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Parallel propagation of dispersive optical solitons by extended trial equation method

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ABSTRACT

This paper applied extended trial equation algorithm to obtain dispersive optical solitons in DWDM system that is governed by the Schrödinger–Hirota equation with Kerr law nonlinearity. Bright and singular soliton solutions are reported.

1. Introduction

Dispersive optical solitons are naturally present in optical fibers whenever there is a dominance of third order and fourth order dispersion (3OD & 4OD) in addition to nonlinear dispersion. This leads to a different model to study soliton propagation through optical fibers. It is known as Schrödinger–Hirota equation (SHE) that is not the usual nonlinear Schrödinger's equation (NLSE). SHE is derivable from NLSE by Lie transform and subsequently ignoring the higher order terms [1–5]. Finally, in order to carry out parallel transmission of soliton molecules through optical fibers, SHE is generalized for DWDM technology with Kerr law nonlinearity.

This paper will study these dispersive solitons with DWDM technology for SHE. There are several integration schemes that are applied nowadays to obtain soliton solutions to such models and other nonlinear evolution equations [1–25]. This paper will implement one such integration algorithm to retrieve soliton solutions to DWDM system with SHE that is studied with cubic nonlinearity. This is the extended trial equation method. After a quick outline of the derivation of SHE from NLSE, the integration algorithm will be revisited and subsequently applied to SHE.

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2. Governing model

The propagation of solitons through optical fibers with 3OD is modeled by NLSE [1–5]:

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u = -i\lambda u_{xxx}. \tag{1}$$

In (1), λ is the coefficient of 3OD. The first term on the left hand side is the linear temporal evolution, while the second term accounts for group velocity dispersion (GVD). Also, on left side, the third term is the self-phase modulation (SPM) with Kerr law nonlinearity. Next, the following Lie symmetry is applied [3,13]:

$$q = u - 3i\lambda\left[u_x + 2u\int_{-\infty}^{x}|u(\xi)|^2 d\xi\right], \tag{2}$$

which transforms (1) to

$$iq_t + \frac{1}{2}q_{xx} + |q|^2 q + i\lambda\left(q_{xxx} + 6|q|^2 q_x\right) = 0, \tag{3}$$

after neglecting higher order terms [1–5,18–20,23]. Eq. (3) is SHE with Kerr law nonlinearity.

Thus, SHE models transmission of dispersive optical solitons through nonlinear fibers. With arbitrary coefficients, SHE can be rewritten as

$$iq_t + aq_{xx} + c|q|^2 q + i\left(\gamma q_{xxx} + \sigma|q|^2 q_x\right) = 0. \tag{4}$$

Here, σ represents nonlinear dispersion. Now, it was pointed out during 2012, that GVD alone makes the governing model ill-posed [15,17]. Therefore, it was suggested that inclusion of spatio-temporal dispersion (STD) introduces well-posedness [15,17]. Hence, SHE with STD is

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i\left(\gamma q_{xxx} + \sigma|q|^2 q_x\right) = 0, \tag{5}$$

where the coefficient of b represents STD. Finally, in presence of perturbation terms, SHE with STD extends to

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i\left(\gamma q_{xxx} + \sigma|q|^2 q_x\right) = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q. \tag{6}$$

In (6), α is the coefficient of intermodal dispersion, λ is the self-steepening term to circumvent shock wave formulation and finally ν is the other form of nonlinear dispersion. This equation of perturbed SHE has been studied earlier in several occasions [18–20,23]. Therefore this paper will simply generalize (5) to DWDM systems in the following section that will be addressed in details by the aid of extended trial equation method.

For DWDM system, Eq. (6), without self-steepening and nonlinear dispersion, generalizes to [23]

$$iq_t^{(l)} + i\alpha_l q_x^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + i\gamma_l q_{xxx}^{(l)} + \left\{c_l\left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N} d_{ln}\left|q^{(n)}\right|^2\right\}q^{(l)} + i\left\{\xi_l\left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N}\eta_{ln}\left|q^{(n)}\right|^2\right\}q_x^{(l)} = 0, \tag{7}$$

where $1 \le l \le N$. The first term in (7) on left hand side is the linear temporal evolution term, while a_l represents the coefficient of GVD and the coefficient of b_l is the STD. Then, the coefficient of γ_l is the 3OD. Also, c_l is the SPM while d_{ln} gives cross-phase modulation. Finally, ξ_l and η_{ln} are from nonlinear dispersions.

In order to solve (7) for solitons, the following phase-amplitude form of decomposition for the wave profile $q^{(l)}(x, t)$ is carried out.

$$q^{(l)}(x, t) = P^{(l)}(s)e^{i\phi_l(x,t)}, \tag{8}$$

where

$$s = x - vt, \tag{9}$$

and the phase component ϕ_l is given by

$$\phi_l(x, t) = -\kappa_l x + \omega_l t + \theta_l, \tag{10}$$

where $1 \le l \le N$. In (8) and (9), $P^{(l)}(x, t)$ represents the amplitude portion of the soliton and v is the speed of the wave. From (10), κ_l is the frequency of the soliton, ω_l is the wave number of the soliton and finally θ_l is the phase constant. Substituting (8) into (7) and decomposing into real and imaginary parts lead to

$$\begin{aligned}&(a_l + 3\gamma_l\kappa_l - b_l v)\left(P^{(l)}\right)'' + \left\{\alpha_l\kappa_l + \omega_l(b_l\kappa_l - 1) - a_l\kappa_l^2 - \gamma_l\kappa_l^3\right\}P^{(l)} + \left(c_l + \kappa_l\xi_l\right)\left(P^{(l)}\right)^3\\ &+\left\{\sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l)\left(P^{(n)}\right)^2\right\}P^{(l)} = 0,\end{aligned} \tag{11}$$

and

$$\left\{\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2-v(1-b_l\kappa_l)\right\}\left(P^{(l)}\right)'+\gamma_l\left(P^{(l)}\right)'''+\left(P^{(l)}\right)'\left\{\xi_l\left(P^{(l)}\right)^2+\sum_{n\neq l}^{N}\eta_{ln}\left(P^{(n)}\right)^2\right\}=0. \tag{12}$$

Using the balancing principle gives

$$P^{(l)}=P^{(n)}. \tag{13}$$

Consequently, Eqs. (11) into (12) reduce to

$$(a_l+3\gamma_l\kappa_l-b_l v)\left(P^{(l)}\right)''+\left\{\alpha_l\kappa_l+\omega_l(b_l\kappa_l-1)-a_l\kappa_l^2-\gamma_l\kappa_l^3\right\}P^{(l)}+\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^{N}(d_{ln}+\eta_{ln}\kappa_l)\right\}\left(P^{(l)}\right)^3=0, \tag{14}$$

and

$$\left\{\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l^2-v(1-b_l\kappa_l)\right\}\left(P^{(l)}\right)'+\gamma_l\left(P^{(l)}\right)'''+\left(P^{(l)}\right)^2\left(P^{(l)}\right)'\left\{\xi_l+\sum_{n\neq l}^{N}\eta_{ln}\right\}=0. \tag{15}$$

From (15), it shows that third order dispersion must vanish, namely

$$\gamma_l=0, \tag{16}$$

for $1\le l\le N$, so that Eqs. (14) and (15) becomes

$$(a_l-b_l v)\left(P^{(l)}\right)''+\left\{\alpha_l\kappa_l+\omega_l(b_l\kappa_l-1)-a_l\kappa_l^2\right\}P^{(l)}+\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^{N}(d_{ln}+\eta_{ln}\kappa_l)\right\}\left(P^{(l)}\right)^3=0, \tag{17}$$

and

$$\left\{\alpha_l+b_l\omega_l+2a_l\kappa_l-v(1-b_l\kappa_l)\right\}\left(P^{(l)}\right)'+\left(P^{(l)}\right)^2\left(P^{(l)}\right)'\left\{\xi_l+\sum_{n\neq l}^{N}\eta_{ln}\right\}=0. \tag{18}$$

Eq. (16) shows that dispersive optical solitons in DWDM systems will exist provided the third order dispersion coefficient is zero. Next, setting the coefficients of the linearly independent functions, in (15), to zero is possible to retrieve the speed of the soliton

$$v=\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}, \tag{19}$$

as long as the constraints

$$b_l\kappa_l\neq 1, \tag{20}$$

$$\xi_l+\sum_{n\neq l}^{N}\eta_{ln}=0, \tag{21}$$

remain valid. The extended trial scheme approach will now be applied, in the subsequent subsection, to Eq. (17) to retrieve bright, dark and singular soliton solutions to DWDM system given by (7).

3. Extended trial equation method

To start off with extended trial scheme approach, the following initial assumption for the solution structure of (17) is made:

$$P^{(l)}=\sum_{i=0}^{\varsigma}\tau_i^{(l)}\Psi^i, \tag{22}$$

where

$$(\Psi')^2=\Lambda(\Psi)=\frac{\Phi(\Psi)}{\Upsilon(\Psi)}=\frac{\mu_\sigma\Psi^\sigma+\cdots+\mu_1\Psi+\mu_0}{\chi_\rho\Psi^\rho+\cdots+\chi_1\Psi+\chi_0}. \tag{23}$$

Here $\tau_0^{(l)}, \ldots, \tau_\varsigma^{(l)}; \mu_0, \ldots, \mu_\sigma$ and $\chi_0, \ldots, \chi_\rho$ are constants to be determined later. Using the relations (22) and (23), we derive the terms $((P^{(l)})')^2$ and $(P^{(l)})''$ as below:

$$((P^{(l)})')^2 = \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i^{(l)}\Psi^{i-1}\right)^2, \tag{24}$$

and

$$(P^{(l)})'' = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i^{(l)}\Psi^{i-1}\right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i(i-1)\tau_i^{(l)}\Psi^{i-2}\right), \tag{25}$$

where $\Phi(\Psi)$ and $\Upsilon(\Psi)$ are polynomials of Ψ. We reduce Eq. (23) to the elementary integral form as follows:

$$\pm(s - s_0) = \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}} = \int \sqrt{\frac{\Upsilon(\Psi)}{\Phi(\Psi)}}\, d\Psi. \tag{26}$$

According to the balance principle, we determine a relation of σ, ρ and ς is given by

$$\sigma = \rho + 2\varsigma + 2. \tag{27}$$

When $\sigma = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (27), we then assume that Eq. (17) have the following formal solutions:

$$P^{(l)} = \tau_0^{(l)} + \tau_1^{(l)}\Psi, \tag{28}$$

where $\tau_0^{(l)}$ and $\tau_1^{(l)}$ are constants to be determined later such that $\tau_1^{(l)} \neq 0$, and Ψ satisfies Eq. (23). Substituting these formal solutions and their necessary derivatives into Eq. (17), and solving the resulting system of algebraic equations one recovers the following solution set:

$$\begin{aligned}
&\mu_0 = \mu_0,\ \mu_2 = \mu_2,\ \chi_0 = \chi_0,\ \tau_0^{(l)} = \tau_0^{(l)},\ \tau_1^{(l)} = \tau_1^{(l)},\\
&\mu_1 = \frac{2\tau_0^{(l)}\left(\mu_2(a_l - b_l v) + 2\chi_0\left(\tau_0^{(l)}\right)^2\left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{\ln} + \eta_{\ln}\kappa_l)\right\}\right)}{\tau_1^{(l)}(a_l - b_l v)},\\
&\mu_3 = -\frac{2\chi_0\tau_0^{(l)}\tau_1^{(l)}\left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{\ln} + \eta_{\ln}\kappa_l)\right\}}{a_l - b_l v},\\
&\mu_4 = -\frac{\chi_0\left(\tau_1^{(l)}\right)^2\left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{\ln} + \eta_{\ln}\kappa_l)\right\}}{2(a_l - b_l v)},\\
&\omega_l = \frac{b_l v\mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2\left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{\ln} + \eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l - 1)}.
\end{aligned} \tag{29}$$

Substituting the solution set (29) into Eqs. (23) and (26), we recover that

$$\pm(s - s_0) = Q\int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}}, \tag{30}$$

where

$$\Lambda(\Psi) = \Psi^4 + \frac{\mu_3}{\mu_4}\Psi^3 + \frac{\mu_2}{\mu_4}\Psi^2 + \frac{\mu_1}{\mu_4}\Psi + \frac{\mu_0}{\mu_4},\ Q = \sqrt{\frac{\chi_0}{\mu_4}}. \tag{31}$$

As a result, we obtain the traveling wave solutions to Eq. (7) as follows:

When $\Lambda(\Psi) = (\Psi - \lambda_1)^4$, rational function solutions are:

$$\begin{aligned}
q^{(l)}(x,t) = &\left\{\tau_0^{(l)} + \tau_1^{(l)}\lambda_1 \pm \frac{\tau_1^{(l)}Q}{x - \left\{\frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l}\right\}t - s_0}\right\}\\
&\times \exp\left[i\left\{-\kappa_l x + \left(\frac{b_l v\mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2\left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{\ln} + \eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l - 1)}\right)t + \theta_l\right\}\right].
\end{aligned} \tag{32}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^3(\Psi-\lambda_2)$ and $\lambda_2>\lambda_1$, plane wave solutions are:

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1+\frac{4\tau_1^{(l)}Q^2(\lambda_2-\lambda_1)}{4Q^2-\left[(\lambda_1-\lambda_2)\left(x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t-s_0\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa_lx+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^N(d_{\ln}+\eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l-1)}\right)t+\theta_l\right\}\right]. \tag{33}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)^2$, hyperbolic function solutions are:

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_2+\frac{\tau_1^{(l)}(\lambda_2-\lambda_1)}{\exp\left[\frac{\lambda_1-\lambda_2}{Q}\left(x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t-s_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_lx+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^N(d_{\ln}+\eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l-1)}\right)t+\theta_l\right\}\right], \tag{34}$$

and

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)}{\exp\left[\frac{\lambda_1-\lambda_2}{Q}\left(x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t-s_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa_lx+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^N(d_{\ln}+\eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l-1)}\right)t+\theta_l\right\}\right]. \tag{35}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)(\Psi-\lambda_3)$ and $\lambda_1>\lambda_2>\lambda_3$, bright soliton solutions are:

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_1-\frac{2\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{2\lambda_1-\lambda_2-\lambda_3+(\lambda_3-\lambda_2)\cosh\left[\frac{\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{Q}\left[x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t\right]\right]}\right\}\times\exp\left[i\left\{-\kappa_lx+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^N(d_{\ln}+\eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l-1)}\right)t+\theta_l\right\}\right]. \tag{36}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)(\Psi-\lambda_3)(\Psi-\lambda_4)$ and $\lambda_1>\lambda_2>\lambda_3>\lambda_4$, Jacobi elliptic function solutions are:

$$q^{(l)}(x,t)=\left\{\tau_0^{(l)}+\tau_1^{(l)}\lambda_2+\frac{\tau_1^{(l)}(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2Q}\left[x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t-s_0\right],m\right]}\right\}\times\exp\left[i\left\{-\kappa_lx+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^N(d_{\ln}+\eta_{\ln}\kappa_l)\right\}\right)}{\chi_0(b_l\kappa_l-1)}\right)t+\theta_l\right\}\right], \tag{37}$$

where

$$m^2=\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}. \tag{38}$$

Note that λ_j for $j=1,\ldots,4$ are the roots of the following equation:

$$\Lambda(\Psi)=0. \tag{39}$$

When $\tau_0^{(l)} = -\tau_1^{(l)}\lambda_1$ and $s_0 = 0$, solutions (32)–(36) are reduced to plane wave solutions

$$q^{(l)}(x,t) = \left\{ \pm \frac{\tau_1^{(l)} Q}{x - \left\{ \frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l} \right\} t} \right\} \times \exp\left[i \left\{ -\kappa_l x + \left(\frac{b_l \nu \mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2 \left\{ c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l) \right\}\right)}{\chi_0(b_l\kappa_l - 1)} \right) t + \theta_l \right\} \right], \tag{40}$$

$$q^{(l)}(x,t) = \left\{ \frac{4\tau_1^{(l)} Q^2 (\lambda_2 - \lambda_1)}{4Q^2 - \left[(\lambda_1 - \lambda_2)\left(x - \left\{ \frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l} \right\} t \right) \right]^2} \right\} \times \exp\left[i \left\{ -\kappa_l x + \left(\frac{b_l \nu \mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2 \left\{ c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l) \right\}\right)}{\chi_0(b_l\kappa_l - 1)} \right) t + \theta_l \right\} \right], \tag{41}$$

singular soliton solutions

$$q^{(l)}(x,t) = \frac{\tau_1^{(l)}(\lambda_2 - \lambda_1)}{2} \left\{ 1 \mp \coth\left[\frac{\lambda_1 - \lambda_2}{2Q} \left(x - \left\{ \frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l} \right\} t \right) \right] \right\} \times \exp\left[i \left\{ -\kappa_l x + \left(\frac{b_l \nu \mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2 \left\{ c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l) \right\}\right)}{\chi_0(b_l\kappa_l - 1)} \right) t + \theta_l \right\} \right], \tag{42}$$

and bright solitons

$$q^{(l)}(x,t) = \left\{ \frac{R}{T + \cosh\left[S\left(x - \left\{ \frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l} \right\} t \right) \right]} \right\} \times \exp\left[i \left\{ -\kappa_l x + \left(\frac{b_l \nu \mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2 \left\{ c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l) \right\}\right)}{\chi_0(b_l\kappa_l - 1)} \right) t + \theta_l \right\} \right], \tag{43}$$

where

$$R = \frac{2\tau_1^{(l)}(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}{\lambda_3 - \lambda_2},\ S = \frac{\sqrt{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}}{Q},\ T = \frac{2\lambda_1 - \lambda_2 - \lambda_3}{\lambda_3 - \lambda_2}. \tag{44}$$

We note that R is the amplitude of the soliton, while S is the inverse width of the soliton. So, the solitons are valid for $\tau_1^{(l)} < 0$. Furthermore, when $\tau_0^{(l)} = -\tau_1^{(l)}\lambda_2$ and $s_0 = 0$, the solutions (37) are simplified below:

$$q^{(l)}(x,t) = \left\{ \frac{R_1}{T_1 + \mathrm{sn}^2\left[S_j \left[x - \left\{ \frac{\alpha_l + b_l\omega_l + 2a_l\kappa_l}{1 - b_l\kappa_l} \right\} t \right], \frac{(\lambda_2 - \lambda_3)(\lambda_1 - \lambda_4)}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)} \right]} \right\} \times \exp\left[i \left\{ -\kappa_l x + \left(\frac{b_l \nu \mu_2 + a_l\left(\chi_0\kappa_l^2 - \mu_2\right) - \chi_0\left(\alpha_l\kappa_l + 3\left(\tau_0^{(l)}\right)^2 \left\{ c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l) \right\}\right)}{\chi_0(b_l\kappa_l - 1)} \right) t + \theta_l \right\} \right], \tag{45}$$

where

$$R_1 = \frac{\tau_1^{(l)}(\lambda_1 - \lambda_2)(\lambda_4 - \lambda_2)}{\lambda_1 - \lambda_4},\ T_1 = \frac{\lambda_4 - \lambda_2}{\lambda_1 - \lambda_4},\ S_j = \frac{(-1)^j\sqrt{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)}}{2Q}, \quad (j = 1, 2). \tag{46}$$

Remark 1. When the modulus $m \to 1$, singular optical soliton solutions emerge:

$$q^{(l)}(x,t)=\left\{\frac{R_1}{T_1+\tanh^2\left[S_j\left(x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_l x+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^{N}(d_{\text{ln}}+\eta_{\text{ln}}\kappa_l)\right\}\right)}{\chi_0\left(b_l\kappa_l-1\right)}\right)t+\theta_l\right\}\right], \tag{47}$$

where $\lambda_3=\lambda_4$.

Remark 2. However, if $m\to 0$, periodic wave solutions are obtained as

$$q^{(l)}(x,t)=\left\{\frac{R_1}{T_1+\sin^2\left[S_j\left(x-\left\{\frac{\alpha_l+b_l\omega_l+2a_l\kappa_l}{1-b_l\kappa_l}\right\}t\right)\right]}\right\}\times\exp\left[i\left\{-\kappa_l x+\left(\frac{b_l\nu\mu_2+a_l\left(\chi_0\kappa_l^2-\mu_2\right)-\chi_0\left(\alpha_l\kappa_l+3\left(\tau_0^{(l)}\right)^2\left\{c_l+\kappa_l\xi_l+\sum_{n\neq l}^{N}(d_{\text{ln}}+\eta_{\text{ln}}\kappa_l)\right\}\right)}{\chi_0\left(b_l\kappa_l-1\right)}\right)t+\theta_l\right\}\right], \tag{48}$$

where $\lambda_2=\lambda_3$.

4. Conclusions

This paper secured bright and singular dispersive optical solitons to DWDM systems that is governed by SHE with Kerr law nonlinearity with STD. The extended trial equation scheme was implemented successfully to retrieve these soliton solutions. The results came up with a number of constraints that must hold for these solutions to exist and these conditions are listed. The dark solitons were not obtainable from this algorithm. This is the drawback of this integration scheme. The results of this paper carry a lot of future prospects. The algorithm can be applied to various other situations such as Bragg gratings, optical couplers and other cases. Those results will be later available. This is just a tip of the spear.

Conflict of interest

The authors also declare that there is no conflict of interest.

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References

[1] A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, W.N. Manrakhan, M. Savescu, A. Biswas, Dispersive optical solitons with Schrödinger–Hirota equation, J. Nonlinear Opt. Phys. Mater. 23 (1) (2014).
[2] A. Biswas, Optical solitons: quasi-stationarity versus Lie transform, Opt. Quant. Electron. 35 (10) (2003) 979–998.
[3] A. Biswas, Stochastic perturbation of optical solitons in Schrödinger–Hirota equation, Opt. Commun. 239 (4–6) (2004) 457–462.
[4] A. Biswas, S. Konar, Introduction to Non-Kerr Law Optical Solitons, CRC Press, Boca Raton, FL, 2006.
[5] A. Biswas, A.J.M. Jawad, W.N. Manrakhan, A.K. Sarma, K.R. Khan, Optical solitons and complexions of the Schrödinger–Hirota equation, Opt. Laser Technol. 44 (7) (2012) 2265–2269.
[6] A. Biswas, K.R. Khan, A. Rahman, A. Yildirim, T. Hayat, O.M. Aldossary, Bright and dark optical solitons in birefringent fibers with Hamiltonian perturbations and Kerr law nonlinearity, J. Optoelectron. Adv. Mater. 14 (7–8) (2012) 571–576.
[7] A. Biswas, M. Mirzazadeh, M. Eslami, Dispersive optical solitons with Schrödinger–Hirota equation by G'/G-expansion approach in power law medium, Optik 125 (16) (2014) 4215–4218.
[8] C.Q. Dai, J.F. Zhang, New solitons for the Hirota equation and generalized higher-order nonlinear Schrödinger equation with variable coefficients, J. Phys. A 30 (4) (2004) 723–737.
[9] R.K. Dowluru, P.R. Bhima, Influences of third order dispersion on linear birefringent optical soliton transmission systems, J. Opt. 40 (3) (2011) 132–142.
[10] M. Ekici, M. Mirzazadeh, M. Eslami, Solitons and other solutions to Boussinesq equation with power law nonlinearity and dual dispersion, Nonlinear Dyn. 84 (2) (2016) 669–676.
[11] M. Eslami, M.A. Mirzazadeh, A. Neirameh, New exact wave solution for Hirota equation, Pramana 84 (1) (2015) 3–8.
[12] X. Geng, Y. Lv, Darboux transformation for an integrable generalization of the nonlinear Schrödinger equation, Nonlinear Dyn. 69 (4) (2012) 1621–1630.
[13] Y. Gurefe, E. Misirli, A. Sonmezoglu, M. Ekici, Extended trial equation method to generalized nonlinear partial differential equations, Appl. Math. Comput. 219 (2013) 5253–5260.

[14] Y. Kodama, M. Romagnoli, M. Midrio, S. Wabnitz, Role of third order dispersion on soliton instabilities and interactions in optical fibers, Opt. Lett. 19 (3) (1994) 165–167.
[15] S. Kumar, K. Singh, R.K. Gupta, Coupled Higgs field equation and Hamiltonian amplitude equation: Lie classical approach and G'/G-expansion method, Pramana 79 (1) (2012) 41–60.
[16] C.S. Liu, Representations and classification of traveling wave solutions to sinh-Gordon equation, Commun. Theor. Phys. 49 (1) (2008) 153–158.
[17] C.S. Liu, Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations, Comput. Phys. Commun. 181 (2) (2010) 317–324.
[18] M. Mirzazadeh, M. Eslami, M. Savescu, A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, A. Biswas, Optical solitons in DWDM system with spatio-temporal dispersion, J. Nonlinear Opt. Phys. Mater. 24 (1) (2015).
[19] M. Mirzazadeh, M. Eslami, Q. Zhou, A. Biswas, M. Belic, Optical solitons in DWDM system by G'/G-expansion scheme, J. Optoelectron. Adv. Mater. 17 (3–4) (2015) 499–504.
[20] M. Savescu, A.A. Alshaery, E.M. Hilal, A.H. Bhrawy, Q. Zhou, A. Biswas, Optical solitons in DWDM system with four-wave mixing, Optoelectron. Adv. Mater. Rapid Commun. 9 (1–2) (2015) 14–19.
[21] J.J. Shu, Exact N-envelope-soliton solutions of the Hirota equation, Opt. Appl. 33 (2–3) (2003) 539–546.
[22] S. Tinggen, Propagation characteristics of dark soliton study in optical fibers with slowly decreasing dispersion, J. Comput. Phys. 13 (1996) 115–118.
[23] M.Z. Ullah, A. Biswas, S.P. Moshokoa, Q. Zhou, M. Mirzazadeh, M. Belic, Dispersive optical solitons in DWDM systems, Optik 132 (2017) 210–215.
[24] P. Wang, B. Tian, W.J. Liu, M. Li, K. Sun, Soliton solutions for a generalized inhomogeneous variable-coefficient Hirota equation with symbolic computation, Stud. Appl. Math. 125 (2) (2010) 213–222.
[25] Q. Zhou, M. Ekici, A. Sonmezoglu, M. Mirzazadeh, M. Eslami, Optical solitons with Biswas–Milovic equation by extended trial equation method, Nonlinear Dyn. 84 (4) (2016) 1883–1900.