x,0) - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}Ł_\alpha\{u_n(x,t)\}}{\partial x^{2\alpha}}\right) \\ &= \frac{1}{s^\alpha}u_n(x,0) + \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}Ł_\alpha\{u_n(x,t)\}}{\partial x^{2\alpha}}. \end{aligned} \quad (41)$$

In view of (28), the initial value reads as follows:

$$u_0(x,t) = u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (42)$$

Hence, we get the first approximation; namely,

$$\begin{aligned} &Ł_\alpha\{u_1(x,t)\} \\ &= \frac{1}{s^\alpha}u_0(x,0) + \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}Ł_\alpha\{u_0(x,t)\}}{\partial x^{2\alpha}} \\ &= \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}Ł_\alpha\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{1}{s^\alpha}+\frac{1}{s^{2\alpha}}\right). \end{aligned} \quad (43)$$

Thus,

$$\begin{aligned} u_1(x,t) &= Ł_\alpha^{-1}\left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left[\frac{1}{s^\alpha}+\frac{1}{s^{2\alpha}}\right]\right) \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(1+\frac{t^\alpha}{\Gamma(1+\alpha)}\right). \end{aligned} \quad (44)$$

The second approximation reads as follows:

$$\begin{aligned} &Ł_\alpha\{u_2(x,t)\} \\ &= \frac{1}{s^\alpha}u_1(x,0) + \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}Ł_\alpha\{u_1(x,t)\}}{\partial x^{2\alpha}} \\ &= \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \\ &\quad + \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}Ł_\alpha\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(1+\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{1}{s^\alpha}+\frac{1}{s^{2\alpha}}+\frac{1}{s^{3\alpha}}\right). \end{aligned} \quad (45)$$

Therefore, we get

$$\begin{aligned} u_2(x,t) &= Ł_\alpha^{-1}\left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left[\frac{1}{s^\alpha}+\frac{1}{s^{2\alpha}}+\frac{1}{s^{3\alpha}}\right]\right) \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(1+\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\cdots \end{aligned} \quad (46)$$

Consequently, the local fractional series solution is

$$\begin{aligned} u(x,t) &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(Ł_\alpha\{u_n(x,t)\}\right) \\ &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{k=0}^{n}\frac{1}{s^{(n+1)\alpha}}\right) \\ &= \lim_{n\to\infty}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{k=0}^{n}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{k=0}^{\infty}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)} = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha(t^\alpha). \end{aligned} \quad (47)$$

The result is the same as the one which is obtained by the local fractional series expansion method [38].

Example 3. Let us consider the following wave equation on Cantor set:

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}} = 0, \quad 0<\alpha\le 1, \quad (48)$$

with the initial value conditions being as follows:

$$u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \qquad \frac{\partial^\alpha u(x,0)}{\partial t^\alpha} = 0. \quad (49)$$

Using relation (26), we structure the iterative relation as

$$\begin{aligned}
&Ł_\alpha \{u_{n+1}(x,t)\} \\
&= Ł_\alpha \{u_n(x,t)\} \\
&\quad - Ł_\alpha \left\{\frac{t^\alpha}{\Gamma(1+\alpha)}\right\} Ł_\alpha \left\{\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}\right\} \\
&= Ł_\alpha \{u_n(x,t)\} \\
&\quad - \frac{1}{s^{2\alpha}} \left(s^{2\alpha} Ł_\alpha \{u_n(x,t)\} - s^\alpha u_n(x,0) - u_n^{(\alpha)}(x,0) - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}} \right) \\
&= \frac{1}{s^\alpha} u_n(x,0) + \frac{1}{s^{2\alpha}} u_n^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}}.
\end{aligned} \tag{50}$$

In view of (28), the initial value reads as follows:

$$u_0(x,t) = u(x,0) + \frac{t^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{51}$$

Hence, we get the first approximation; namely,

$$\begin{aligned}
Ł_\alpha \{u_1(x,t)\} &= \frac{1}{s^\alpha} u_0(x,0) + \frac{1}{s^{2\alpha}} u_0^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_0(x,t)\}}{\partial x^{2\alpha}} \\
&= \frac{1}{s^\alpha} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right\} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{1}{s^\alpha} + \frac{1}{s^{3\alpha}}\right).
\end{aligned} \tag{52}$$

Thus,

$$\begin{aligned}
u_1(x,t) &= Ł_\alpha^{-1} \left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left[\frac{1}{s^\alpha} + \frac{1}{s^{3\alpha}}\right] \right) \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(1 + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right).
\end{aligned} \tag{53}$$

The second approximation reads as follows:

$$\begin{aligned}
&Ł_\alpha \{u_2(x,t)\} \\
&= \frac{1}{s^\alpha} u_1(x,0) + \frac{1}{s^{2\alpha}} u_1^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_1(x,t)\}}{\partial x^{2\alpha}} \\
&= \frac{1}{s^\alpha} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(1 + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\right\} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{1}{s^\alpha} + \frac{1}{s^{3\alpha}} + \frac{1}{s^{5\alpha}}\right).
\end{aligned} \tag{54}$$

Therefore, we get

$$\begin{aligned}
u_2(x,t) &= Ł_\alpha^{-1} \left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left[\frac{1}{s^\alpha} + \frac{1}{s^{3\alpha}} + \frac{1}{s^{5\alpha}}\right] \right) \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(1 + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{4\alpha}}{\Gamma(1+4\alpha)}\right) \cdots .
\end{aligned} \tag{55}$$

Consequently, the local fractional series solution is

$$\begin{aligned}
u(x,t) &= \lim_{n\to\infty} Ł_\alpha^{-1} \left(Ł_\alpha \{u_n(x,t)\}\right) \\
&= \lim_{n\to\infty} Ł_\alpha^{-1} \left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sum_{k=0}^{n} \frac{1}{s^{(2n+1)\alpha}} \right) \\
&= \lim_{n\to\infty} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sum_{k=0}^{n} \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sum_{k=0}^{\infty} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)} = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \cosh_\alpha(t^\alpha).
\end{aligned} \tag{56}$$

The result is the same as the one which is obtained by the local fractional Adomian decomposition method and local fractional variational iteration method in [34].

Example 4. Let us consider the following wave equation on Cantor set:

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \quad 0 < \alpha \le 1, \tag{57}$$

with the initial value conditions being as follows:

$$u(x,0) = 0, \quad \frac{\partial^\alpha u(x,0)}{\partial t^\alpha} = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{58}$$

Using relation (26), we structure the iterative relation as

$$\begin{aligned}
&Ł_\alpha \{u_{n+1}(x,t)\} \\
&= Ł_\alpha \{u_n(x,t)\} \\
&\quad - Ł_\alpha \left\{\frac{t^\alpha}{\Gamma(1+\alpha)}\right\} Ł_\alpha \left\{\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}\right\} \\
&= Ł_\alpha \{u_n(x,t)\} \\
&\quad - \frac{1}{s^{2\alpha}} \left(s^{2\alpha} Ł_\alpha \{u_n(x,t)\} - s^\alpha u_n(x,0) - u_n^{(\alpha)}(x,0) - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}}\right) \\
&= \frac{1}{s^\alpha} u_n(x,0) + \frac{1}{s^{2\alpha}} u_n^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_n(x,t)\}}{\partial x^{2\alpha}}.
\end{aligned} \tag{59}$$

In view of (28), the initial value reads as follows:

$$\begin{aligned}
&u_0(x,t) \\
&= u(x,0) + \frac{t^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{t^\alpha}{\Gamma(1+\alpha)}.
\end{aligned} \tag{60}$$

Hence, we get the first approximation; namely,

$$\begin{aligned}
Ł_\alpha \{u_1(x,t)\} &= \frac{1}{s^\alpha} u_0(x,0) + \frac{1}{s^{2\alpha}} u_0^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_0(x,t)\}}{\partial x^{2\alpha}} \\
&= \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right\} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}}\right).
\end{aligned} \tag{61}$$

Thus,

$$\begin{aligned}
u_1(x,t) &= Ł_\alpha^{-1} \left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left[\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}}\right]\right) \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right).
\end{aligned} \tag{62}$$

The second approximation reads as follows:

$$\begin{aligned}
&Ł_\alpha \{u_2(x,t)\} \\
&= \frac{1}{s^\alpha} u_1(x,0) + \frac{1}{s^{2\alpha}} u_1^{(\alpha)}(x,0) \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} Ł_\alpha \{u_1(x,t)\}}{\partial x^{2\alpha}} \\
&= \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \\
&\quad + \frac{1}{s^{2\alpha}} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} Ł_\alpha \left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \cdot \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right)\right\} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} + \frac{1}{s^{6\alpha}}\right).
\end{aligned} \tag{63}$$

Therefore, we get

$$\begin{aligned}
&u_2(x,t) \\
&= Ł_\alpha^{-1} \left(\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left[\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} + \frac{1}{s^{6\alpha}}\right]\right) \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)}\right) \cdots .
\end{aligned} \tag{64}$$

Consequently, the local fractional series solution is

$$\begin{aligned}
u(x,t) &= \lim_{n\to\infty} Ł_\alpha^{-1} \left(Ł_\alpha \{u_n(x,t)\}\right) \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sum_{k=0}^{\infty} \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1))} \\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sinh_\alpha(t^\alpha).
\end{aligned} \tag{65}$$

5. Conclusions

The local fractional Laplace variational iteration method was applied to the diffusion and wave equations defined on Cantor sets with the fractal conditions. The local fractional Laplace variational iteration method was proved to be effective and very reliable for analytic purposes. Further, the same problems are solved by local fractional expansion series method (LFESM), local fractional variational iteration method (LFVIM), and local fractional Adomian decomposition method (LFADM). The results obtained by the four methods are in agreement and, hence, this technique may be considered an alternative and efficient method for finding approximate solutions of both linear and nonlinear fractional differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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